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ARISTOTLE UNIVERSITY OF THESSALONIKI

Algorithmic Complexity & the Hard Limits of Artificial Intelligence

(a tale of Chaitin’s Omega, Fractal Basins & Bayesian Inference)

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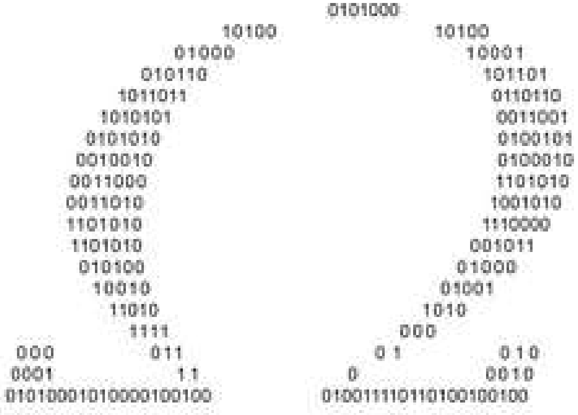
30th SUMMER SCHOOL-CONFERENCE ON DYNAMICAL SYSTEMS AND COMPLEXITY ,
ΑΠΘ University Camping at Kalandra Chalkidiki, 28/8 - 6/9/2024



Science & Technology FORESIGHT from society to research

National Research Council of Italy

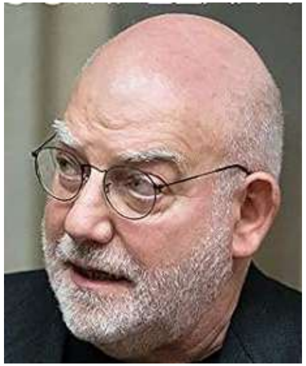
Interdisciplinary Studies

- Complex Systems
- Nonlinear Dynamics
- Quantum Cognition
- Data science (Biomemetics)

Yukio-Pegio Gunji
Waseda University

Pier-Francesco Moretti
CNR Roma HQ

Gregory Chaitin



1931: Gödel (later joined by Turing, Chaitin et al) terminates the search of the "Vienna Circle" (Hilbert, Frege et al) for an absolute, consistent & complete, mechanical formal logic

He formally proves that truth and provability are distinct

Then Gödel raised the "final question":

"Does our physical and biochemical substratum permit a mechanical one-to-one interpretation of all the functions of life and of the mind ?"

Manifestations of the Uncomputable:

Challenge to conventional frameworks:

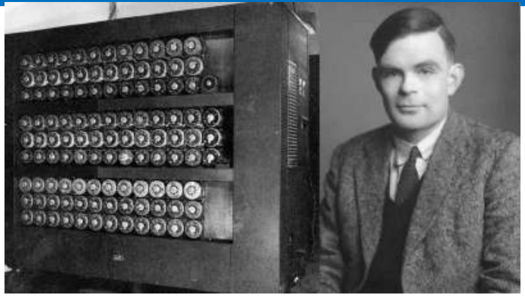
paradoxical situations, anomalies ...

Challenge: broaden the concept of computation

"All Cretans are liars, said Epimenides the Cretan"
"A: This statement (A) is false"

Kurt Gödel destroys Positivism: "There are always Undecidable Propositions in any formal Logic"

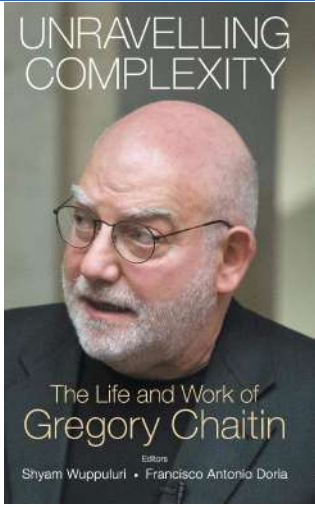
"Proof Does Not Lead to Truth, Truth Leads to Proof"



Alan Turing, The Halting Problem is undecidable: No algorithm can tell whether a computer program & an input, will halt, or run forever.

Every halting probability, *Chaitin's* **Ω,** is a normal and transcendental real number that is non-computable, which means there *cannot be* any algorithm to compute its digits.

'There are definable numbers that are uncomputable'



Manifestations of Uncomputable: Gödel

"A: This statement (A) is false"

Arithmetizing Meta-mathematics:

Gödel *numbering.*

Constant sign	Gödel number	Usual Meaning
~	1	not
∨	2	or
⊃	3	if...then...
∃	4	there is an...
=	5	equals
0	6	zero
S	7	the successor of
(	8	punctuation mark
)	9	punctuation mark
,	10	punctuation mark
+	11	plus
×	12	times

$$0 = 0 \rightarrow 2^6 \times 3^5 \times 5^6$$

$$G(0=0) = 243,000,000.$$

G(2+2=5) = 5344390000

x,y,z … etc, map onto prime numbers > 12 (x→13, y→17, z →19, … etc).

$$\mathrm{enc}(x_1, x_2, x_3, \ldots, x_n) = 2^{x_1} \cdot 3^{x_2} \cdot 5^{x_3} \cdots p_n^{x_n}$$

$$
\begin{array}{lcl}
s_1 & = & 0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,\ldots \\
s_2 & = & 1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,\ldots \\
s_3 & = & 0\,1\,0\,1\,0\,1\,0\,1\,0\,1\,0\,\ldots \\
s_4 & = & 1\,0\,1\,0\,1\,0\,1\,0\,1\,0\,1\,\ldots \\
s_5 & = & 1\,1\,0\,1\,0\,1\,1\,0\,1\,0\,1\,\ldots \\
s_6 & = & 0\,0\,1\,1\,0\,1\,1\,0\,1\,1\,0\,\ldots \\
s_7 & = & 1\,0\,0\,0\,1\,0\,0\,0\,1\,0\,0\,\ldots \\
s_8 & = & 0\,0\,1\,1\,0\,0\,1\,1\,0\,0\,1\,\ldots \\
s_9 & = & 1\,1\,0\,0\,1\,1\,0\,0\,1\,1\,0\,\ldots \\
s_{10} & = & 1\,1\,0\,1\,1\,1\,0\,0\,1\,0\,1\,\ldots \\
s_{11} & = & 1\,1\,0\,1\,0\,1\,0\,0\,1\,0\,0\,\ldots \\
\vdots & & \vdots\,\vdots\,\vdots\,\vdots\,\vdots\,\vdots\,\vdots\,\vdots\,\vdots\,\vdots\,\vdots\,\ddots
\end{array}
$$

$$s = 1\,0\,1\,1\,1\,0\,1\,0\,0\,1\,1\,\ldots$$

The set of Real numbers, $\mathbb{R}$, is uncountable

Cantor's diagonal argument

Provability Function P(*) :

if F if G-valid => P(F) is true

Negation function Not(*):

Not(f) = ~f

Manifestations of Uncomputable: Gödel

"A: This statement (A) is false"
Arithmetizing Meta-mathematics:

["This statement is false", is false]

'False': this statement is not provable in Principia Mathematica..

		Input Value					
#	**Function**	x = 1	x = 2	x = 3	x = 4	...	**x = g**
1	$F_1(x) = x$	1	2	3	4		
2	$F_2(x) = 2x$	2	4	6	8		
3	$F_3(x) = x^2$	1	4	9	16		
4	$F_4(x) = 2x^2$	2	8	18	32		
...	...						
g	$\mathbf{F_g(x) = Not(P(F_x(x)))}$						$\mathbf{F_g(g) = Not(P(F_g(g)))}$

Statement $F_g(g)$ is not provable in Principia Mathematica.
or ...
This statement is not provable in Principia Mathematica.
or ...

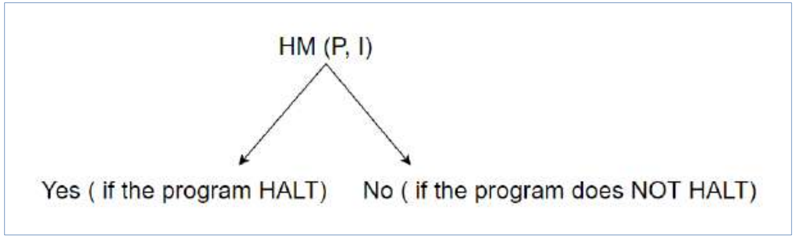
["This statement is false", is false]

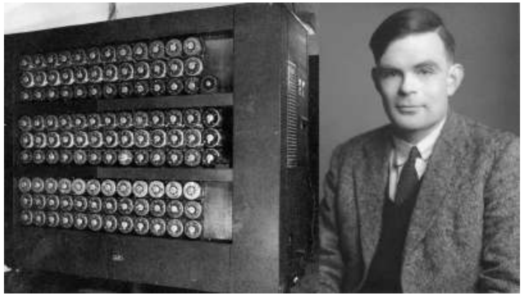
Table from: mybrainsthoughts.com/?p=302

Manifestations of the Uncomputable: Turing:

Proof by contradiction …

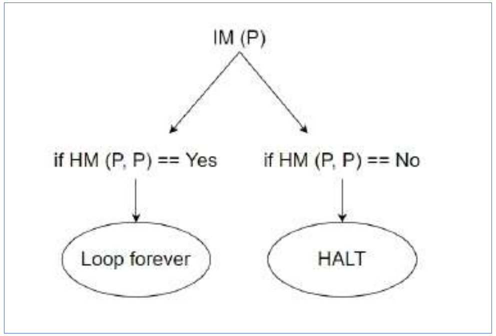
Assume there is a program, HM(P,I) ,a ‘Halting Machine’, that can decide if another program P with input I stops or not, and stops:





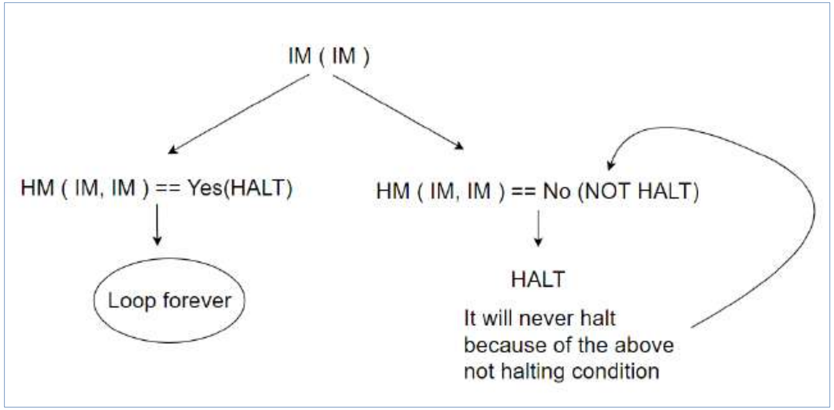
Alan Turing, The Halting Problem is undecidable: No algorithm can tell whether a computer program & an input, will halt, or run forever.

From this make IM the Inverting Machine’



and feedback it to itself
IM(IM)
then HM(IM,IM) *cannot* stop
… contradiction!
Therefore, no such HM exists

...QED



Manifestations of the Uncomputable Chaitin:

Challenge:

broaden the concept of Computation

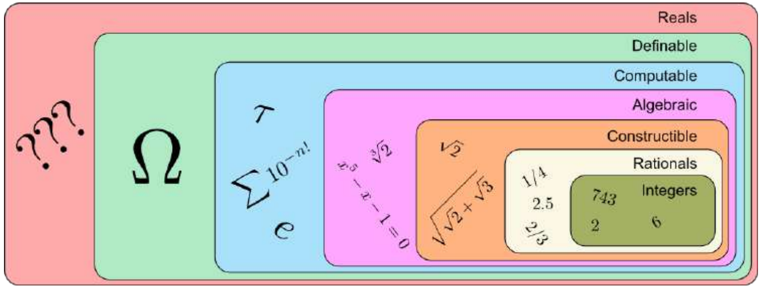
Math isn't the art of answering mathematical questions, it is the art of asking the right questions

The real goal of mathematics is to obtain insight, not just proofs.

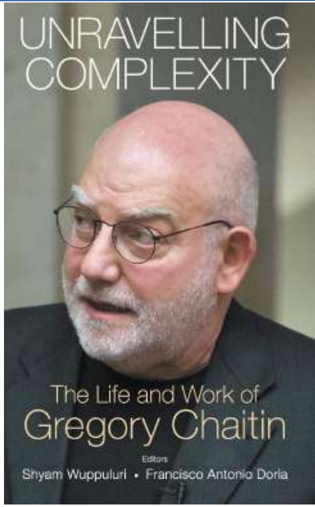
$$\Omega_U \equiv \sum_{p \text{ halts}} 2^{-|p|}$$

Chaitin constant is simultaneously computably enumerable
(the limit of a computable, increasing, converging sequence of rationals),
and algorithmically random
(its binary expansion is an algorithmic random sequence), hence uncomputable.

‘There are definable numbers that are uncomputable’

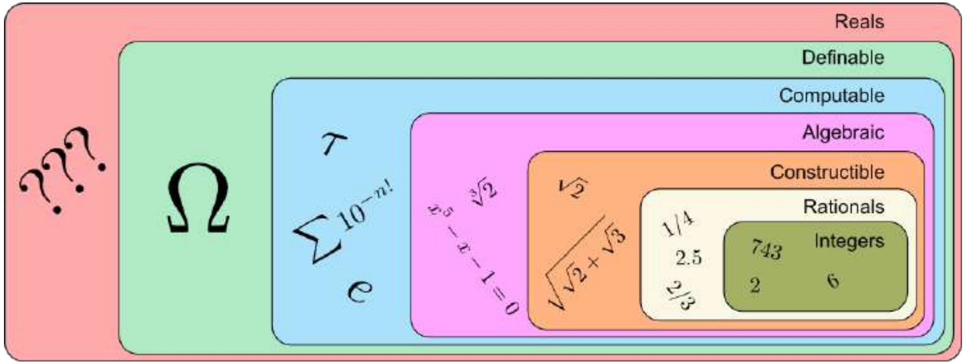


Every halting probability, *Chaitin’s* **Ω,** is a normal, transcendental real number that is non-computable, which means there *cannot be* any algorithm to compute its digits.



Manifestations of the Uncomputable Chaitin:

Chaitin constant algorithmically random
(its binary expansion is an algorithmic random sequence), hence uncomputable.
No algorithm can be constructed to compute it.



$$\pi = \frac{4}{1} - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \frac{4}{11} + \frac{4}{13} - \cdots$$

$$e = \sum_{n=0}^{\infty} \frac{1}{n!} = 1 + \frac{1}{1} + \frac{1}{1 \cdot 2} + \frac{1}{1 \cdot 2 \cdot 3} + \cdots.$$

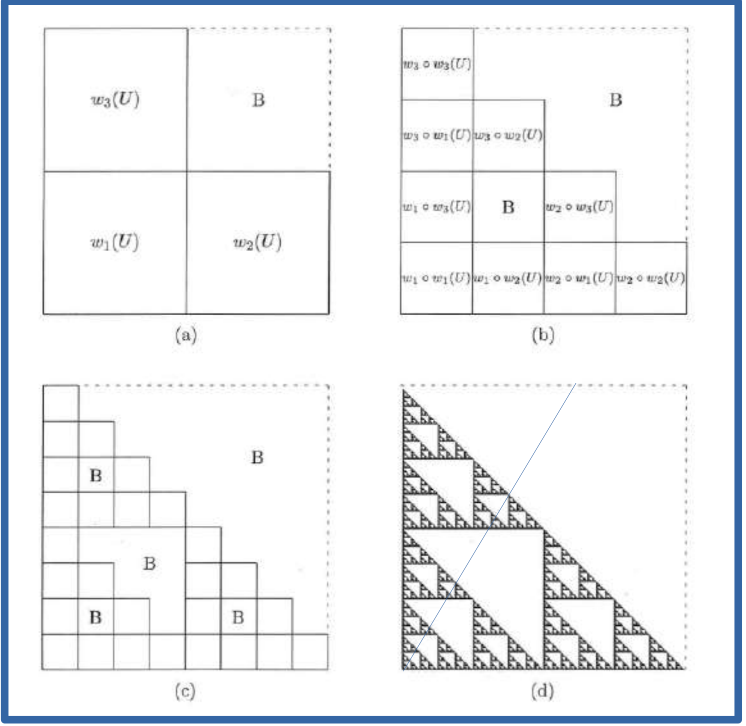
these are computable, many (formulas) algorithms can compress their information.

$$\Omega_U \equiv \sum_{p \text{ halts}} 2^{-|p|}$$

this is algorithmically random: Its first n-bits cannot be compressed in an algorithm shorter than n-bits.

The shortest program to output the first n bits of Ω must be of size at least n − O(1). Where O(1) a prefix depending on the formal scheme, the language of these programs that halt among all those of length at most n.

‘There exist definable numbers that are uncomputable’



Undecidable & Uncomputable Problems in Fractal Geometry: Formally Proven (Dube, 1993)

Whether the attractor of a given IFS intersects with a line segment is undecidable.

Whether a given IFS is totally disconnected is also undecidable.

Strategy: use symbolic dynamics to associate trajectories to symbol sequences (i.e symbolic dynamics, or cellular automata)

Ask the question as a question for symbol sequences comparison

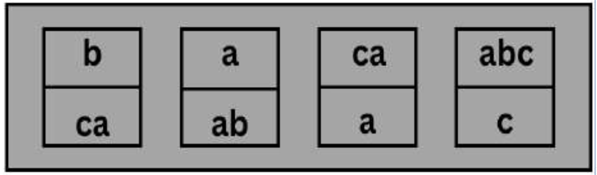
Example: reduce the problem to a tiling or “(Emil) Post Correspondence Problem”, PCP.

IFS $\{[0,1]^2; w_1, w_2, w_3\}$

$$w_1(x,y) = (0.5x, 0.5y)$$
$$w_2(x,y) = (0.5x + 0.5, 0.5y)$$
$$w_3(x,y) = (0.5x, 0.5y + 0.5).$$

$$w_{i_1}(w_{i_2}(\ldots(w_{i_n}(U))\ldots))$$

$$A = \bigcap_{n=1}^{\infty} S_n$$



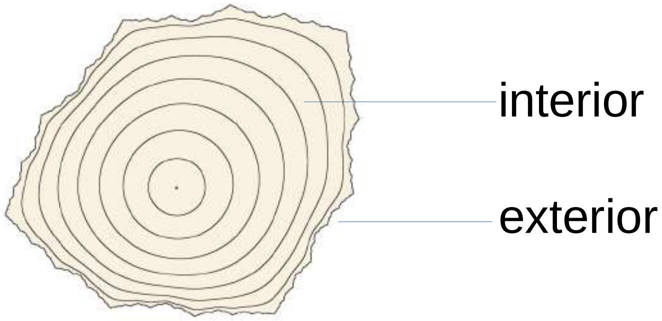
Arrange the dominoes in such a way that the string produced by the denominators & the string produced by the numerators are the same.

more uncomputable things in fractals ...

