Hidden Attractors

New horizons in exploring dynamical systems

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Dynamical systems

Dynamical are called systems that evolve over time

Dynamical systems

Continuous dynamical systems

$$\frac{dx_i}{dt} = f_j(x_i, t), i, j = 1, 2, ..., n,$$

Dynamical systems

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Discrete dynamical systems

$$x_i^{k+1} = f_j(x_i^k),$$
$$i, j = 1, 2, ..., n \text{ and } k = 1, 2, ...$$

Discrete dynamical systems

$$
\begin{aligned}
x_i^{k+1} &= f_j(x_i^k), \\
i, j &= 1, 2, ..., n \text{ and } k = 1, 2, ...
\end{aligned}
$$

Dynamical systems

$$\frac{dx_i}{dt} = f_j(x_i, t), i, j = 1, 2, ..., n,$$

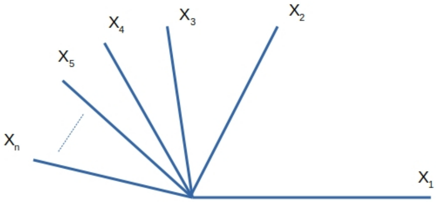


Figura 1: Phase space

Equilibrium points

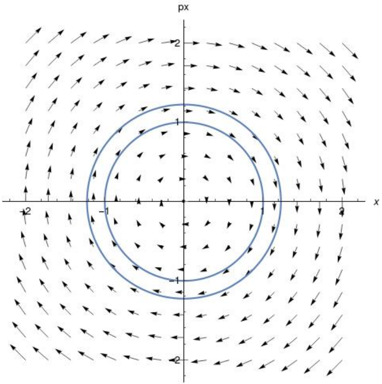


Figura 2: Phase space

Equilibrium points

Equilibrium points

$$\frac{dx_i}{dt} = 0$$

Equilibrium points

- Stable Equilibrium points
- Unstable Equilibrium points
- Non hyperbolic Equilibrium points

Equilibrium points

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, t) \Longrightarrow \frac{d\mathbf{x}}{dt} = \mathbf{Ax}$$

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ . & . & \cdots & . \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}$$

Attractors

What is an Attractor?

The phase space invariant subsets that attract and trap the trajectories on them

Attractors

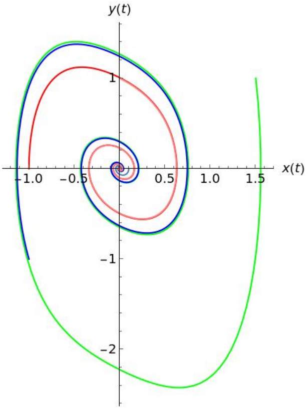


Figura 3: Stable Equilibrium point

Attractors

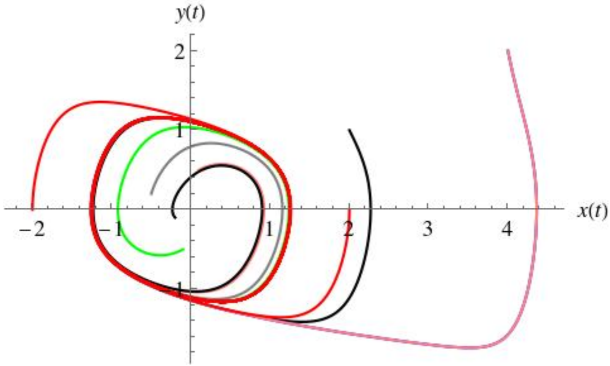


Figura 4: Stable limit cycle

Attractors

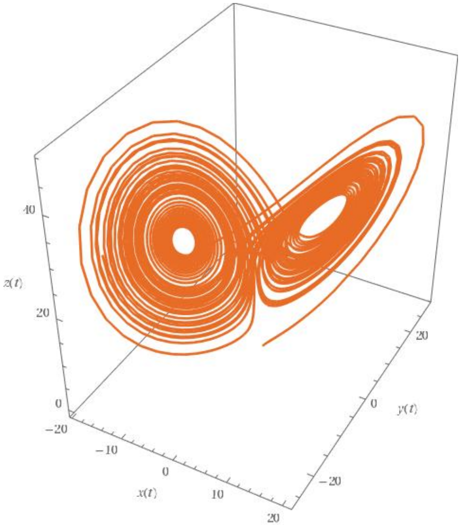


Figura 5: Chaotic attractor

Hidden Attractors

What are Hidden Attractors?

Hidden Attractors

Hidden are called the attractors whose basin of attraction does not intersect with small neighborhoods of the unstable equilibrium point, i.e., the basins of attraction of the hidden attractors do not touch unstable equilibrium points and are located far away from them.

- Hidden Attractors have been known since the 1960s when they were discovered in various nonlinear control systems.
- 2009: Leonov and Kuznetsov named this kind of Attractos as Hidden.
- 2010: Leonov and Kuznetzov discovered a chaotic Hidden Attractor in Chua's circuit.
- Since then, much work has been done.

