Pink Noise is the Canonical Representation of Environmental Variability

John M. Halley
Professor of Ecology
Dept. Biological Applications & Technology
University of Ioannina
Greece

jhalley@uoi.gr

University of Ioannina

Department of Biological Applications and Technology

BET EcoLab

Ecological Models

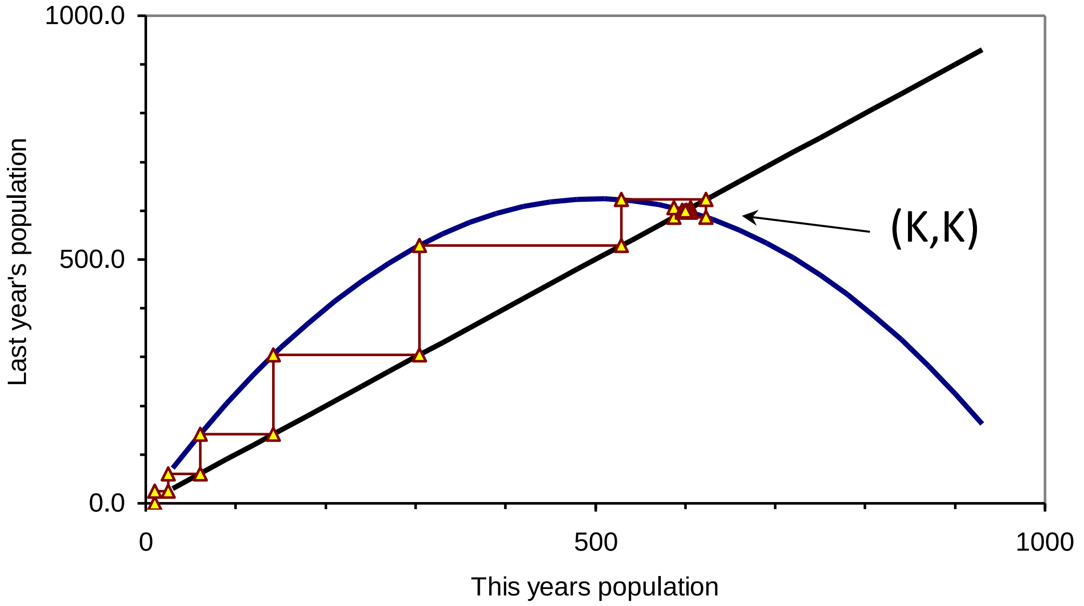
Classical Population Dynamics

A popular approach begins with the ***Logistic Equation***, a simple NL difference Eq.

$$X_{t+1} = \lambda X_t \left[1 - \frac{X_t}{K} \right]$$

'Replacement rate'

'Carrying capacity'



The evolution of Logistic Eq. sequence of population values.

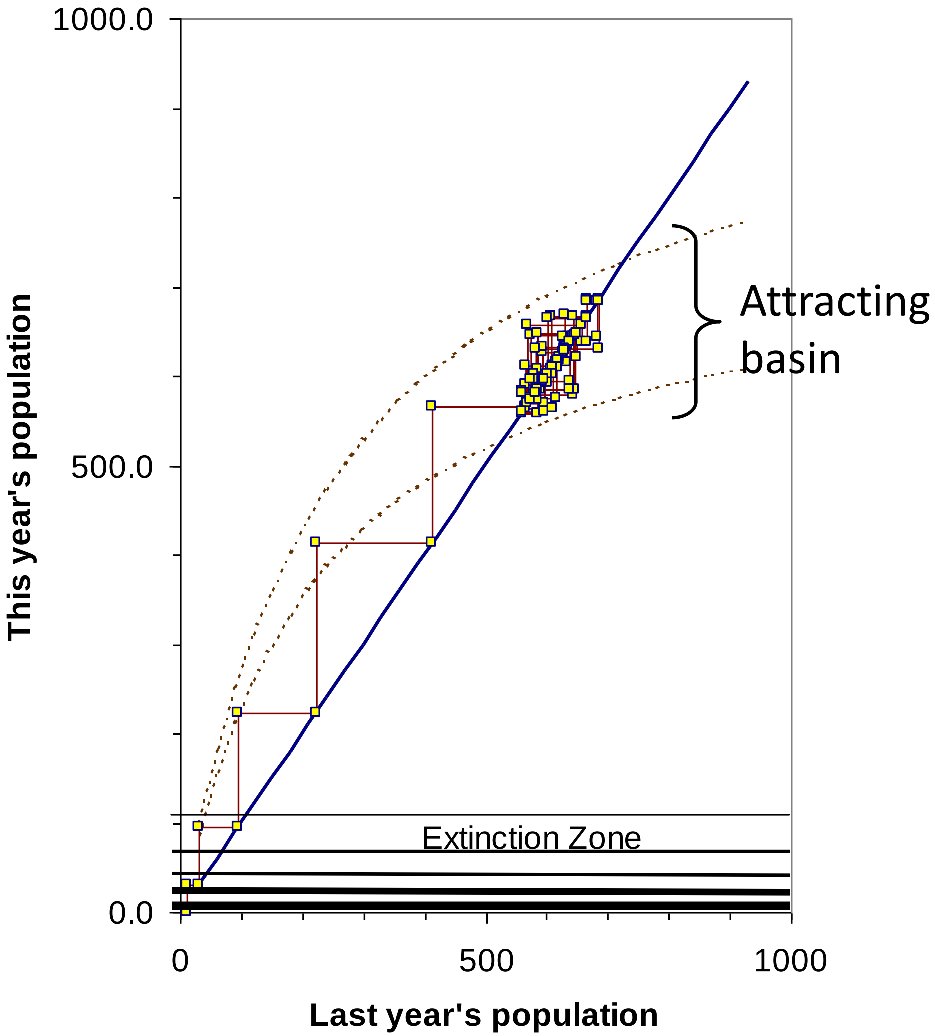
$$X_0 \rightarrow X_1 \rightarrow X_2 \rightarrow X_3 \rightarrow ....$$