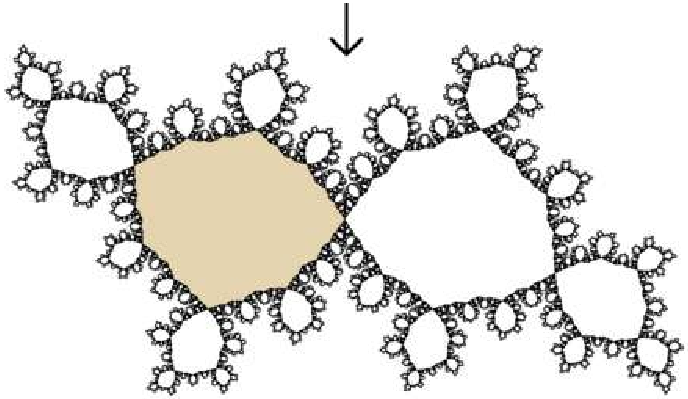
$$z \mapsto \rho z + z^2$$

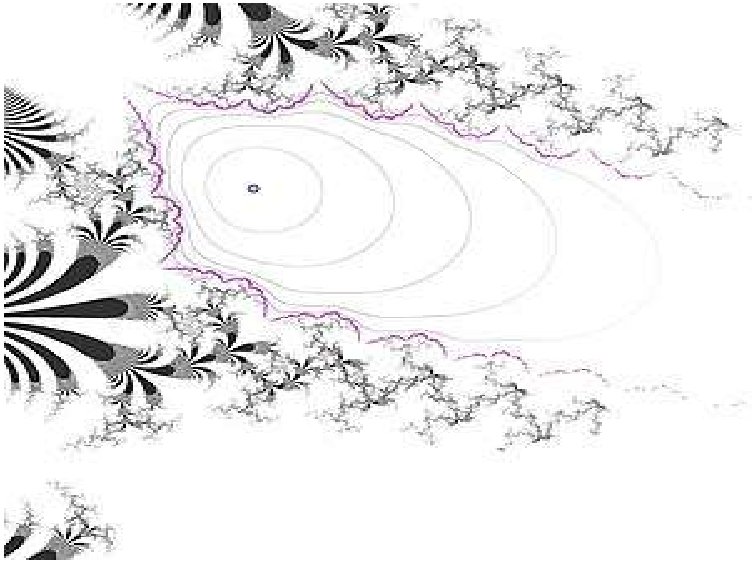
$$\rho = e^{i2\pi\omega}$$

$$\omega = \frac{\sqrt{5}+1}{2}$$



A quadratic golden mean Siegel disk
with its Jordan curve fractal boundary





There exist quadratic polynomials with Siegel disks whose Julia sets are not computable

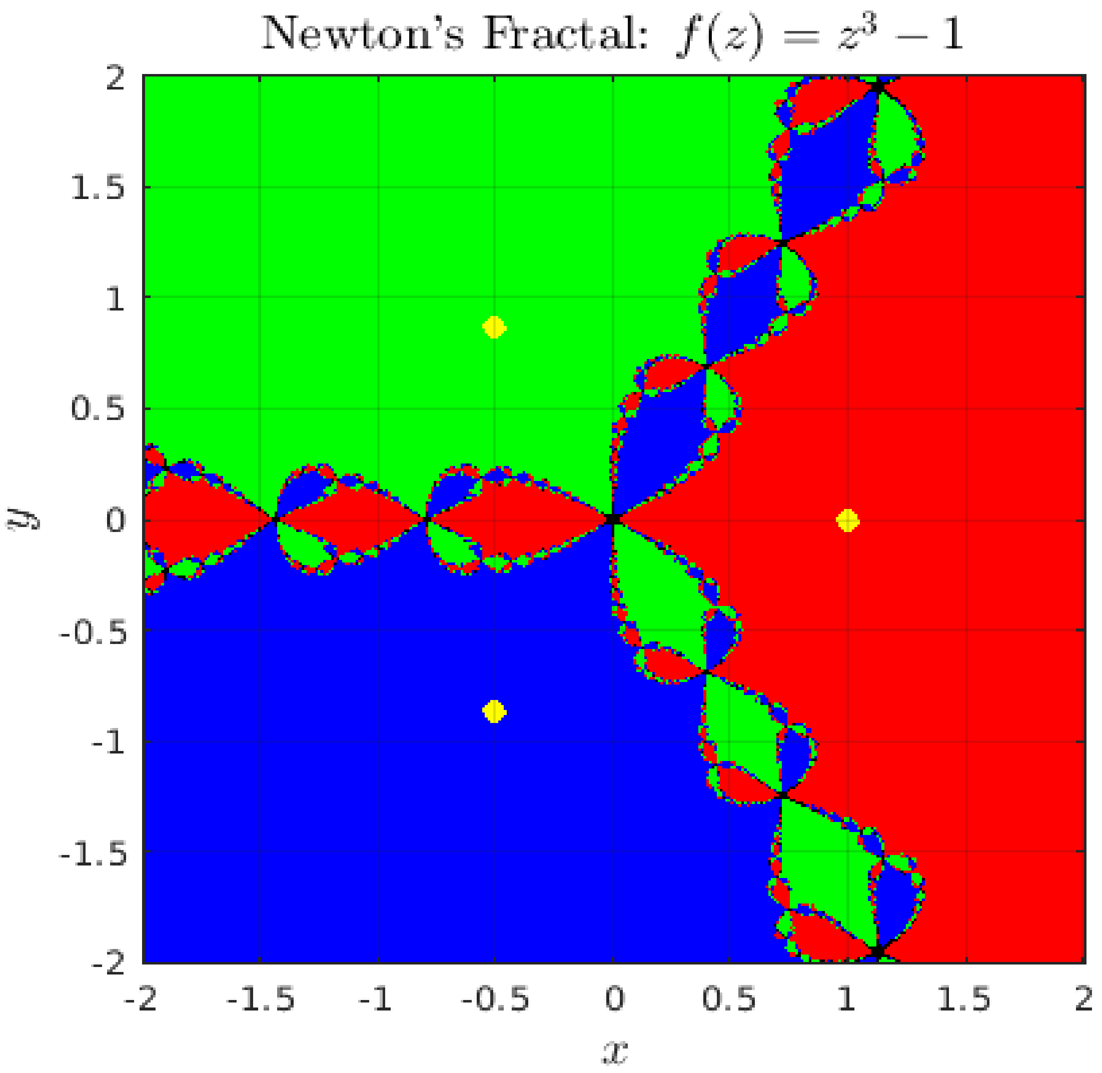
There is no algorithm that can compute arbitrarily good approximations of these Julia sets.

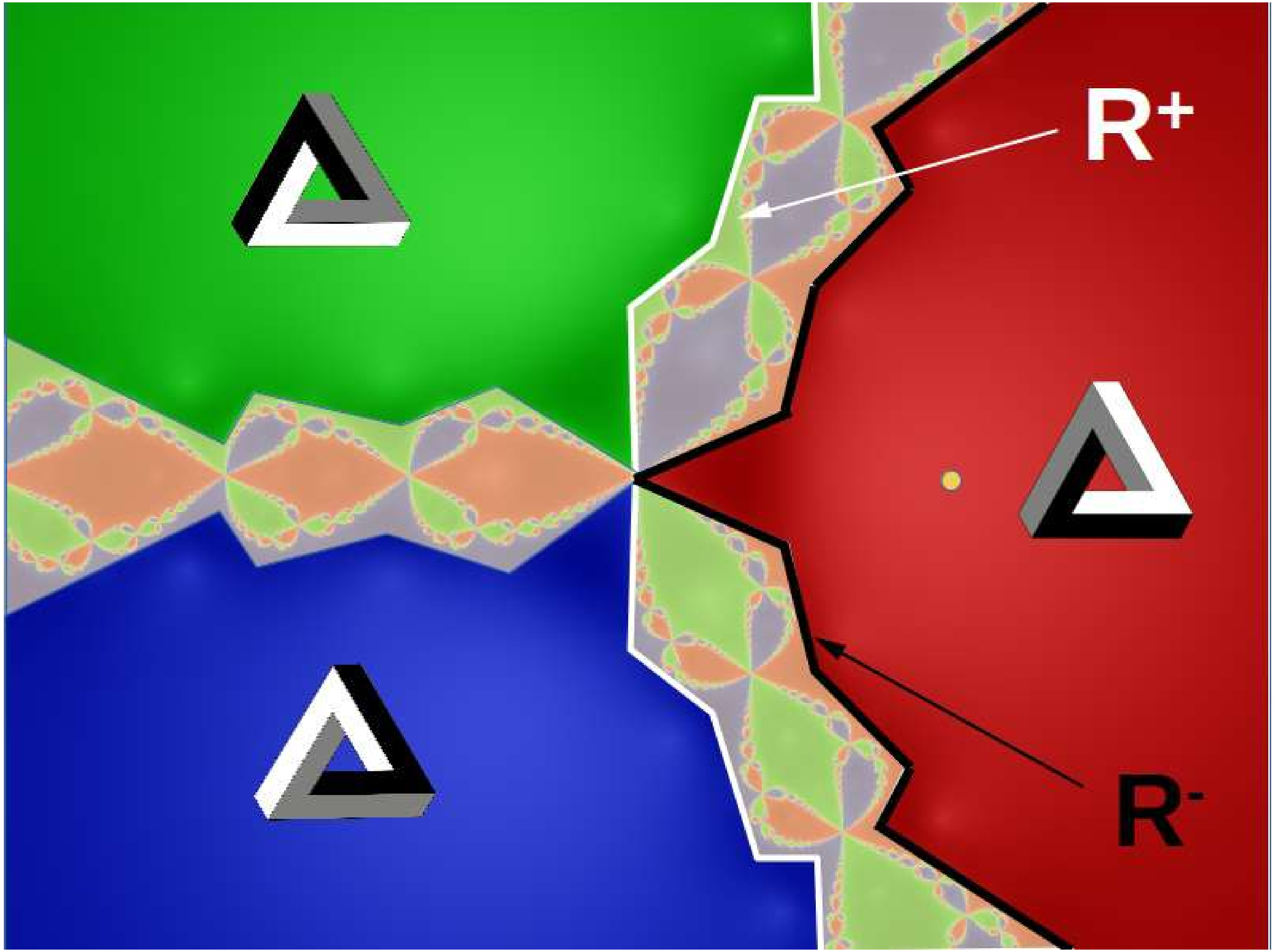
The conformal radius of a quadratic Siegel disk varies continuously with respect to the Hausdorff distance on Julia sets.

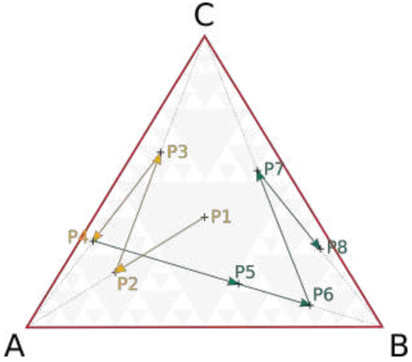
However, small changes in parameters can lead to an "implosion" where the inner radius of the Siegel disk collapses to zero.

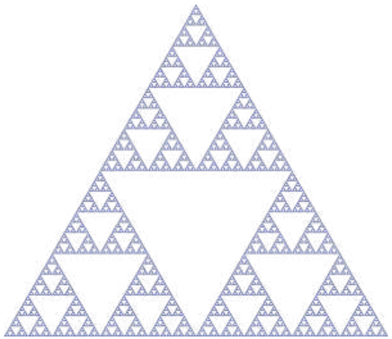
$$f(z) : z_{n+1} := z_n - \frac{f(z_n)}{f'(z_n)}$$

$z \in \mathbb{C}$




R+
R-





Randomness in **IFS**. The **Chaos Game**:

Start tat a random point P1 within the triangle.
Choose one of the three corners of the triangle **at random**
Place P2 in the middle between point P1 the corner point.
Repeat. The result is the Sierpinski triangle fractal.

w	a	b	c	d	e	f	p
f_1	0	0	0	0.16	0	0	0.01
f_2	0.85	0.04	−0.04	0.85	0	1.60	0.85
f_3	0.20	−0.26	0.23	0.22	0	1.60	0.07
f_4	−0.15	0.28	0.26	0.24	0	0.44	0.07

$$f_w(x,y) = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix}$$

w={1,2,3,4} is a random sequence. This is Barnsley's fern

We can use **randomness**
to expand our algorithmic
(computational) capabilities.

We can **imitate** natural information
processes (biomimesis).

We can use **Chaos** constructively.

We can propose and simulate
non-conventional computation
(Nnets, reservoir computing etc.).

We can base decision making &
perception on an **expanded logic**.

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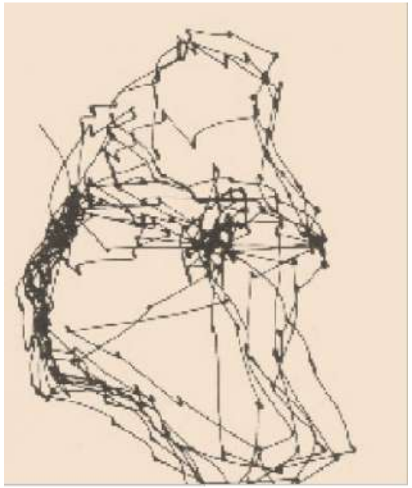
Then Gödel raised the "final question":

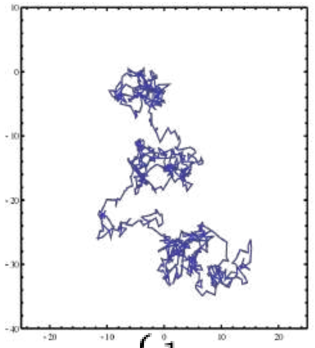
"Does our physical and biochemical substratum permit a mechanical <u>one-to-one</u> interpretation of <u>all</u> the functions of life and of the mind ?"

J.S. Nicolis & I. Tsuda
“The Magical Number 7±2 Revisited”

NEUROSCIENCE, Fourth Edition, Figure 20.1

Biological Information Processing

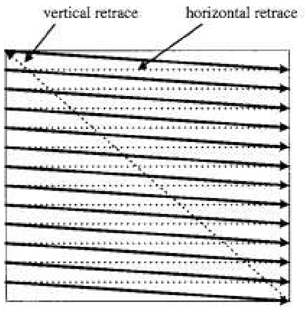




$$\Pr(U > u) = \begin{cases} 1 & : u < 1, \\ u^{-D} & : u \geq 1. \end{cases}$$

Levy Flight
[Fractal, NL-diffusion]

Artificial Information Processing



Scanning
[Linear-sweep]

John S. Nicolis   Ichiro Tsuda

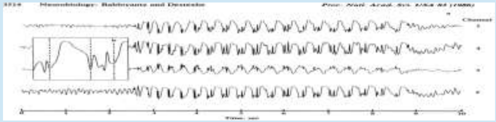
"The smallest biological information processor is the enzyme; the biggest is the (human) brain. They are separated by nine orders of magnitude. Yet their complexity is comparable. ..." (!)

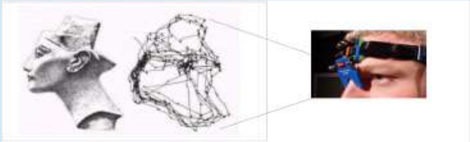
John S. Nicolis (2007)

Chaos & Biological Information Processing

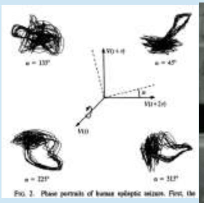
- Chaos and Complexity are the "sine qua non" ingredients for the generation and processing of biological information and communication.
- A reliable biological information processor must allow for chaos.
- Biological information processing spans many orders of magnitude (QM?)...
- It has context, meaning, depth and HISTORICITY (chaotic itinerancy).
- Biological Information Processing is more than mechanical.
- It goes beyond the paradigm of Turing.
- Healthy Brains (hearts etc) MUST have chaotic components.

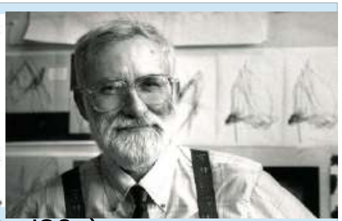
Macroscopic Level
(brain & brain regions)
EEG chaos-order transitions
Chaotic Attractors from EEG
Spatio-Temporal Patterns ... etc





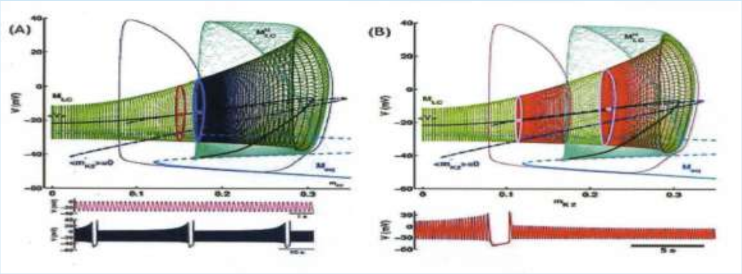
Norton & Stark, Science 1971



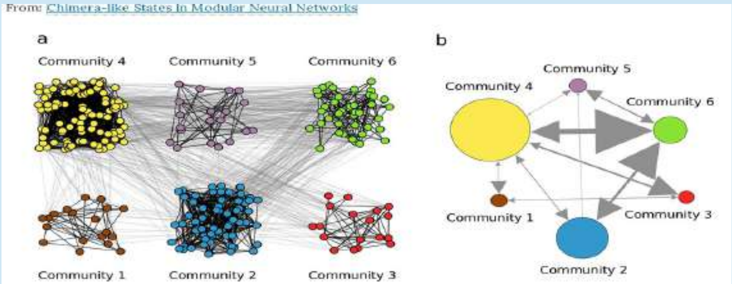


Pioneers (late '80s):
Walter J. Freeman (UCB)
also Agnes Babloyantz (ULB)
& many many others

Microscopic Level
(neurons and small group of neurons)
The more realistic/complex the model
the more allowing for chaos. Novel phenomena,
blue-sky catastrophe and spike trains
Andrey Shilnikov et al



Mesoscopic Level
(communities of neurons and
groups of communities of neurons)
Contemporary challenge, new concepts:
chimera-states, coarse-graining,
Non-local synchronization



Nature Sci. Rep. 6, 19845 (2016)

JS. Nicolis & I. Tsuda

~ 80’s - 90’s

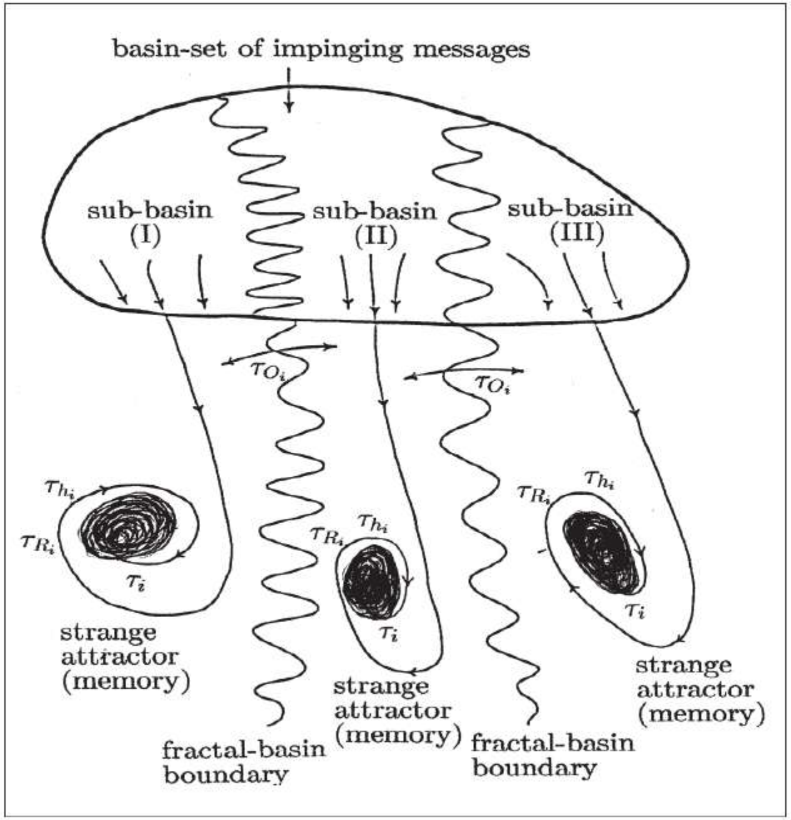


Figure 2 Sketch of a cognitive channel working after the dynamics of chaotic itinerancy (see text); within each attractor a ‘micro’-intermittency may go on as well (as, for example, in the Lorentz system).