Hidden Attractors

No equilibrium points

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= -x^3 - zy, \\ \dot{z} &= y^2 - A, \end{aligned}$$

where $A \in R$ is a constant parameter.

Hidden Attractors

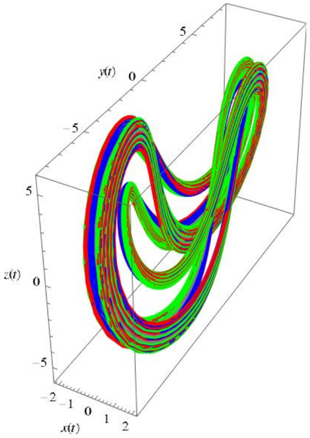


Figura 6: No equilibrium points

Hidden Attractors

One stable equilibrium point

$$
\begin{aligned}
\dot{x} &= yz + 0.006, \\
\dot{y} &= x^2 - y, \\
\dot{z} &= 1 - 4x.
\end{aligned}
$$

Hidden Attractors

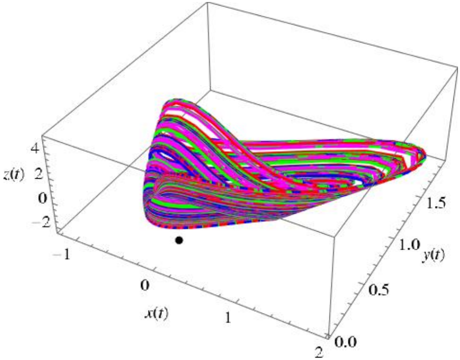


Figura 7: One stable equilibrium point

Hidden Attractors

Line of equilibrium points

$$\begin{aligned}\dot{x} &= y,\\ \dot{y} &= -x + yz,\\ \dot{z} &= -x - 15xy - xz.\end{aligned}$$

Hidden Attractors

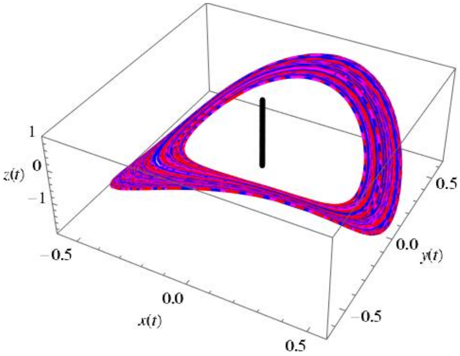


Figura 8: Line of equilibrium point

Hidden Attractors

Some remarks

1. Hidden Attractors might be regular or chaotic.
2. Small basins of attraction.
3. Difficulties in locating them.

The dynamics of a cubic nonlinear system with no equilibrium point

A modified version of the initial Sprott model with a cubic nonlinearity and a constant parameter A.

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= -x^3 - zy, \\ \dot{z} &= y^2 - A, \end{aligned}$$

Maaita, J. O., Volos, C. K., Kyprianidis, I. M., and Stouboulos, I. N. (2015). The dynamics of a cubic nonlinear system with no equilibrium point. Journal of Nonlinear Dynamics, 2015.