Ecological Models

- "The evidence shows that reputable scientists feel ecology to be less credible and weak or increasingly fractious…" R.H.Peters *A Critique for Ecology (1991)* p.6.
- "Despite the weaknesses of the Logistic, theoretical ecology did not abandon the approach, but developed it." *ibid. p.56.*
- "Instead, we should develop simple predictive tools that allow us to propose and confirm observable patterns that are relevant to the biological world…" *ibid..* p.109.
- " there are mathematicians whose idea of collaboration in 'applied' biological research is to spend just enough time with the biologist to be able to write down a set of probability equations which will keep himself amused for the the next few months!" E.Renshaw, *Modelling Biological Populations in Space and Time*, p1.

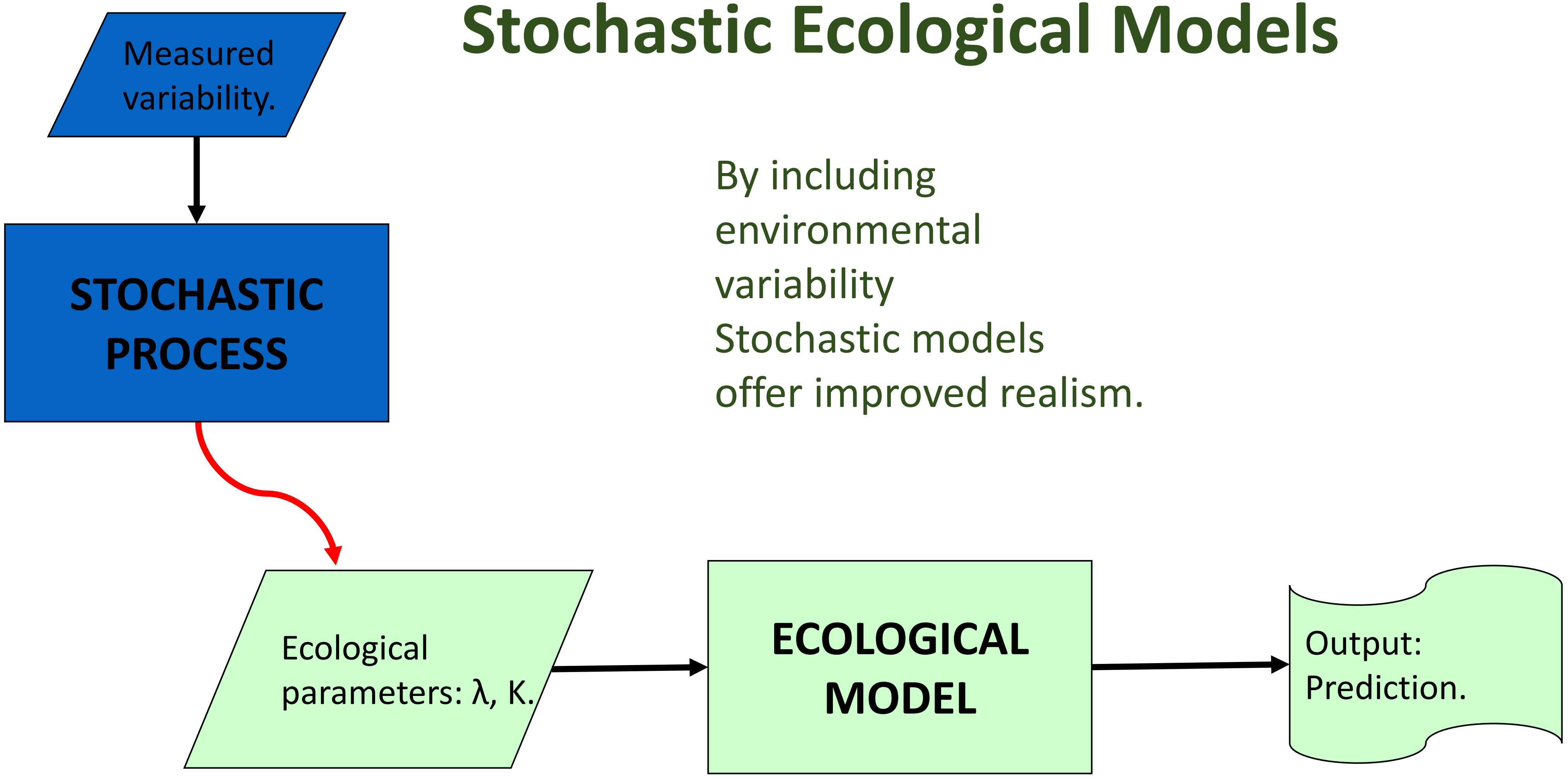
Ecology tends to lack predictive power because ecological interactions are complex and subject to large stochastic (random) influences.