τ_i = relaxation time on an attractor.
τ_{R_i} = residence time before the interruption of the thalamo-cortical pacemaker.
τ_{h_i} = holding time, after the interruption of the thalamo-cortical pacemaker.
τ_{0_i} = transient time between attractors after leaving attractor *i*.

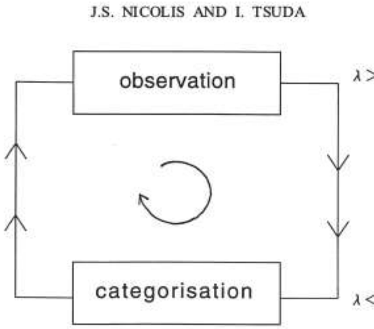
The thalamo-cortical pacemaker is responsible for the jumpings among the co-existing memories-attractors (a multifractal-*inhomogeneous* attractor). The jumpings can be viewed as *chaotic itinerancy*. The processor is partitioning a set

“… To observe you need a priori categories,
but to form categories you need observations...”

Emergent, Non-linear Recurrence:

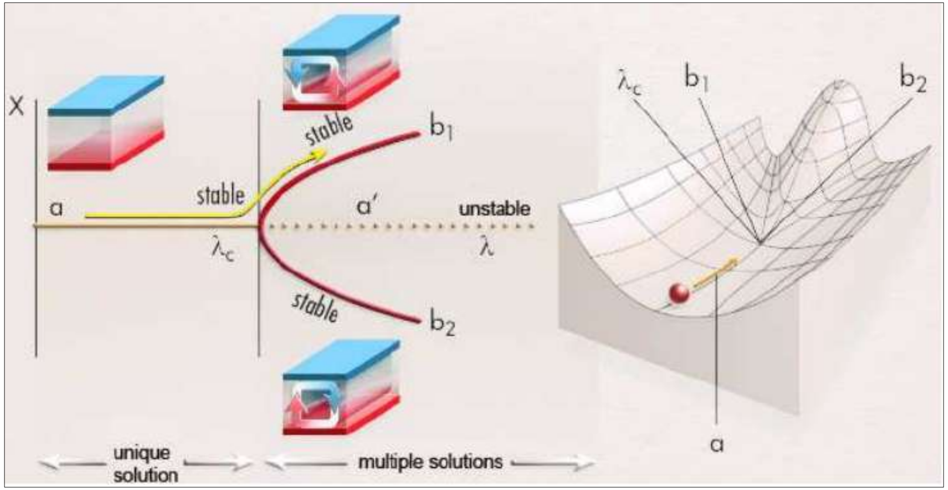
Explore, generate information: DATA
excitatory dynamics, (+) *feedback,*
Chaos: *positive Lyapunov exponents*

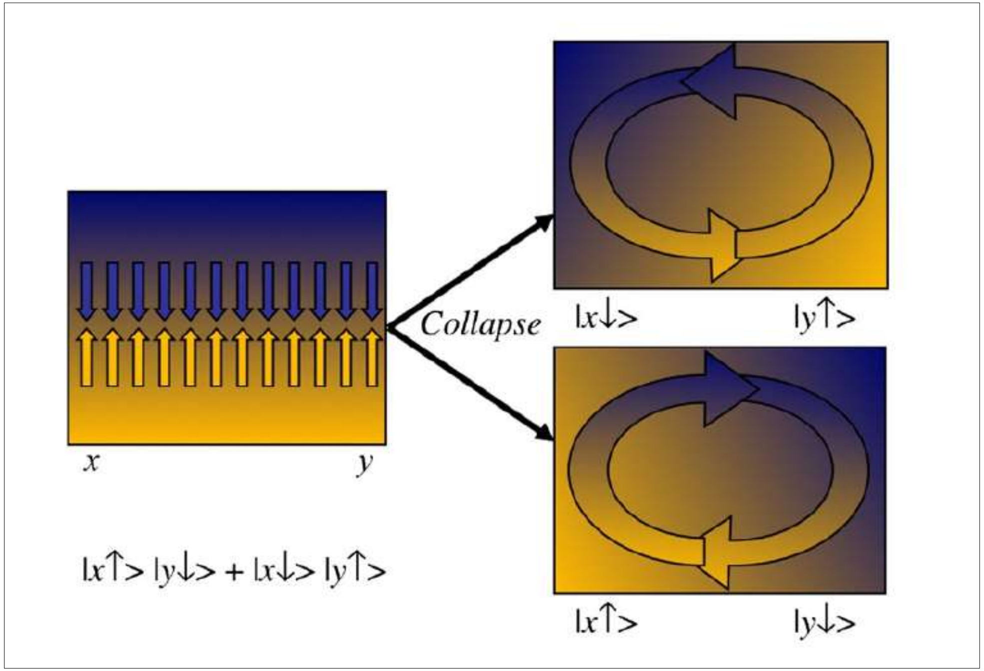
Categorize, retain information: HYPOTHESES
inhibitory dynamics, (-) *feedback,*
Stability: *negative Lyapunov exponents*



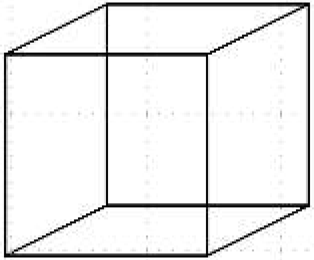
Bifurcation & Symmetry Breaking

Superposition & Collapse



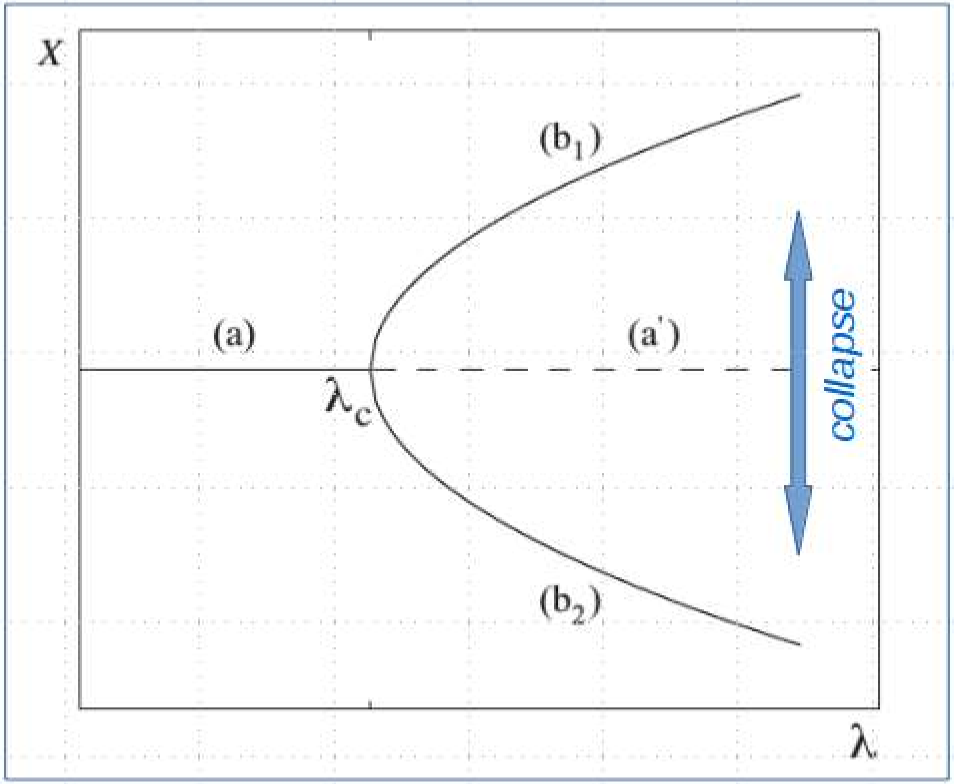


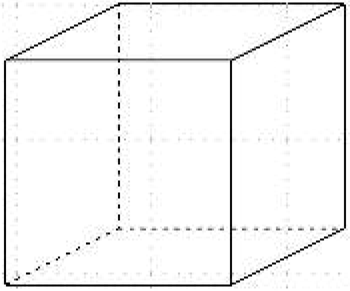
Bifurcations: Superposition & 'Collapse' in Cognition



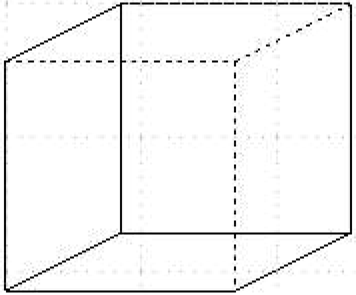
$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$|\alpha|^2 + |\beta|^2 = 1$$

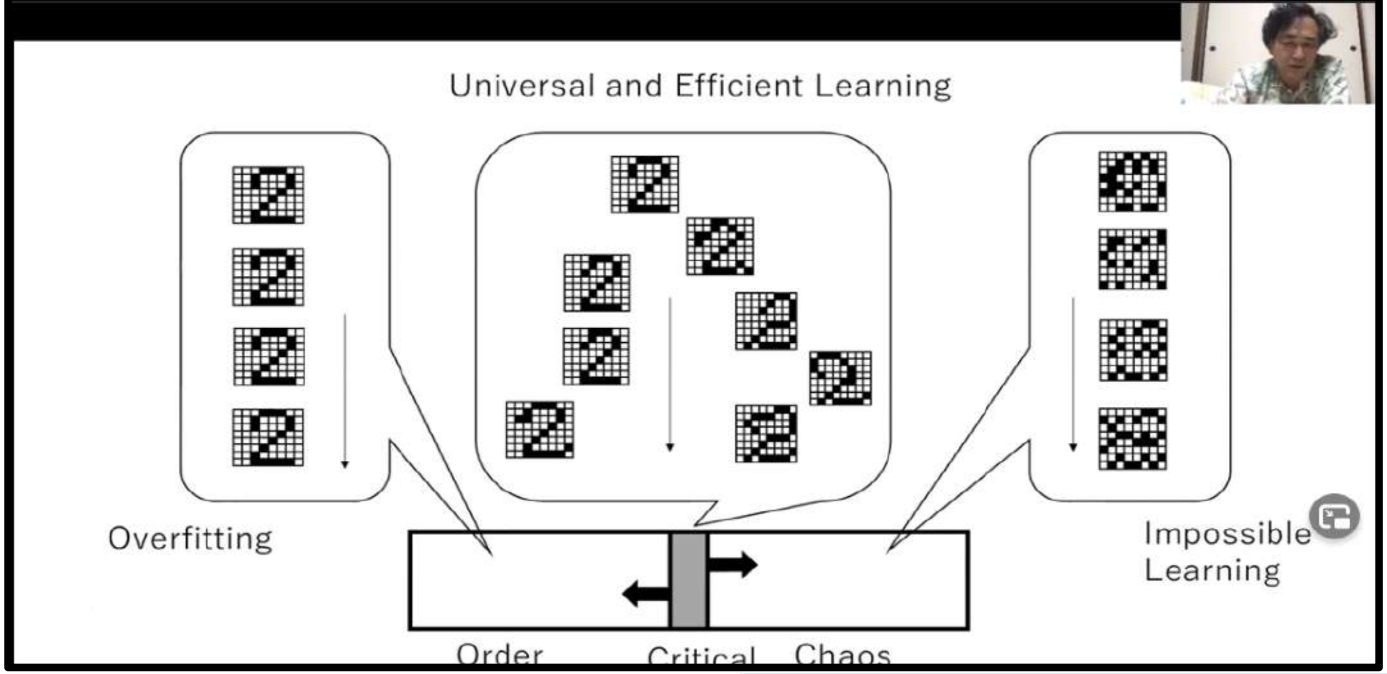




$$|\psi\rangle \rightarrow \alpha|0\rangle$$



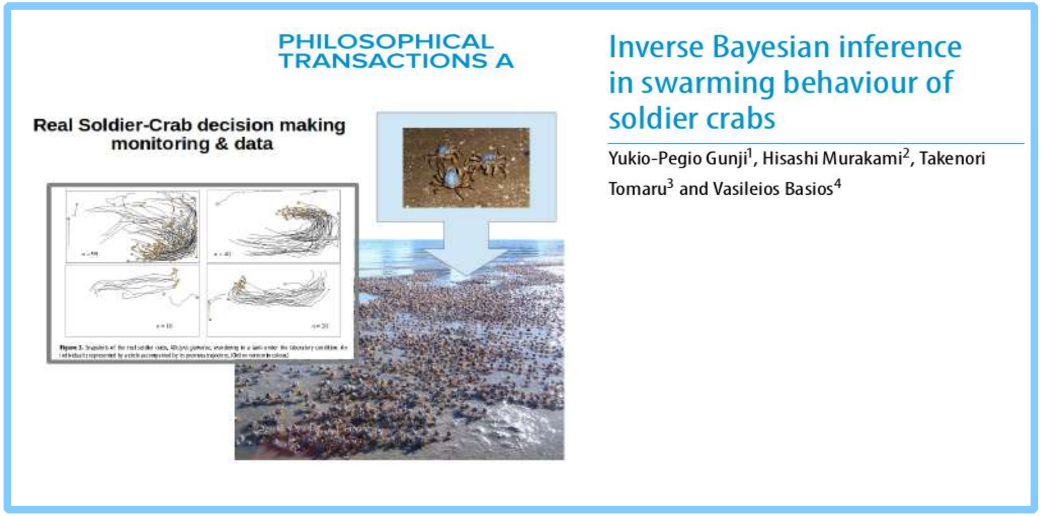
$$|\psi\rangle \rightarrow \beta|1\rangle$$



Order (Linear):
Negative Feedback > Positive Feedback

Criticality (non-linear):
Negative Feedback = Positive Feedback

Chaos (non-linear):
Negative Feedback < Positive Feedback

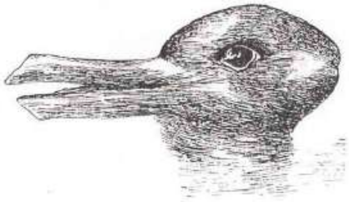


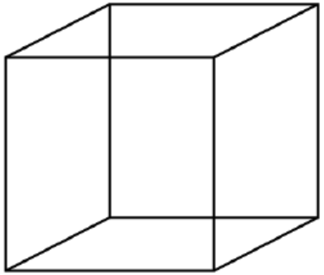
Yuo Cna
Raed Tihs

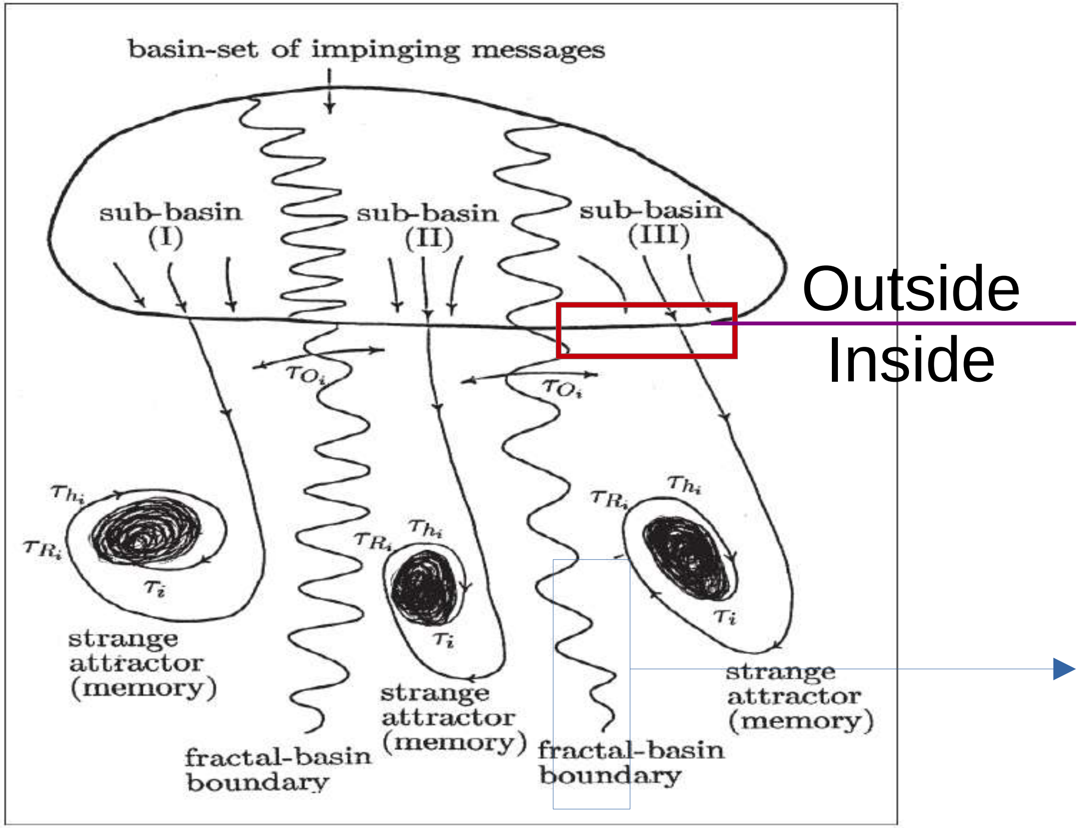
Typoglycemia

"Raeding Wrods With Jubmled Lettres There Is a Cost"

Rayner, K. et al, Psychological Science, 17(3), 192-193, (2006)