The dynamics of a cubic nonlinear system with no equilibrium point

Bifurcation diagram

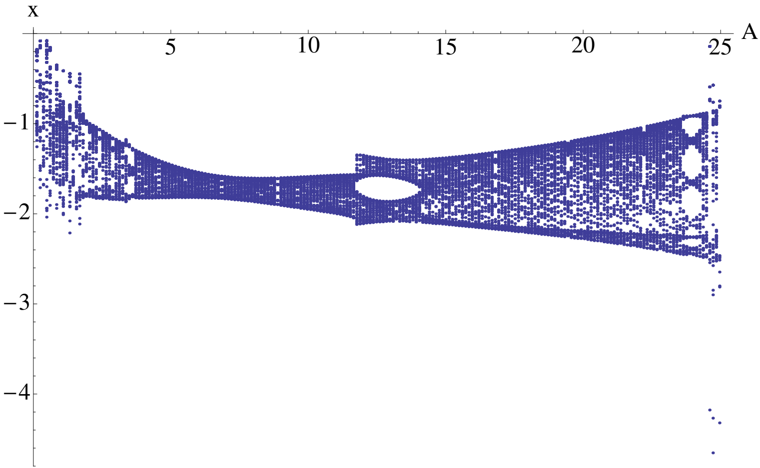


Figura 9: Bifurcation Diagrams for $x_0 = 1.5$, $y_0 = 2.0$, $z_0 = 1.5$

The dynamics of a cubic nonlinear system with no equilibrium point

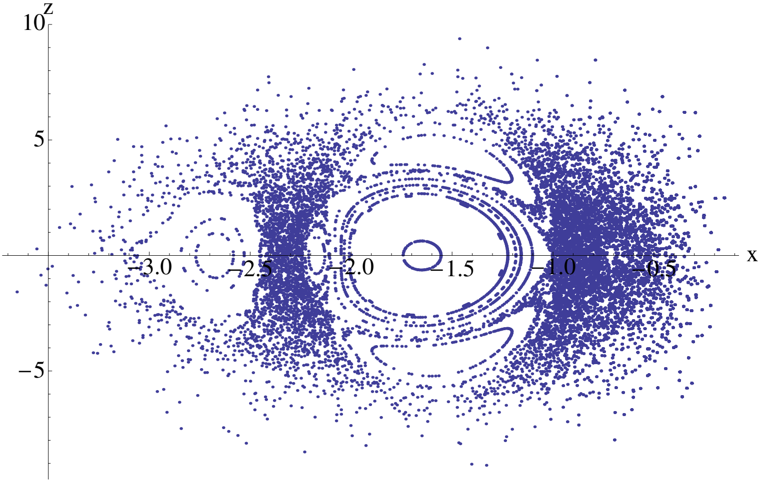


Figura 10: Poincaré section for $A = 5.16$

The dynamics of a cubic nonlinear system with no equilibrium point

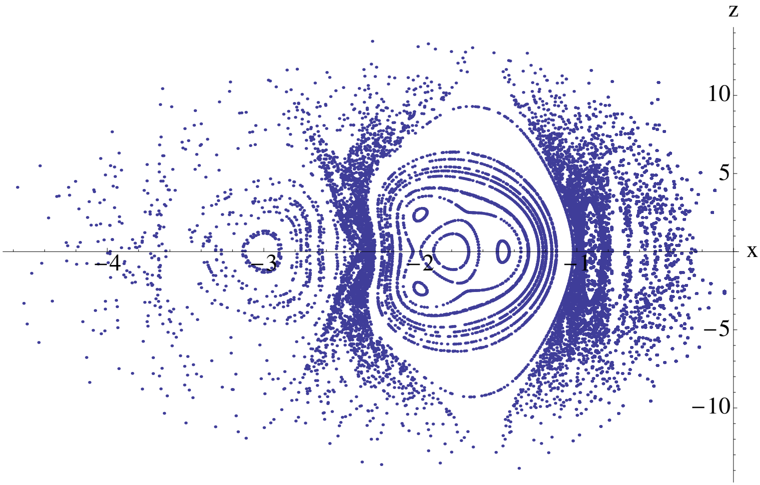


Figura 11: Poincaré section for $A = 12.7$

The dynamics of a cubic nonlinear system with no equilibrium point

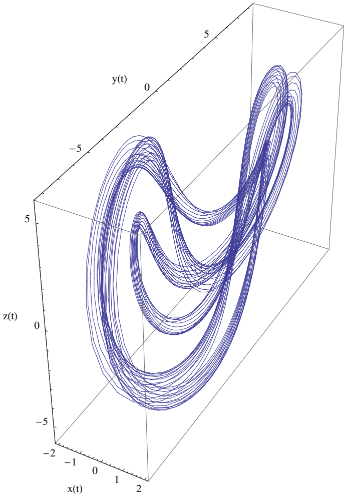


Figura 12: Trajectories for $A = 12.7$ and initial conditions: $x_0 = -1.4$, $y_0 = 0.0$, $z_0 = 0.0$.

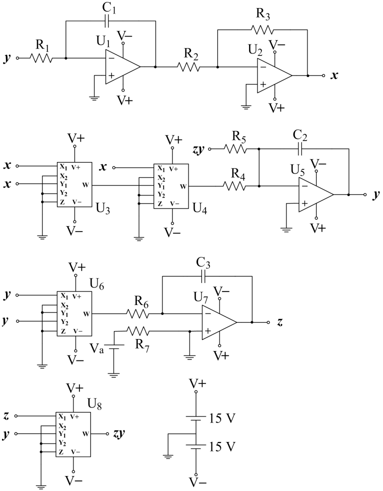
Applications

Are there any Applications?

Applications

- Electromechanical system without equilibria.
- Electromechanical model of the drilling system.
- Lorenz type systems.
- Chua's circuit.
- Circuit implementations.

Circuit implementation



Closing remarks

- Hidden Attractors- Not intersect with unstable equilibrium point.
- Hidden attractors often have small basins of attractions, are strongly chaotic, and have complex dynamics.
- New systems are being proposed and studied numerically and experimentally.
- Nonlinear electrical circuits are a "laboratory"where many dynamical systems with hidden attractors have been tested. Also, it opens a window for new applications.

Closing remarks

Certainly, Hidden attractors open new horizons in exploring and applying dynamical systems. The work that has been done until today is a good base that allows us to take the next steps: Theoretically describe those complex phenomena, derive the analytical and numerical tools, and develop new applications.

Bibliography

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