Ecological Models in a Noisy Environment



Ecology tends to lack predictive power because ecological interactions are complex and subject to large stochastic (random) influences.

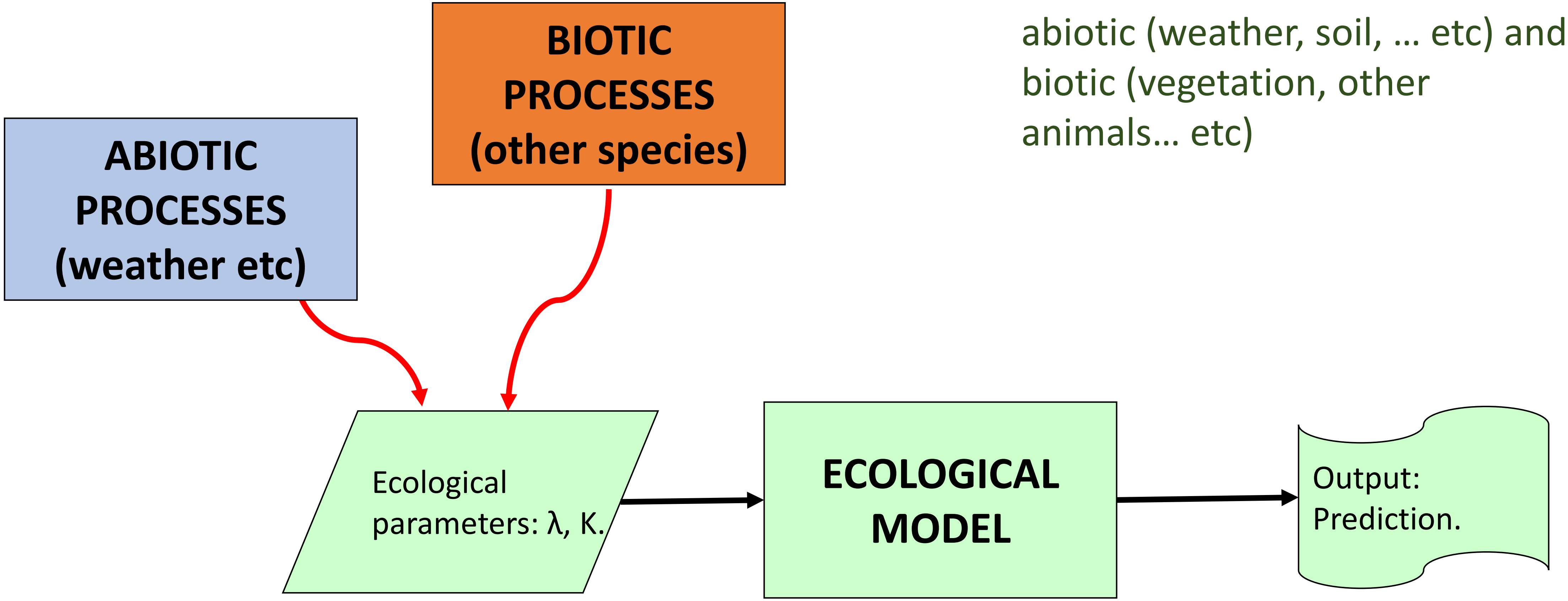
Stochastic Ecological Models



By including environmental variability Stochastic models offer improved realism.

But what kind of Stochastic Model??