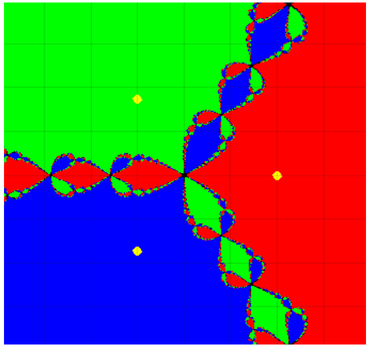
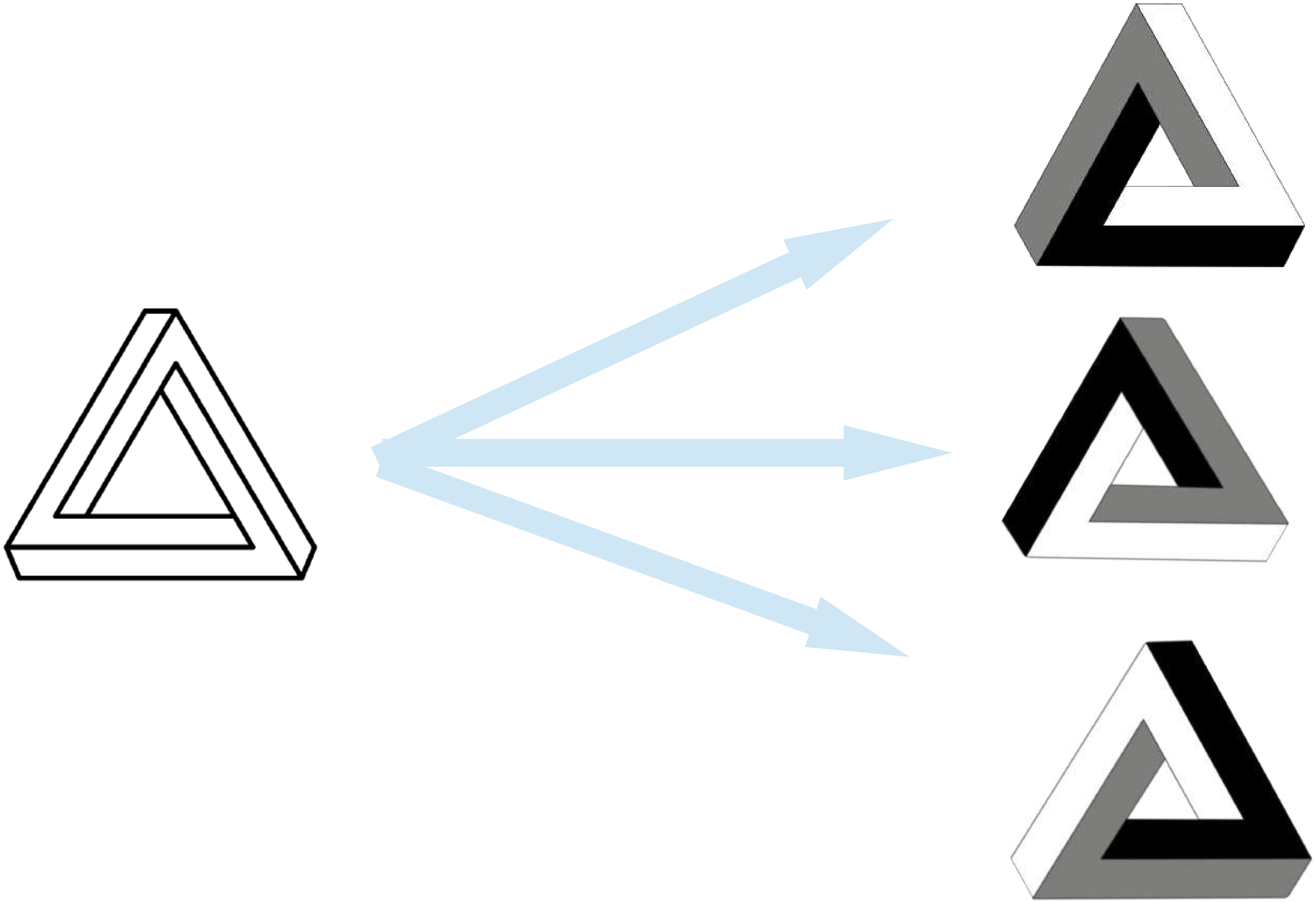


Figure 2 Sketch of a cognitive channel working after the dynamics of chaotic itinerancy (see text); within each attractor a 'micro'-intermittency may go on as well (as, for example, in the Lorentz system).

τ_i = relaxation time on an attractor.
τ_{R_i} = residence time before the interruption of the thalamo-cortical pacemaker.
τ_{h_i} = holding time, after the interruption of the thalamo-cortical pacemaker.
τ_{0_i} = transient time between attractors after leaving attractor *i*.

The thalamo-cortical pacemaker is responsible for the jumpings among the co-existing memories-attractors (a multifractal-*inhomogeneous* attractor). The jumpings can be viewed as *chaotic itinerancy*. The processor is partitioning a set



Extended Bayesian Inference

(questioning assumptions)

$$\frac{P(H|E)}{P(E|H)} = \frac{P(H)}{P(E)}$$

Probability of hypothesis H given evidence E (called the posterior belief)

Probability of this specific evidence, E, given this hypothesis H, times the overall probability of this hypothesis (called the prior belief)

Probability evidence E and hypothesis H would occur together

$$P(H|E) = \frac{P(E|H)P(H)}{P(E)} = \frac{P(E,H)}{P(E)} = \frac{P(E,H)}{P(E|H)P(H) + P(E|H')P(H')}$$

Total probability of encountering this evidence

which is a sum over all the hypotheses (H, H') that are compatible with the evidence

Bayesian Inference

"How often have I said to you that when you have excluded the impossible, whatever remains, however improbable, must be the truth"

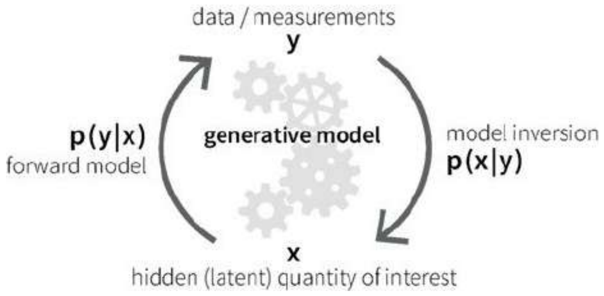
– Sherlock Holmes

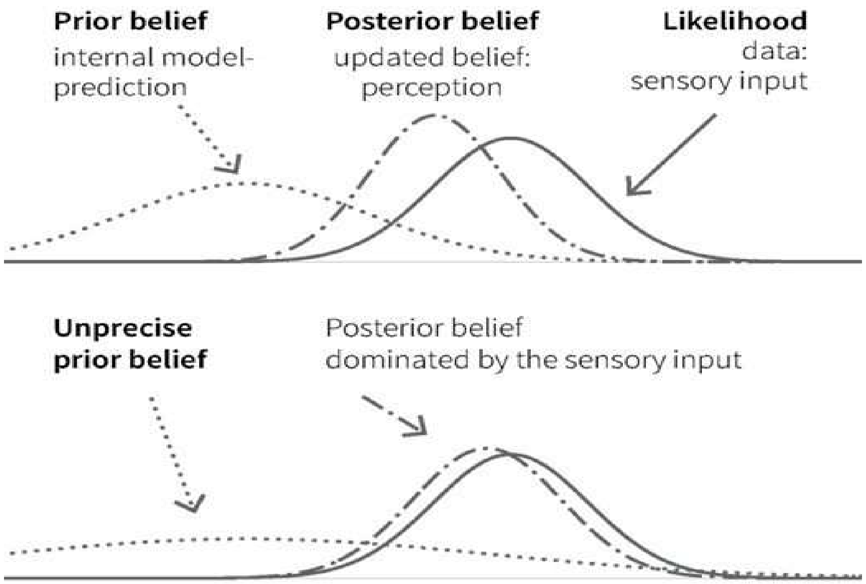
"How often have I said to you that when all other θ yield $P(x|\theta)$ of 0, whatever remains, however low its $P(\theta)$, must have $P(\theta|x) = 1$?"

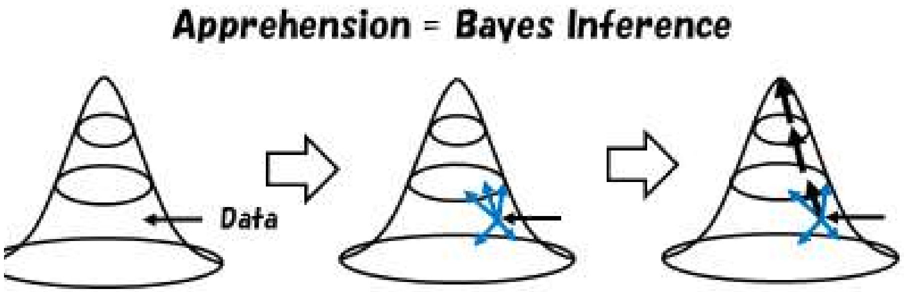
– Sherlock Holmes, paraphrased

in Kruschke, J. K., & Liddell, T. (2005)

"Bayesian data analysis for newcomers"







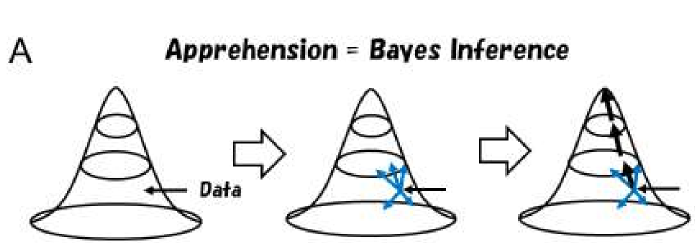
BI = climbing up ONE mountain top
Friston's Free Energy Minimization
(minimizing surprise : iteration)

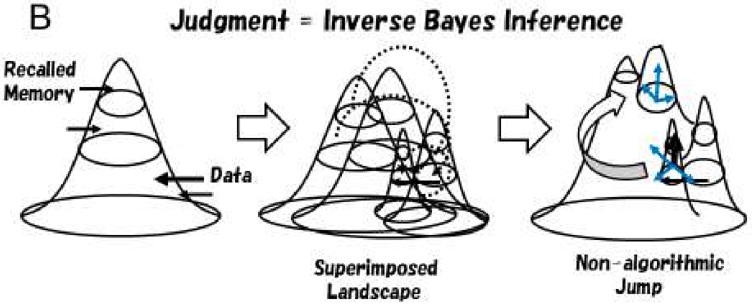
$$F = Energy - Entropy = -\langle \ln p(s,\eta) \rangle_q + \langle \ln q(\eta) \rangle_q$$

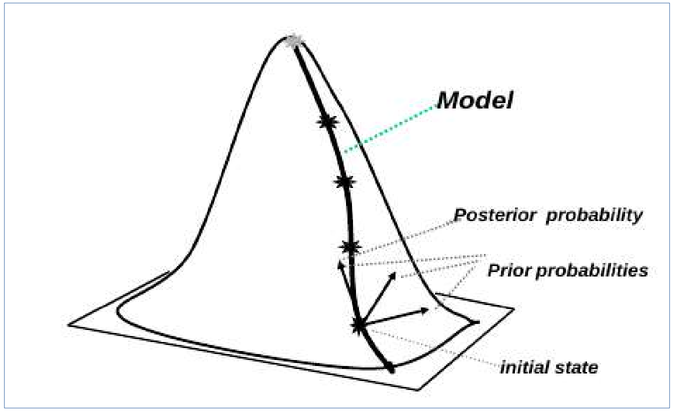
Action to minimise a bound on surprise

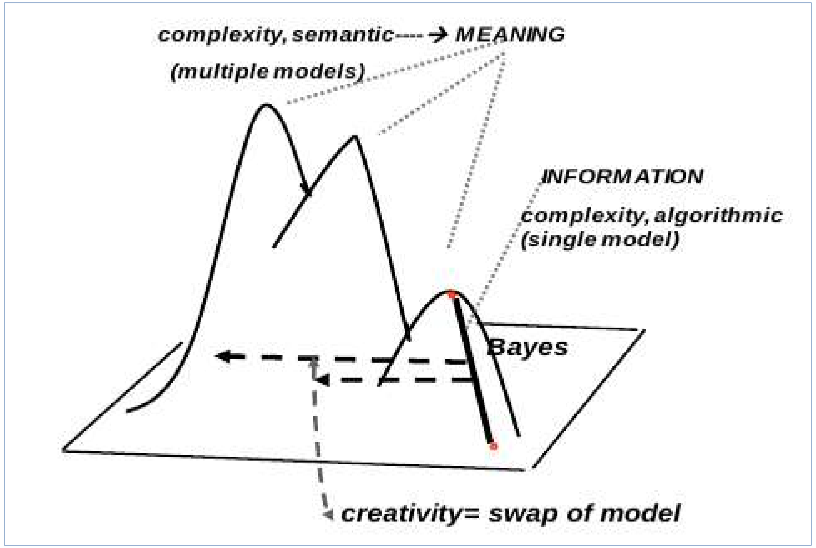
Perception to optimise the bound

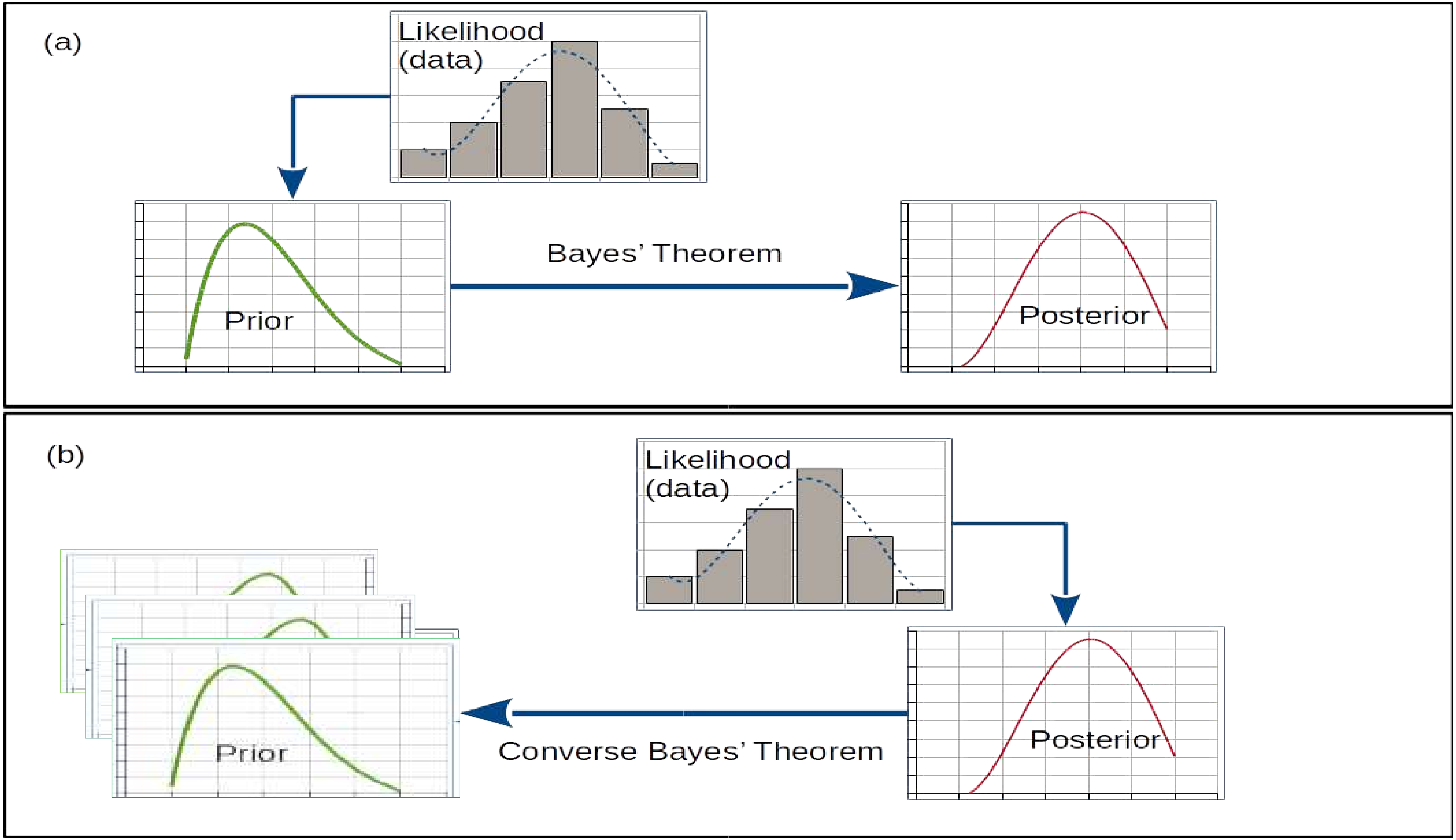
What if the mountain has multiple peaks ?… (Judgement)
! BIB = Bayesian & Inverse Bayesian Inference

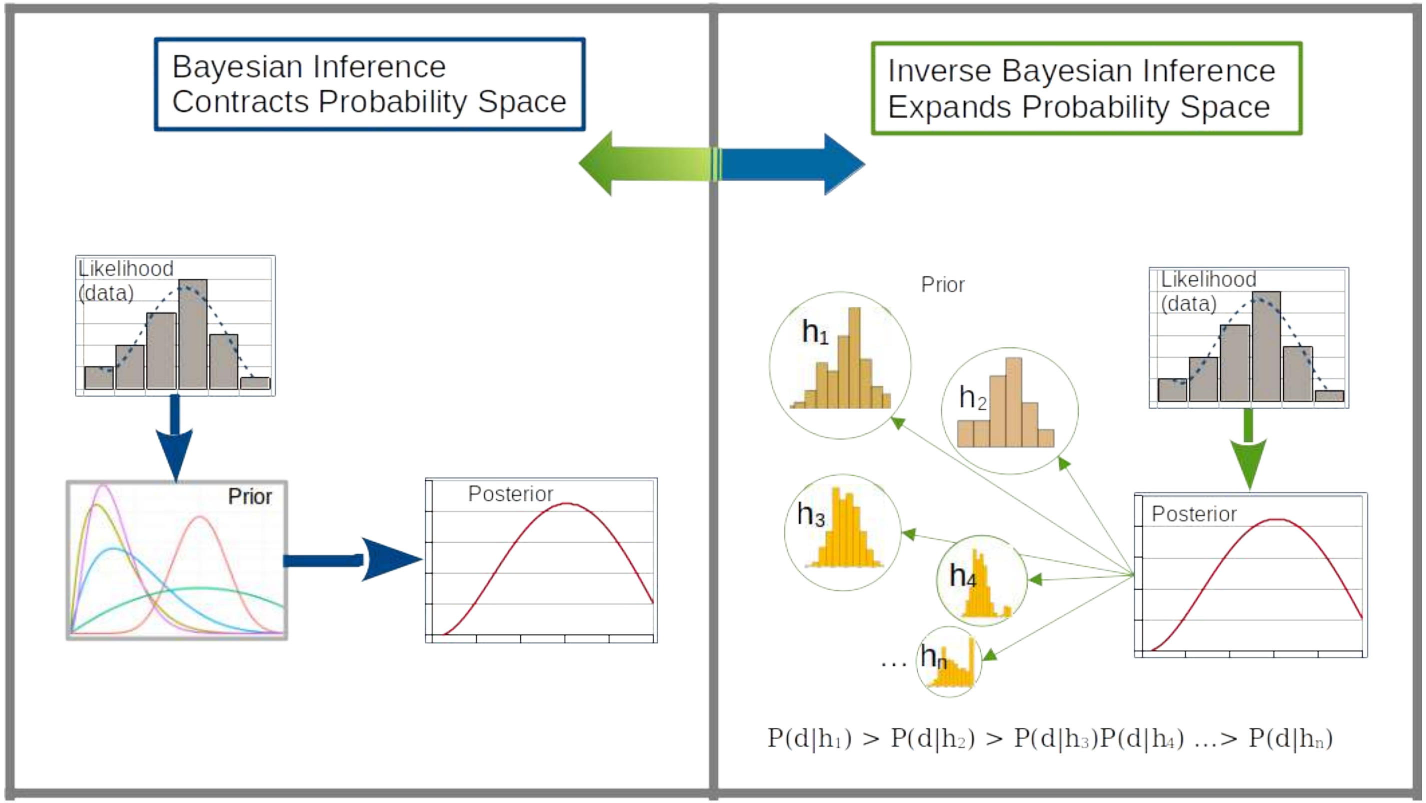




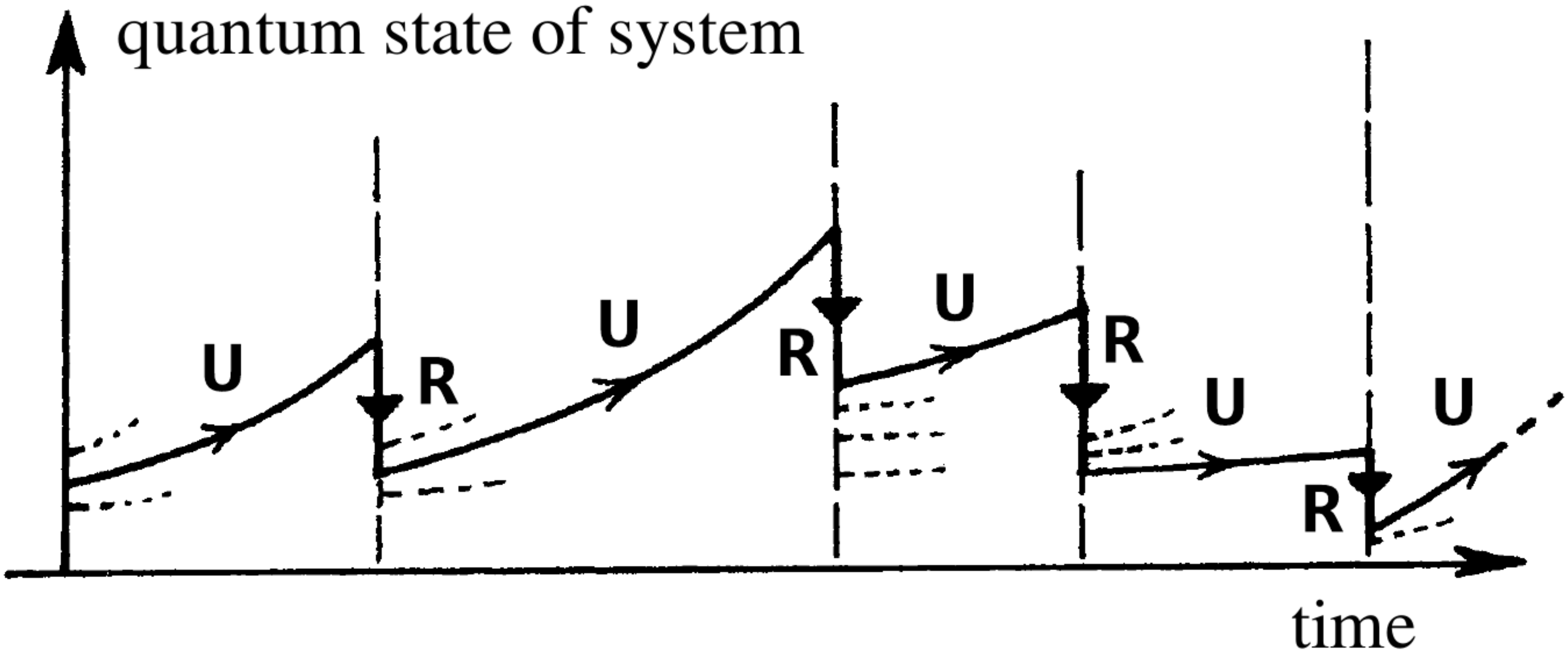





(a)
Likelihood
(data)
Bayes' Theorem
Prior
Posterior
(b)
Likelihood
(data)
Prior
Converse Bayes' Theorem
Posterior



Bayesian Inference
Contracts Probability Space
Inverse Bayesian Inference
Expands Probability Space
Likelihood
(data)
Prior
Posterior
Prior
h1
h2
h3
h4
… hn
Likelihood
(data)
Posterior
P(d|h1) > P(d|h2) > P(d|h3)P(d|h4) …> P(d|hn)



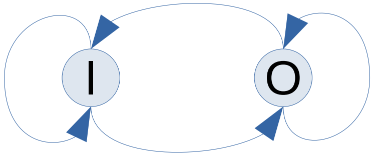
__Sir Roger Penrose (2016, pg 143)__

Figure 2-8: The way that the quantum-theoretic world appears to behave, with stretches of deterministic **U**-evolution, punctuated by moments of probabilistic **R**-action, each of which restores some element of classicality.

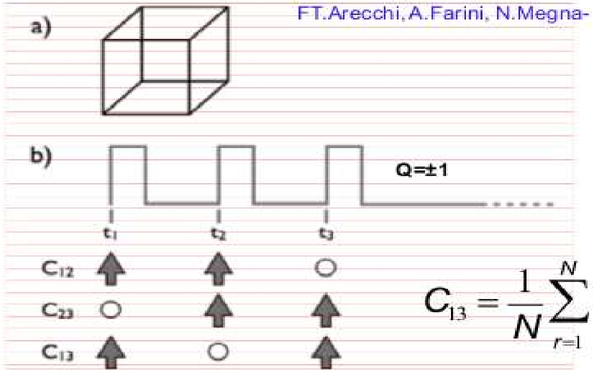
Non-Boolean Logic
&
Quantum Cognition

(the logics of objects and processes)

Apprehension & Judgement in Necker Cube Dynamics



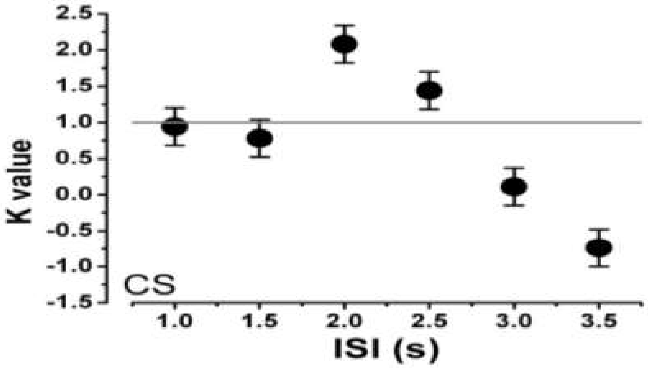
- Non-Markovian with short term memory
- Contextual
- Violates temporal-Bell Inequalities [CHSH and Legget-Garg ineq.]



$$C_{13} = \frac{1}{N}\sum_{r=1}^{N} Q_r(t_1)Q_r(t_3)$$

$$K = \frac{1}{N}\sum_{r=0}^{N}\left(Q(t_1)Q(t_2) + Q(t_2)Q(t_3) - Q(t_1)Q(t_3)\right)$$

Fortunato-Tito Arecchi



QQ : Quantum Question : Order Effects

$$A\,B \neq B\,A$$

$$f(g(x)) \neq g(f(x))$$

No-Commutative

Contextual

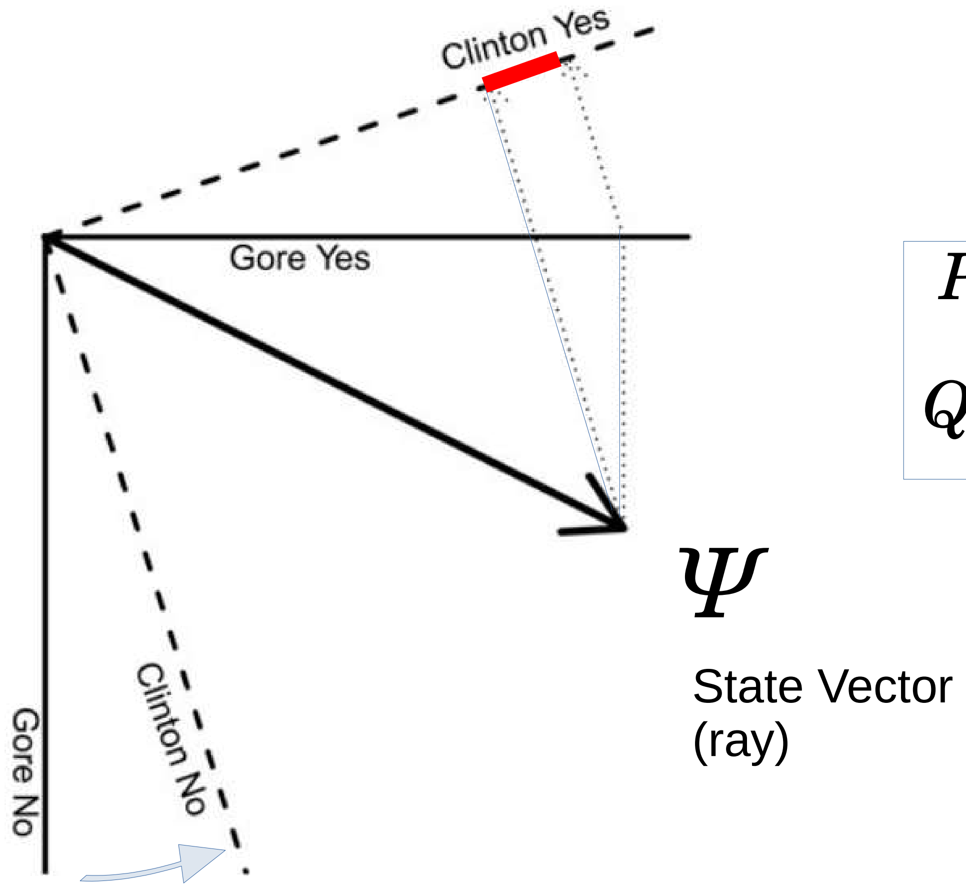
Complementary

-William James (~1860)
-Niels Bohr
-Heisenberg (~1930)


Is Clinton honest?	50%
Is Gore honest?	68%


Is Gore honest?	60%
Is Clinton honest?	57%

Moore (2002), Busemeyer and Wang (2009)



$$P = \Psi^* \Psi = |\Psi|^2$$

$$QQ - Hilbert\ Space$$

Base A: Clinton-Gore
Base B: Gore-Clinton
Bases’ angle = Interference factor

Quantum-like Logic

- **Classical Logic (Boolean Logic):** The ‘distributive law’ holds **COMMUTATIVE OPERATIONS**
  ‘*A and (B or C)’ is equivalent to ‘(A and B) or (A and C)’.*
- **Quantum Logic:** The ‘distributive law’ is broken! **NON-COMMUTATIVE OPERATIONS**
  ‘*A and (B or C)’ is NOT equivalent to ‘(A and B) or (A and C)’.*

Once the distributive law is not observed the three tenets of classical (Aristotelian, Boolean) logic also cannot hold unconditionally.

<u>The law of identity:</u> 'Whatever is, is.' ,

<u>The law of contradiction: </u>'Nothing can both be and not be.'

<u>The law of excluded middle:</u> 'Everything must either be or not be.' .

So … what is reasonable in logic?

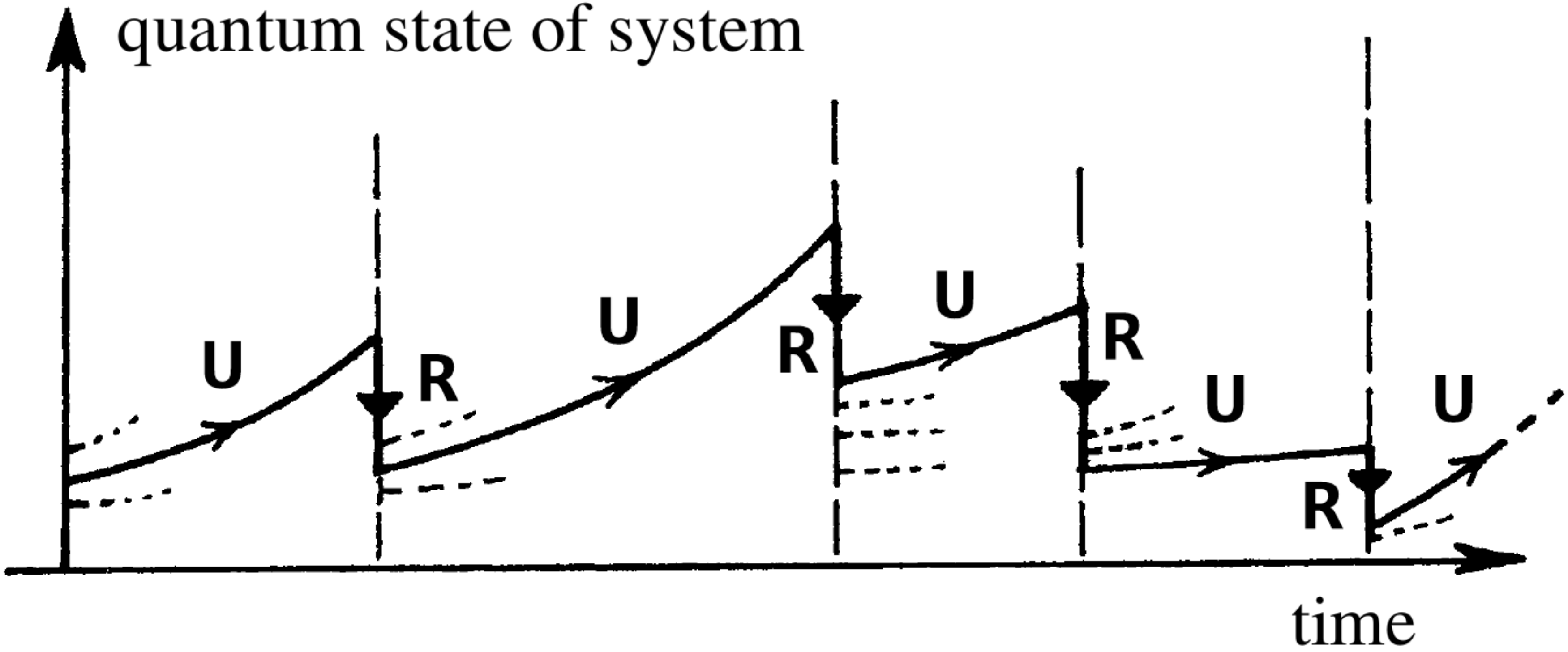
Eggs **.AND.** (Bacon **.OR.** Sausages) = (Eggs **.AND.** Bacon) **.OR.** (Eggs **.AND.** Sausages)

.OR.

Eggs **.AND.** (Bacon **.OR.** Sausages) = (Eggs **.AND.** Bacon**-**Sausages)

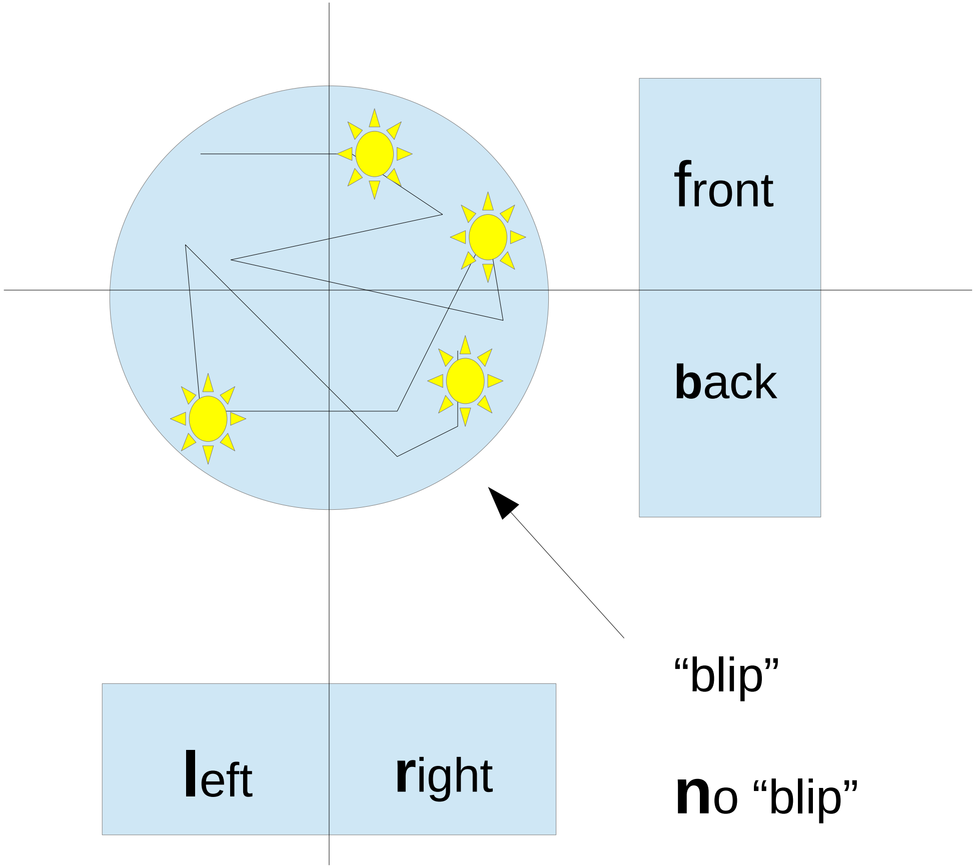
P(bacon)+(1-P)(sausages)

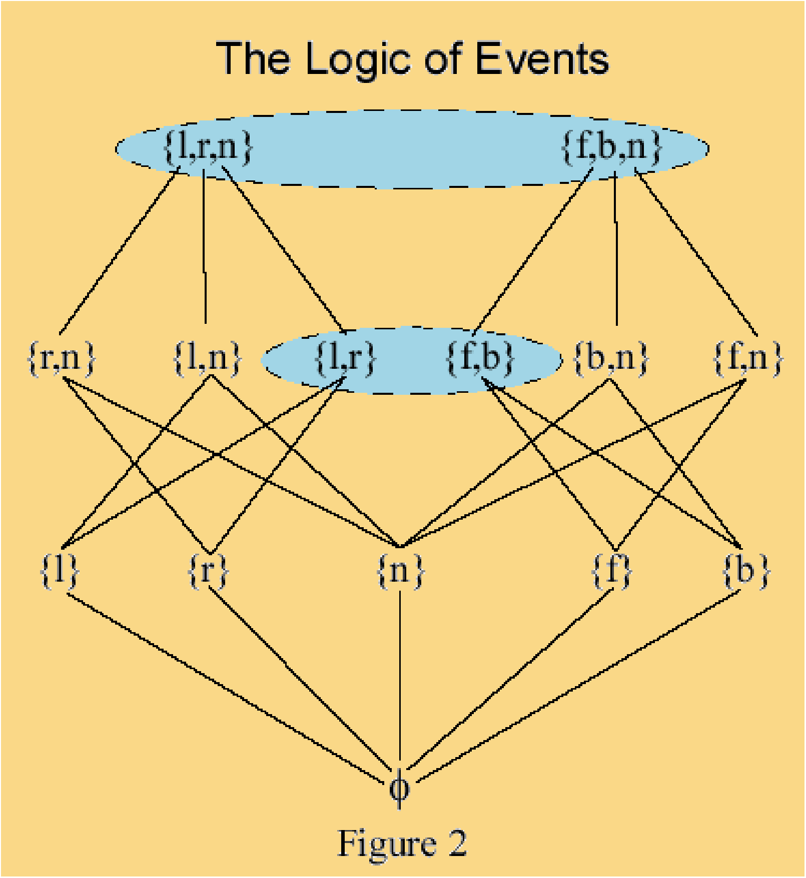
“KNOWING, DOING and BEING”, by Chris Clarke
Imprint Academic (2013)



<u>Sir Roger Penrose (2016, pg 143)</u>
Figure 2-8: The way that the quantum-theoretic world appears to behave, with stretches of deterministic **U**-evolution, punctuated by moments of probabilistic **R**-action, each of which restores some element of classicality.

Foulis' Firefly in a box

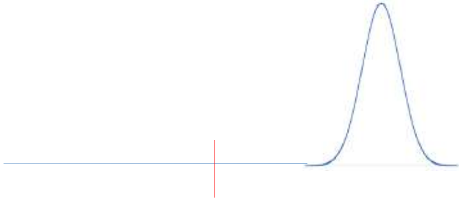




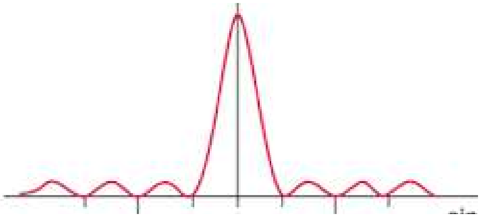
The Guppy Effect as Interference & Concepts in QQ

Q1:
What is a good example of a Pet?

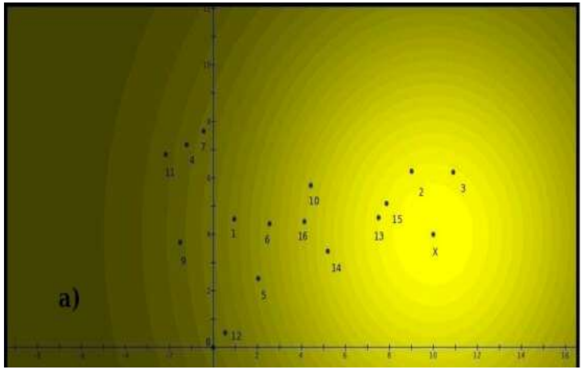
Q2:
What is a good example of a Fish?



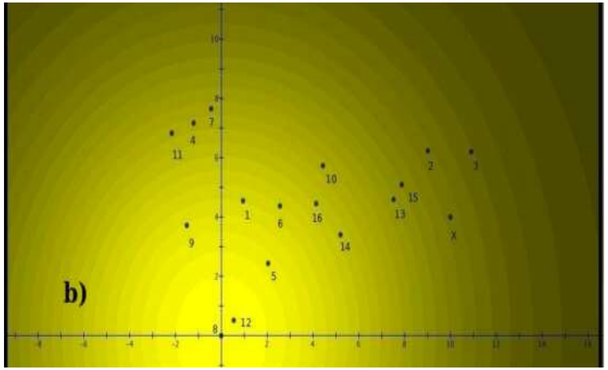
Q1 .AND. Q2:
What is a good example of a Pet and a Fish?



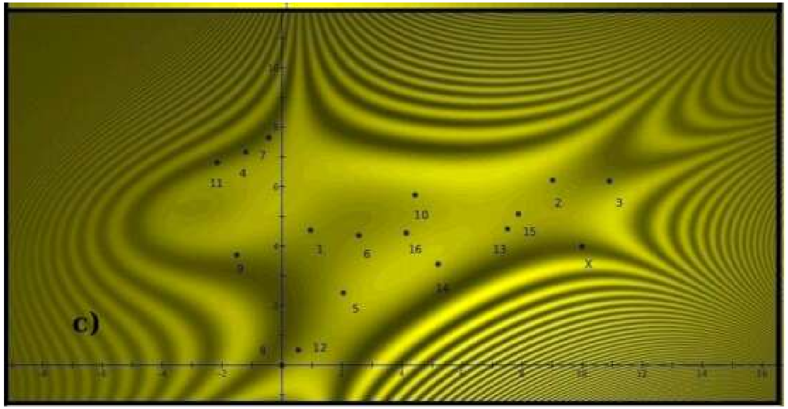
‘**deepai.org**/publication/the-guppy-effect-as-interference’ (Aerts et al)



Q1
a good example
of appliance?



Q2
a good example
of furniture?



Q1 .AND. Q2
a good example
of furniture and appliance?

Visualization of interference probabilities, standard QM formalism:
Hilbert Space: { |A> appliance |B> furniture}, (x,y) labels of objects of given table

$$\frac{1}{2}|\psi_A(x,y)+\psi_B(x,y)|^2 = \frac{1}{2}(|\psi_A(x,y)|^2+|\psi_B(x,y)|^2)+|\psi_A(x,y)\psi_B(x,y)|\cos\theta(x,y)$$

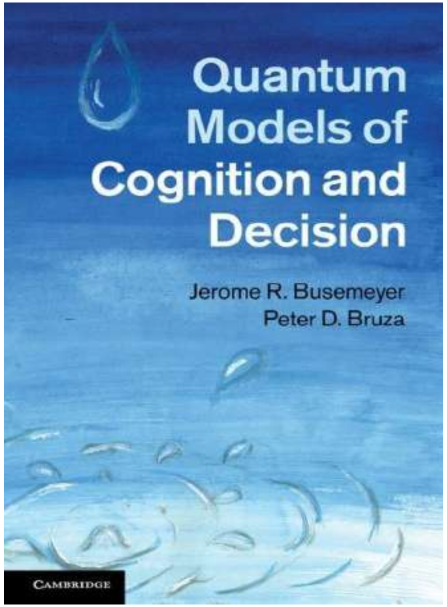
Diederick Aerts

VUB

Andrei Khrenikov
LNU

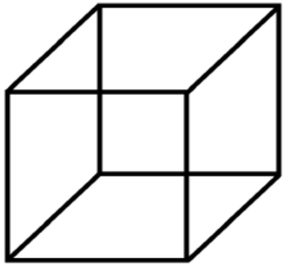
Concepts are "Quantum-like" Entities

"… perceptions & concepts, like objects seem to loose their rigid boundaries ..."



- Entangled
- Complementary
- Inter-penetrating
- Super-positioned
- Context Dependent ...

Our mind works with Quantum Probabilities (Processes) rather than Classical Probabilities (Objects)



Biological Information Processing
Extended Bayesian Inference

(putting it all together from information to action)

ubiquitous ambiguity

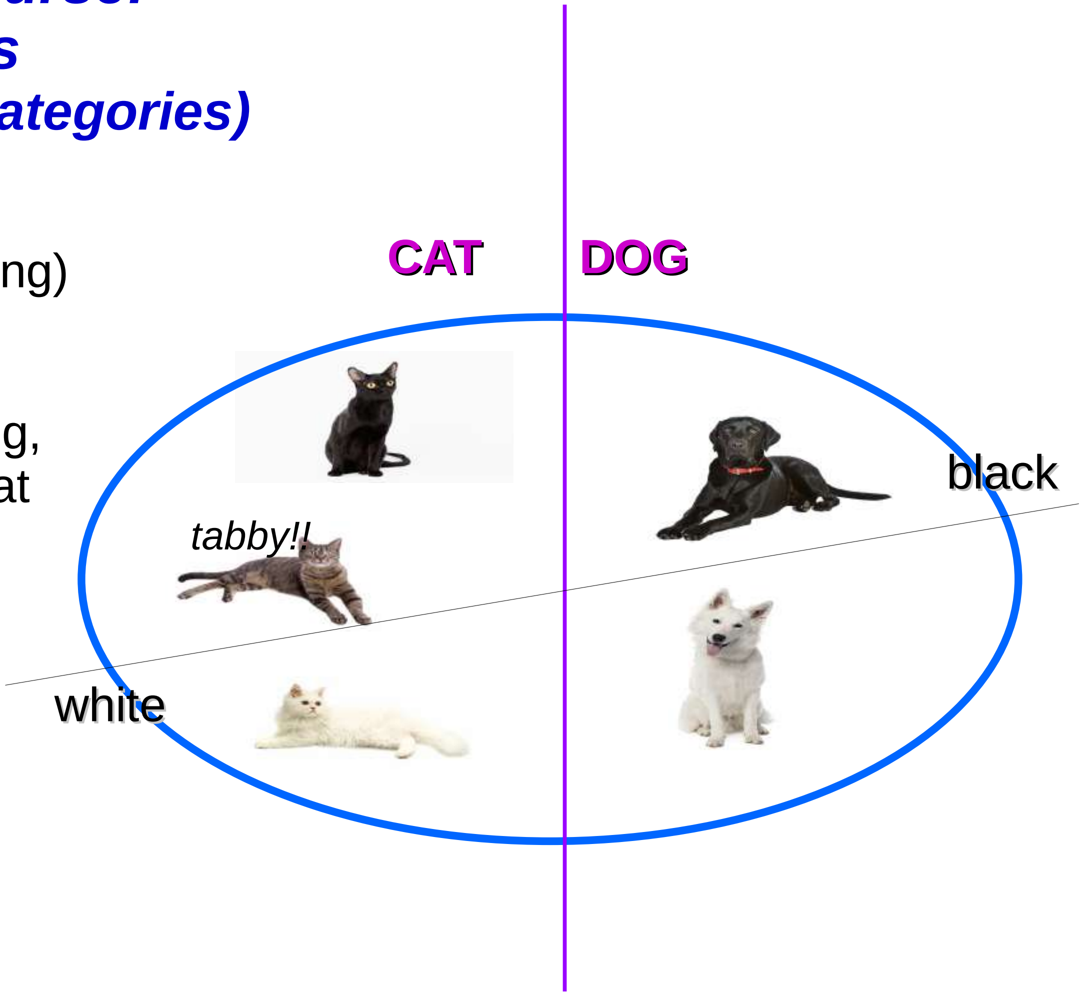
An example of BIB & Q-Logic

e.g. a Universe of Discourse:
Data + Relations
(impinging signals + a priori categories)

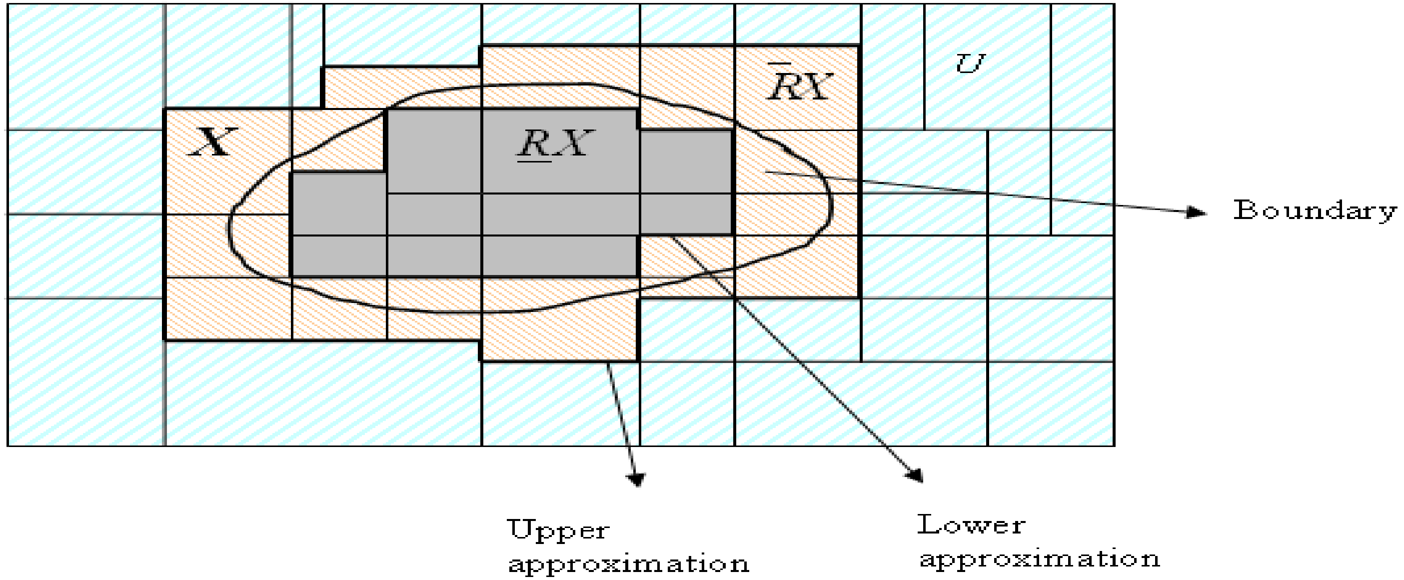
Choosing a representative (… roughly speaking)

Symmetry: if an individual cat is not like a dog,
an individual dog is not like a cat

Locality: two neighbours have
same representative.



Rough-Sets approximation:



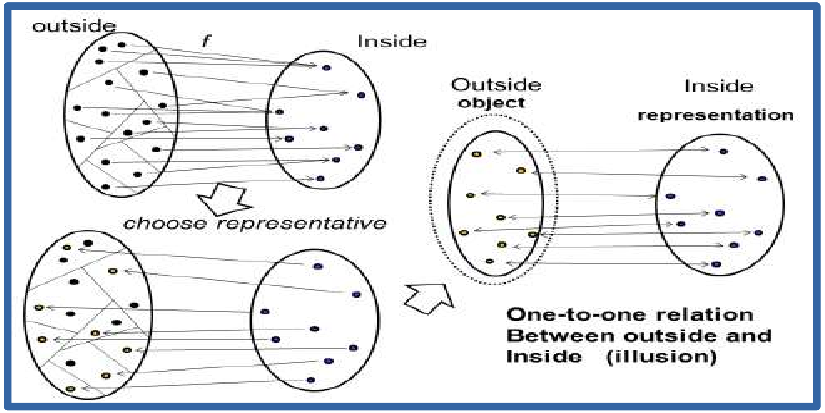
If $\overline{R}X = \underline{R}X$ then, X is definable (the boundary set is empty)

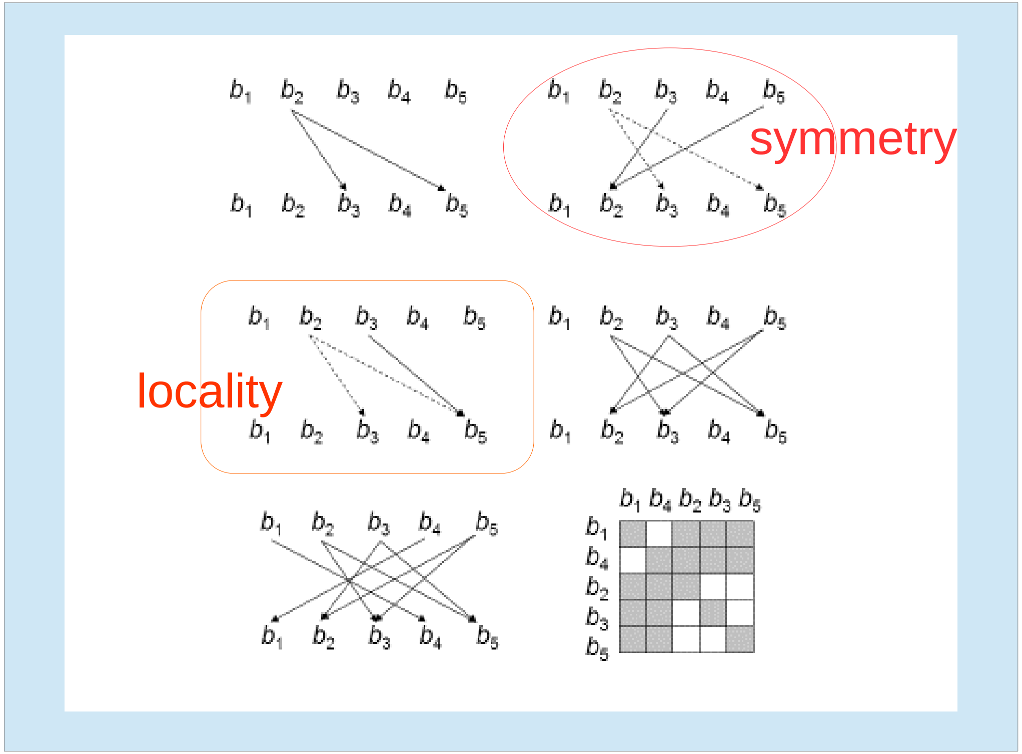
If $\overline{R}X \neq \underline{R}X$ then X is Rough with respect to R.

ACCURACY := Cardinality(Lower)/ Cardinality (Upper)

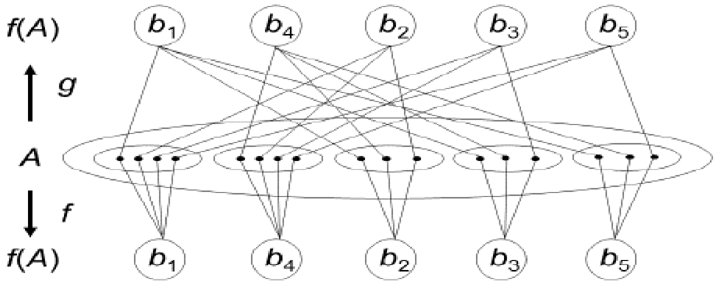
Formal Scheme:

***Y.-P. Gunji** et al. / BioSystems 141 (2016) 55–66*





g: A: → g(A) Inhibitory network construction induces a rough set approximation K: ($K^*(X)$, $K_*(X)$)



f: A → f(A) induces a rough set representation R: ($R^*(X)$, $R_*(X)$)

Y.-P. Gunji, VB et al
Biosystems,
141 (2016) 55-66

Apprehension can be implemented by (forward) Bayes inference (Arecchi, 2015): $P(h^*) = P(h|d) = P(h)P(d|h)/P(d)$,

$P(h)$: a priori probability of hypothesis , *h, d* is data,

$P(d|h)$: a priori probability that *d* results from *h*,

$P(d)$: probability we observe data *d*, and

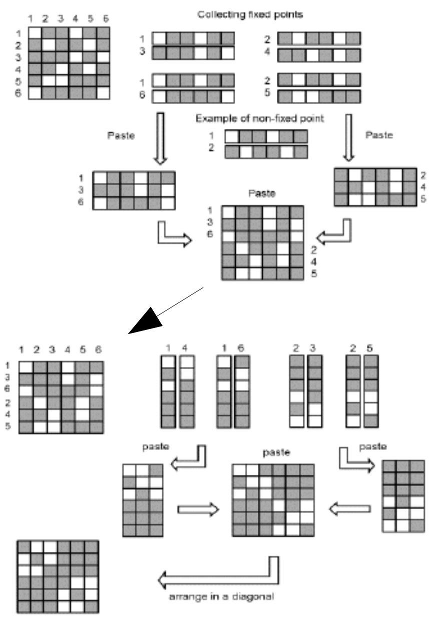
$P(h^*)$: a posteriori probability among a priori Hypothesis.

h is replaced by equivalence relation,

R derived by a particular representation (map, “f”),

$P(d|h)$ is replaced by $R^*(X)$, and $P(h|d) = P(h^*)$ is replaced by $R_*(X)$.

Therefore: **Bayesian inference maps to the process of computing $R^*(X)$ from $R_*(X)$**
(i.e. from a priori to a posteriori)



Collecting Fixed points, sets X, for all f,g (R,K) compositions:

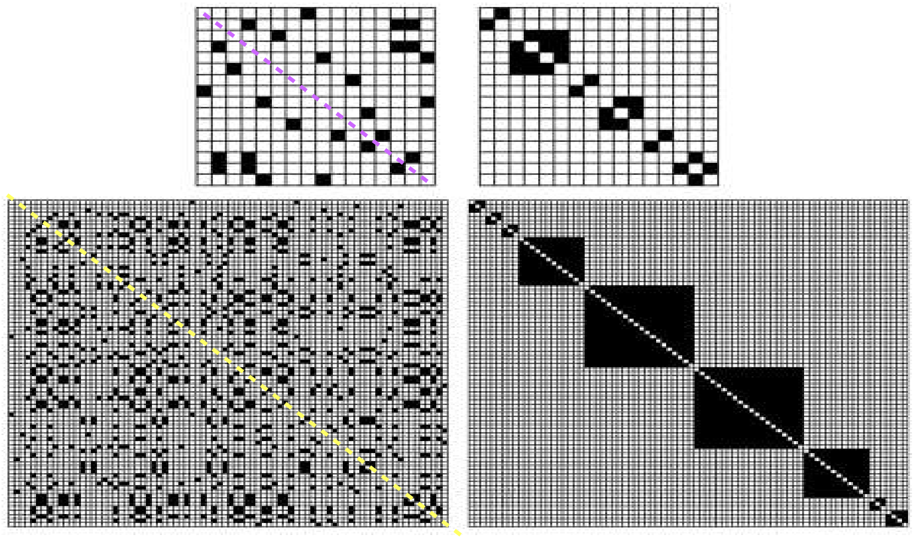
(algorithm based on row-column rearrangement)

$R*K_*(X) = X \quad R*K_*(X) = X$

$K*R_*(X) = X$

$K*R_*(X) = X$

… and can go for larger and larger systems!



Play it once more... with Restricted Boltzmann Machines: (Bayesian-Inverse Bayesian Inference beats simple Bayesian Inference!)

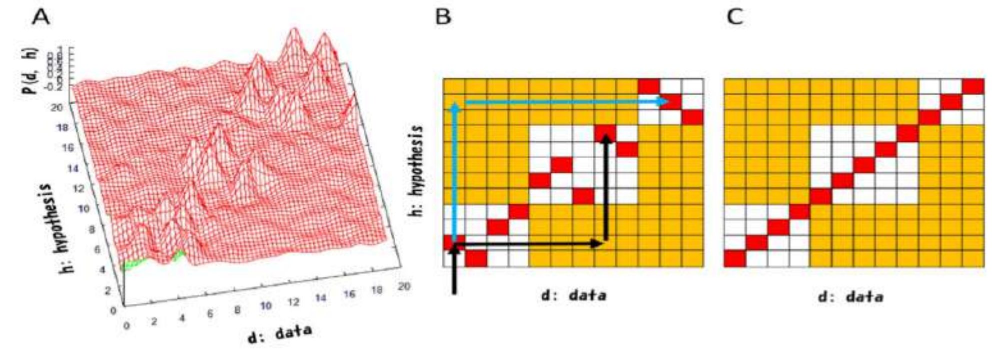


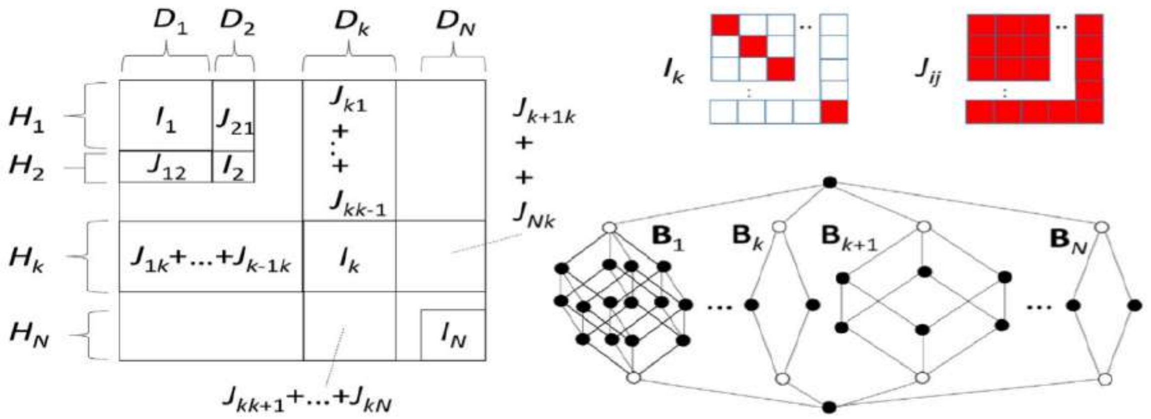
Fig. 10. (A) Distribution of the joint probability $P(d, h)$ plotted against data, d and hypotheses, h for a steady state ($t = 100$). (B) Matrix expression for the joint probability where a matrix consisting of red and white cells is a diagonal matrix area and a matrix consisting of orange cells is a noisy area. Arrows represent probabilistic transition from an attractor to another attractor. (C) Diagonal matrix is obtained by the exchange of rows or columns. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

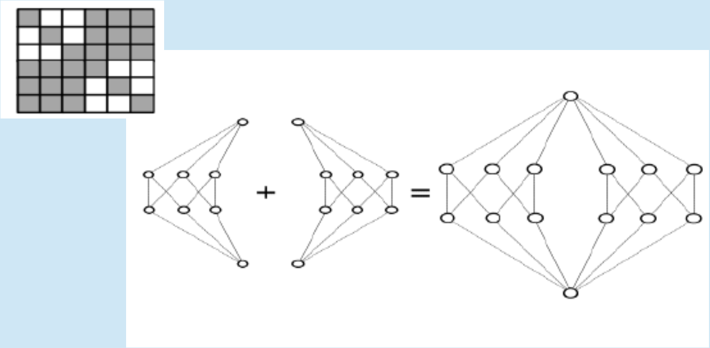
"Inverse Bayesian inference as a key of consciousness featuring a macroscopic quantum logical structure"

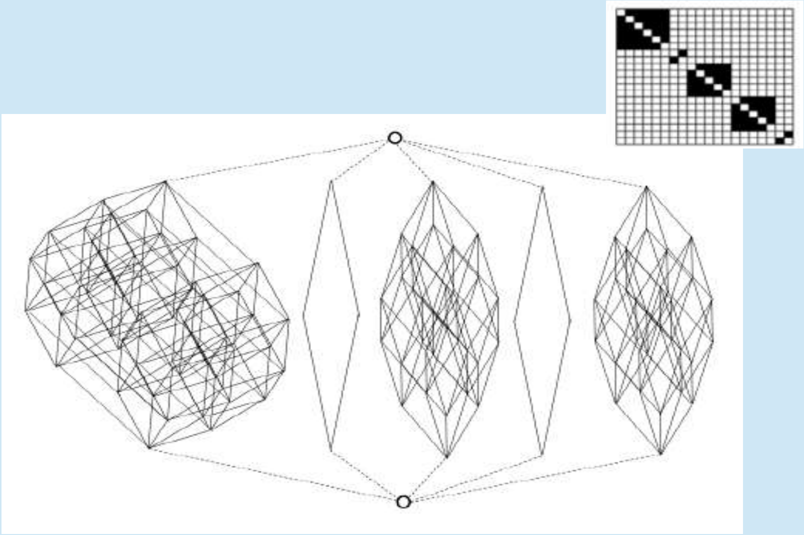
Gunji Y.-P., Shinohara S., Haruna T., Basios V. (2017)

BioSystems, 152 , pp. 44-65.

Y.-P. Gunji et al. / BioSystems 152 (2017) 44–65







Hasse Diagrams of the matrix of equivalence

qualify this logic as a non-Boolean
“multi channel” $\leftrightarrow$ ortho-modular Quantum-Logic

Real Soldier-Crab decision making monitoring & data

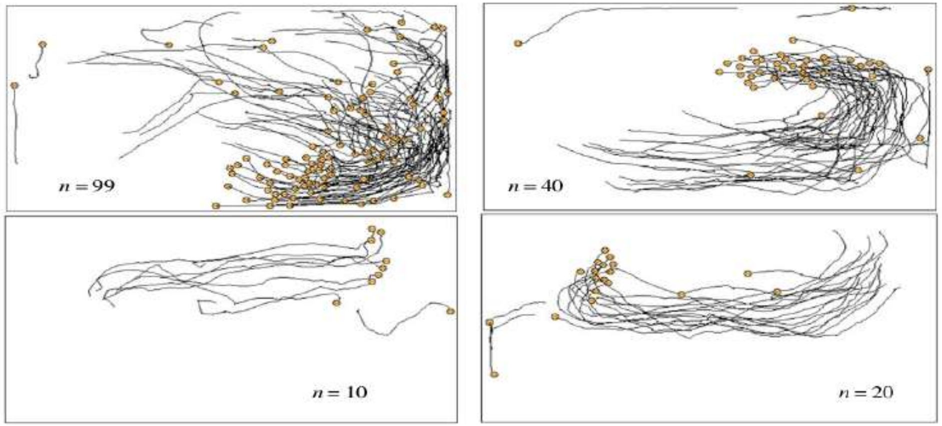
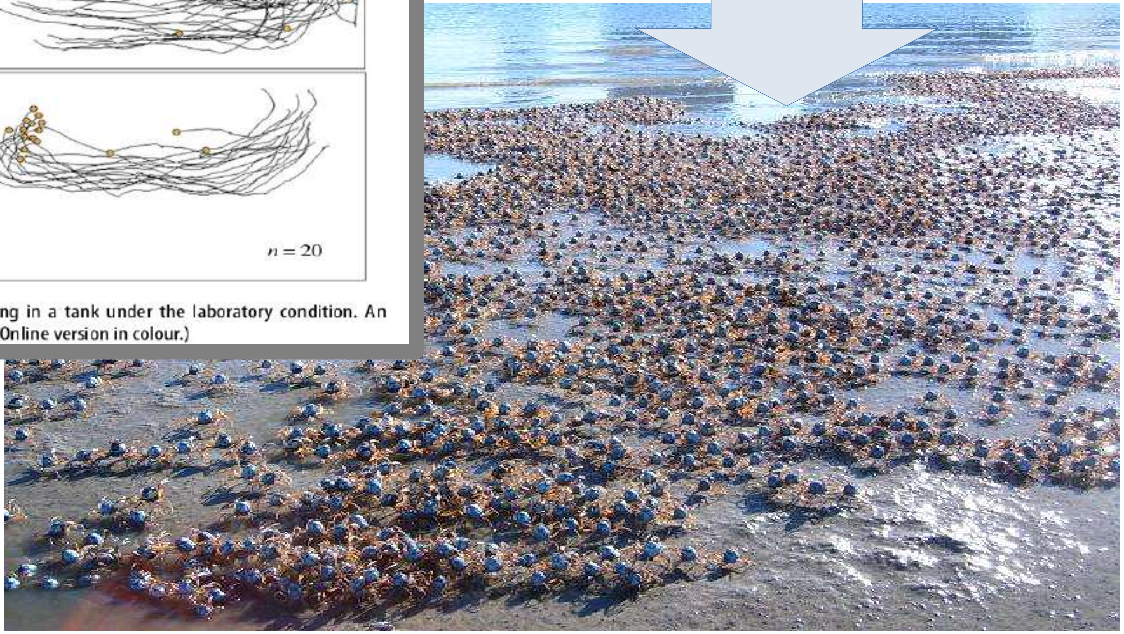


Figure 3. Snapshots of the real soldier crabs, *Mictyris guinotae*, wandering in a tank under the laboratory condition. An individual is represented by a circle accompanied by its previous trajectory. (Online version in colour.)



Modified Vicsek Model
With BIB as internal steering

BIB = Bayesian and Inverse Bayesian Inference Process

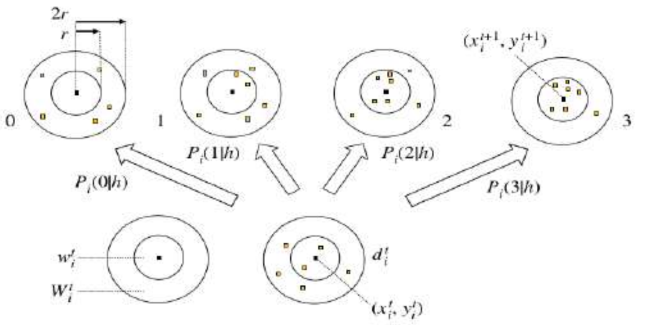


Figure 5. Schematic diagram of data and hypothesis adopted by a time series of real soldier crabs. (Online version in colour.)

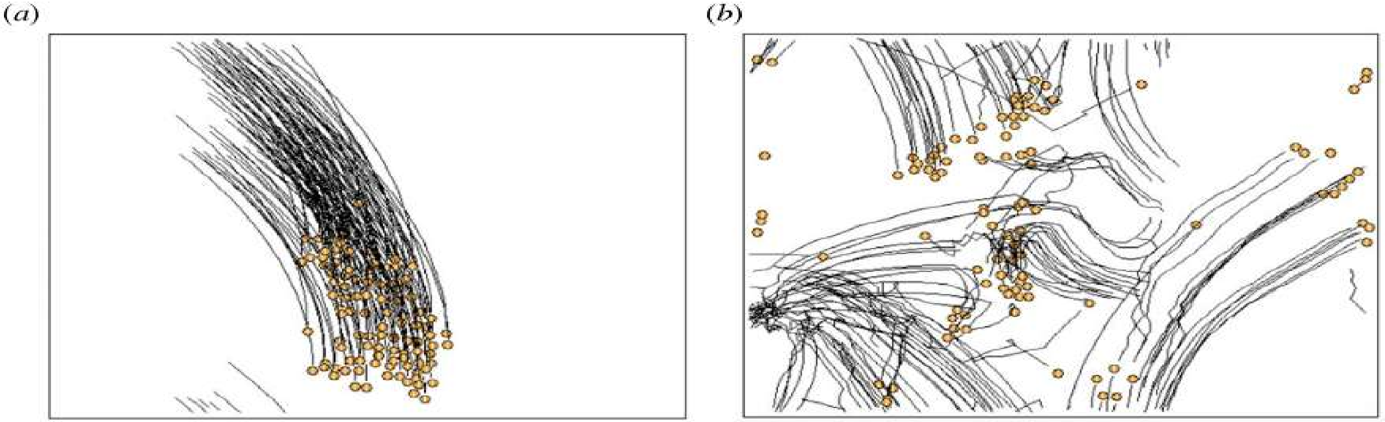
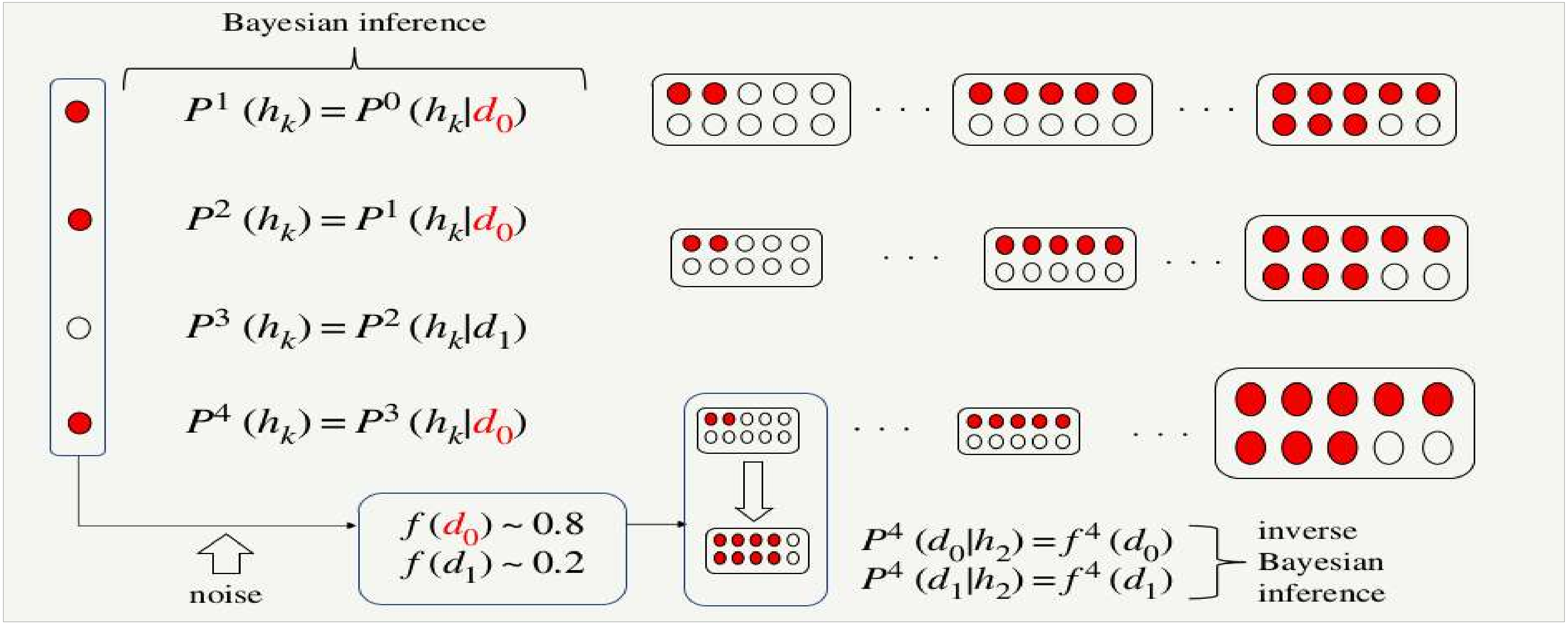


Figure 10. Snapshots of the swarm model based on BIB inference. Swarming phase (*a*) and dispersing phase (*b*). (Online version



BIB & Extended Bayesian Inference code for Levy flight
by S. Shinohara:
https://zenodo.org/record/5018080
Simulation test data & source files in C ++ (uses Qt v5 library)

PHILOSOPHICAL
TRANSACTIONS A

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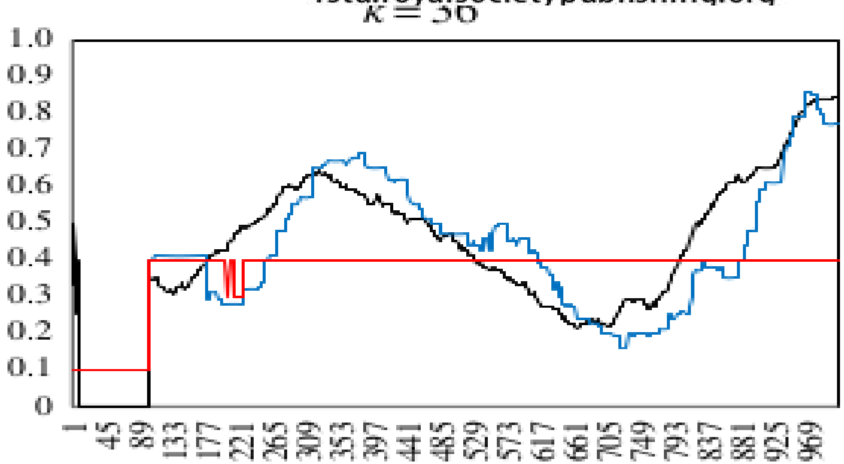
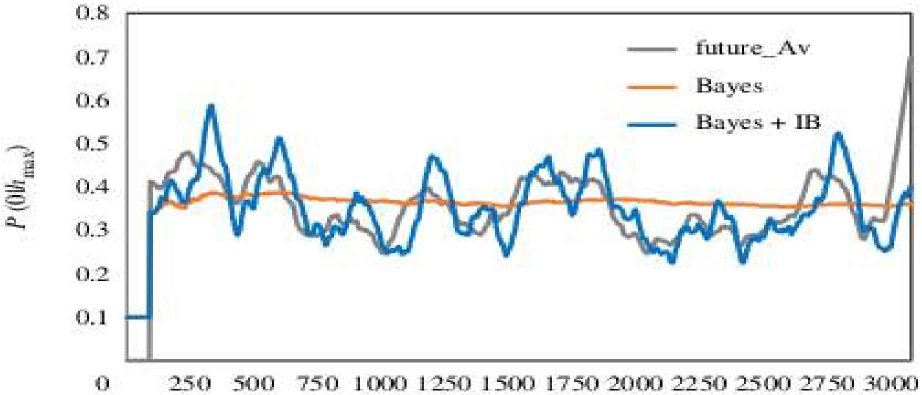
Inverse Bayesian inference in swarming behaviour of soldier crabs

Yukio-Pegio Gunji[1], Hisashi Murakami[2], Takenori Tomaru[3] and Vasileios Basios[4]

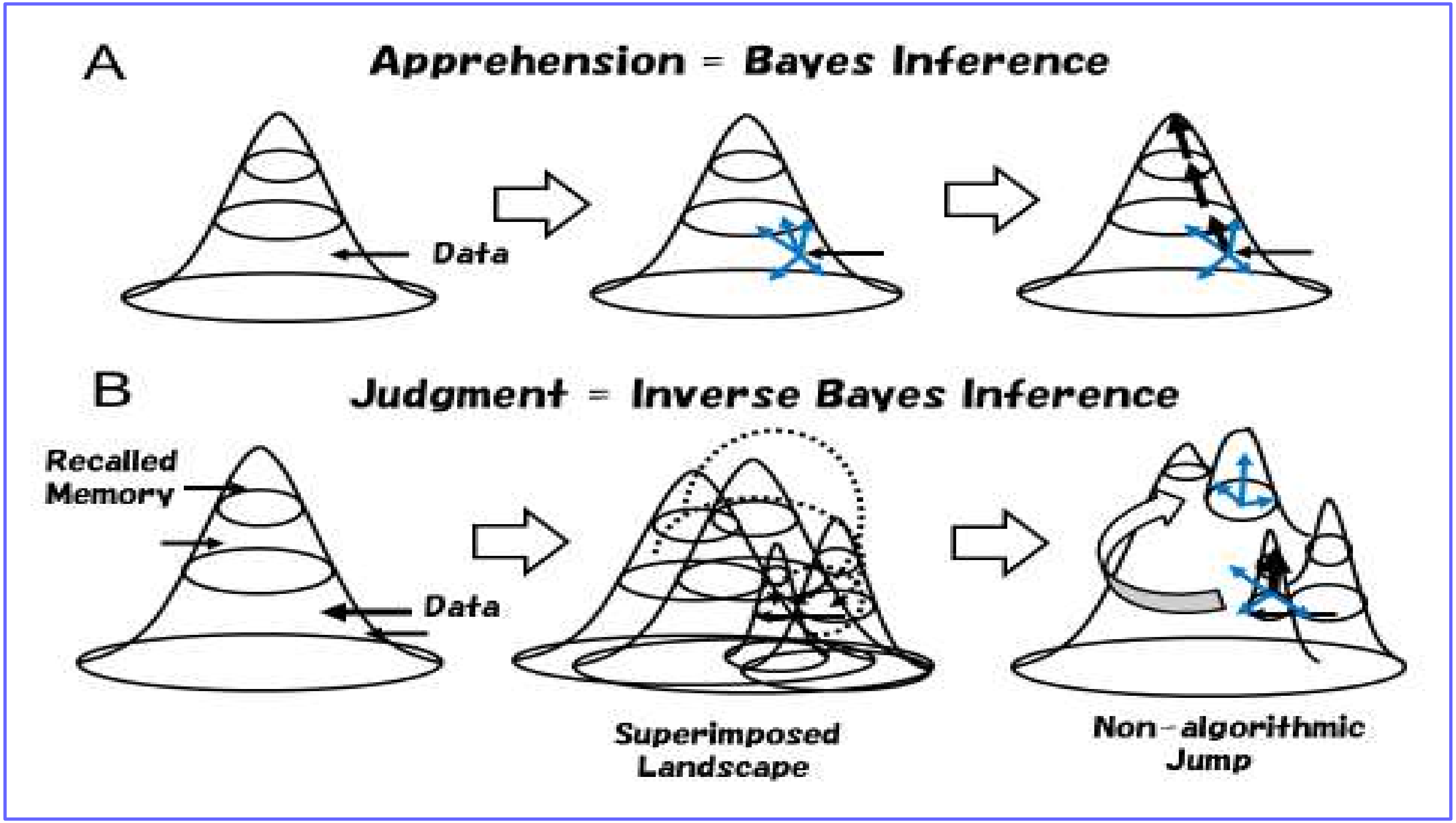




Scores of Prediction
of the next move

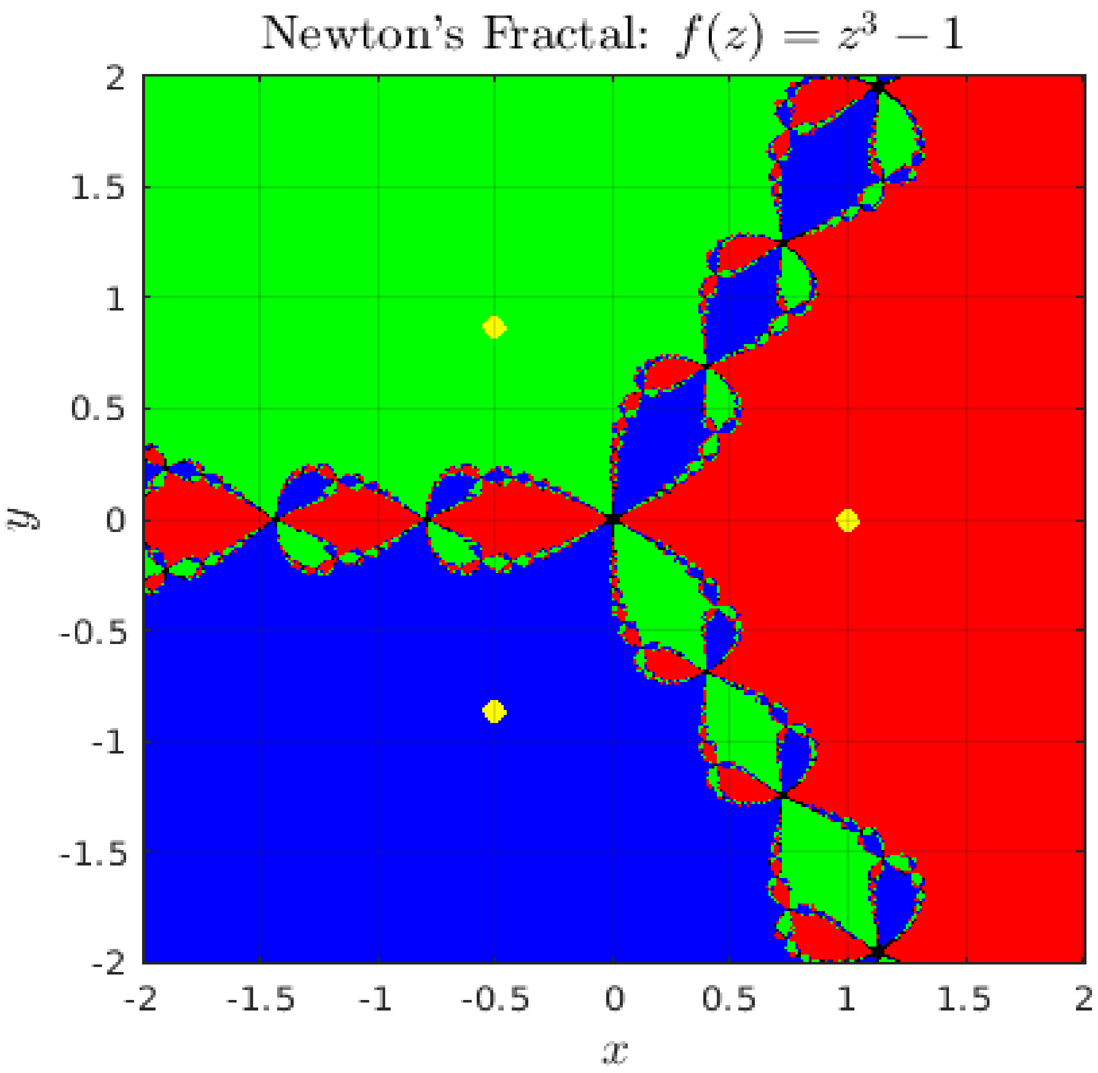
Bayesian
vs
Bayesian Inverse-Bayesian
inferences

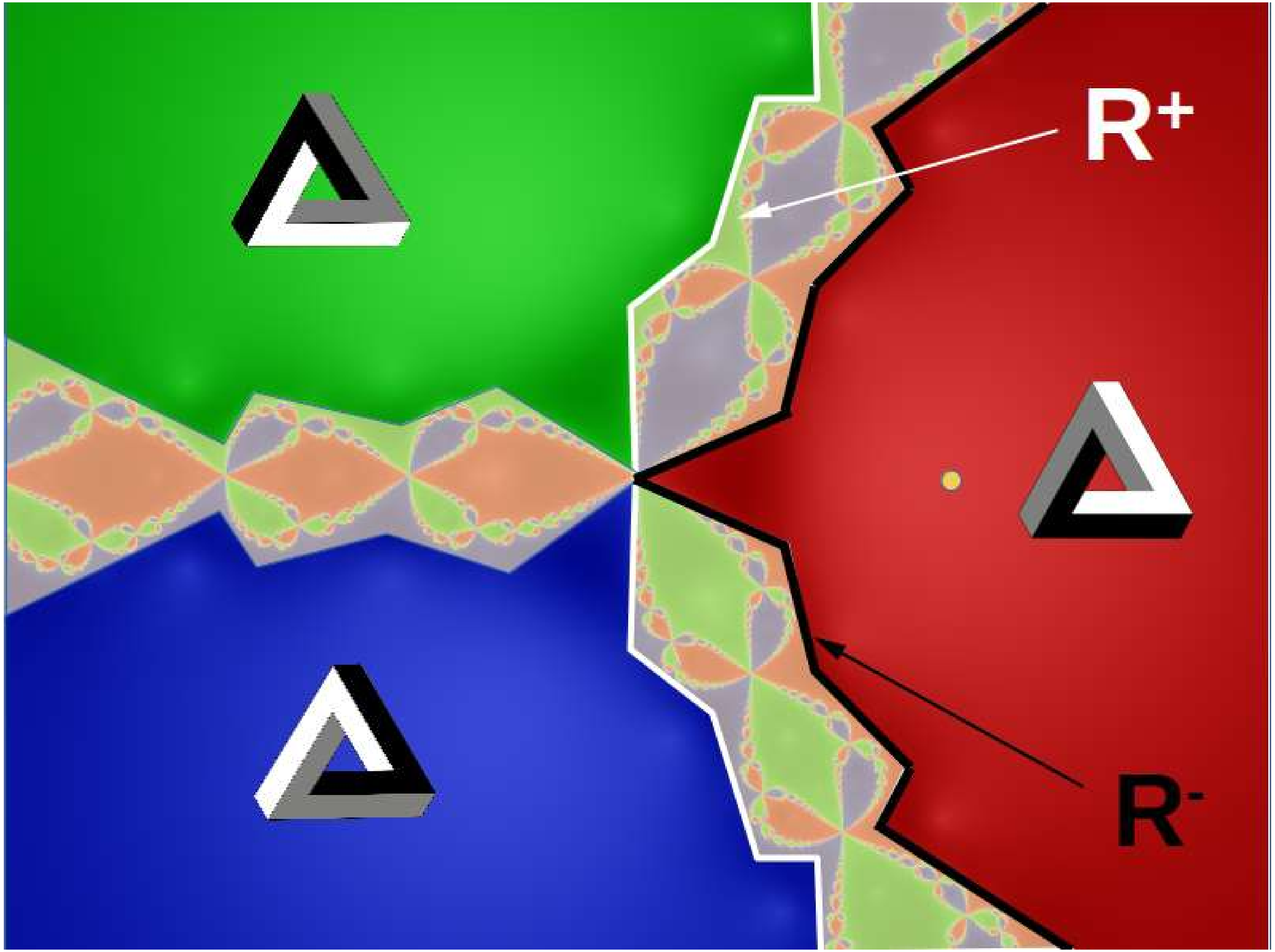
individual crab (up)
average of a collective (down)

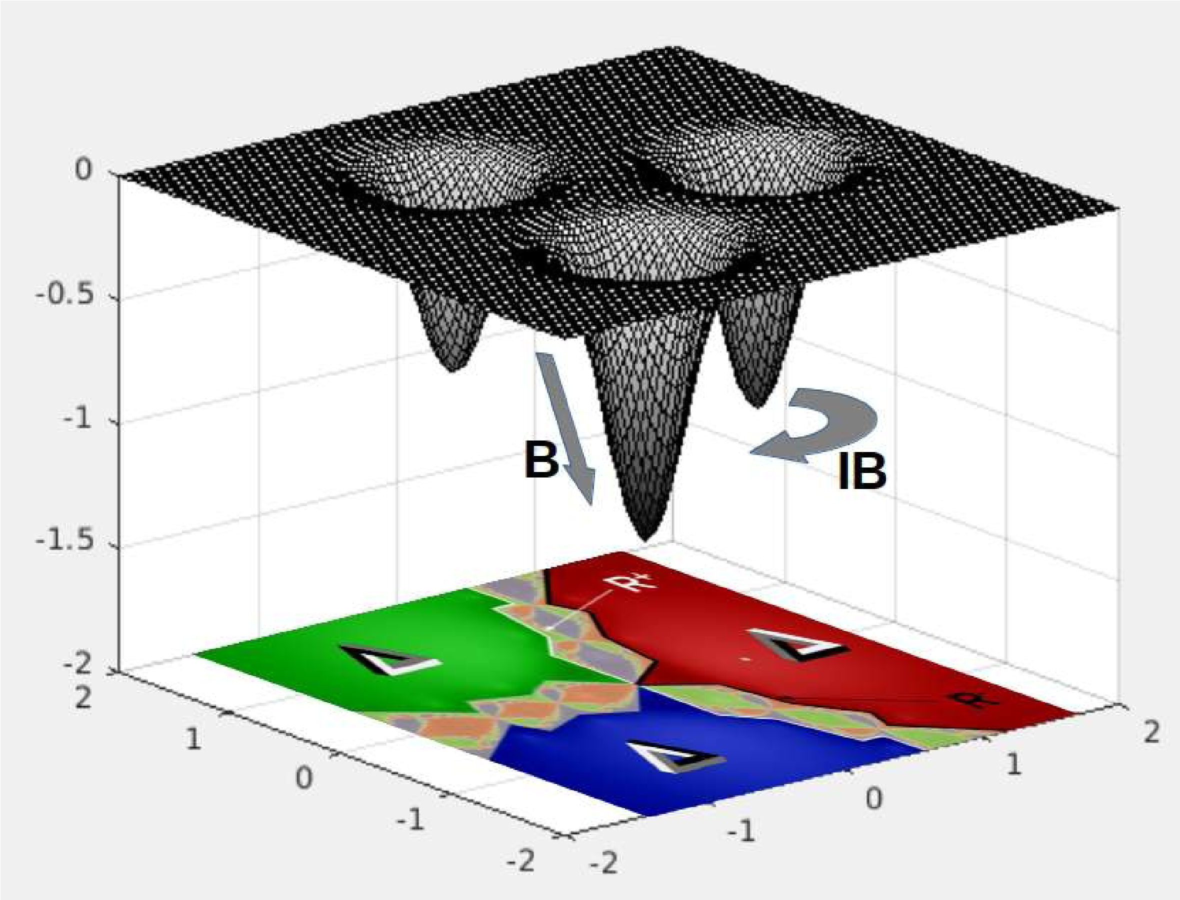

A
Apprehension = Bayes Inference
Data
B
Judgment = Inverse Bayes Inference
Recalled
Memory
Data
Superimposed
Landscape
Non-algorithmic
Jump

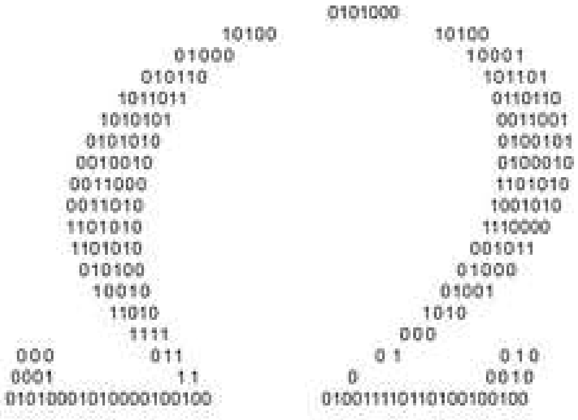
$$f(z) : z_{n+1} := z_n - \frac{f(z_n)}{f'(z_n)}$$

$z \in \mathbb{C}$




R+
R-


0
-0.5
-1
-1.5
-2
B
IB
2
1
0
-1
-2
-2
-1
0
1
2



Science & Technology FORESIGHT from society to research

National Research Council of Italy

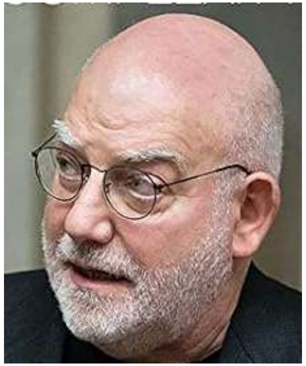
Interdisciplinary Studies

- Complex Systems
- Nonlinear Dynamics
- Quantum Cognition
- Data science (Biomemetics)

Yukio-Pegio Gunji
Waseda University

Pier-Francesco Moretti
CNR Roma HQ

Gregory Chaitin



We can use **randomness**
to expand our algorithmic
(computational) capabilities.

We can **imitate** natural information
processes (biomimesis).

We can use **Chaos** constructively.

We can propose and simulate
non-conventional computation
(Nnets, reservoir computing etc.).

We can base decision making &
perception on an **expanded logic**.

… to be continued

Thank You!

(for your patience & attention)

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