Stochastic Ecological Models



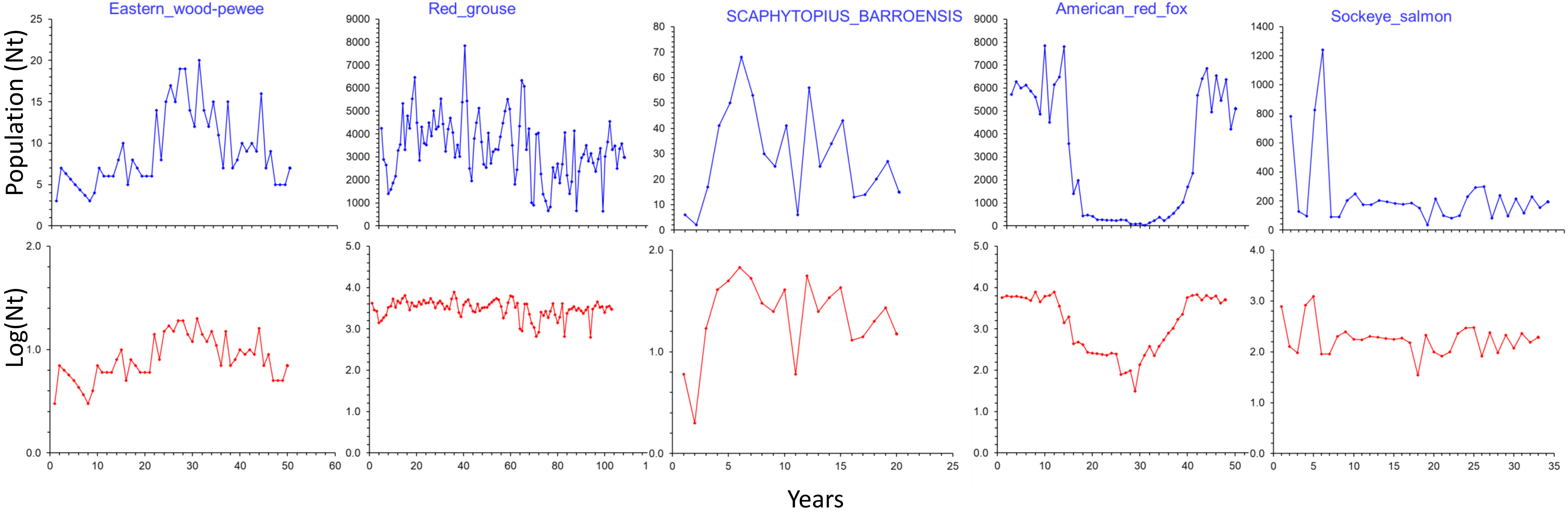
There are two types of environmental fluctuation: abiotic (weather, soil, … etc) and biotic (vegetation, other animals… etc)

Interactions with other species

Other species may be prey, predators, parasites, competitors … So, how do they fluctuate?







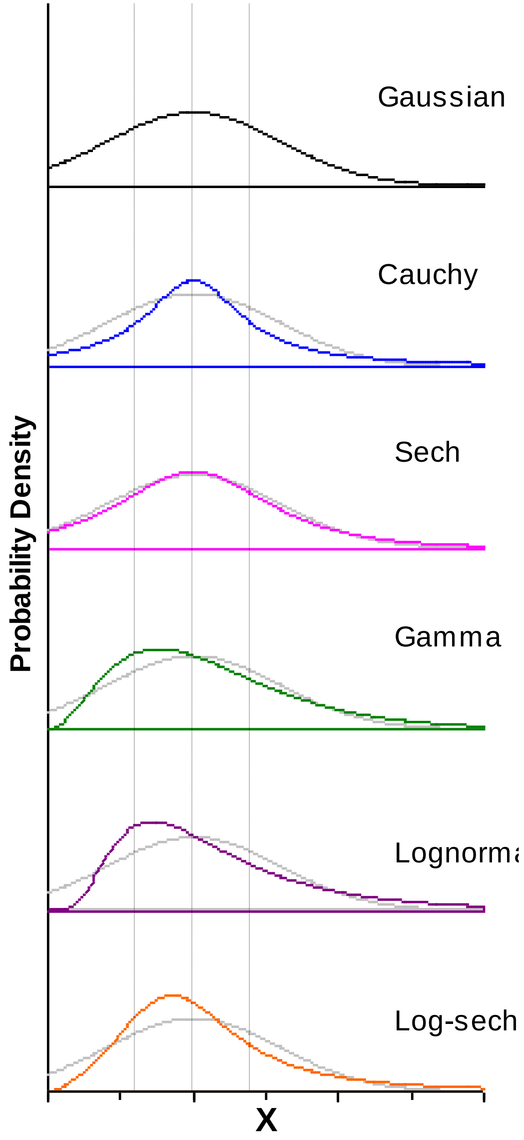
Time series

Much of our data comes as time series.

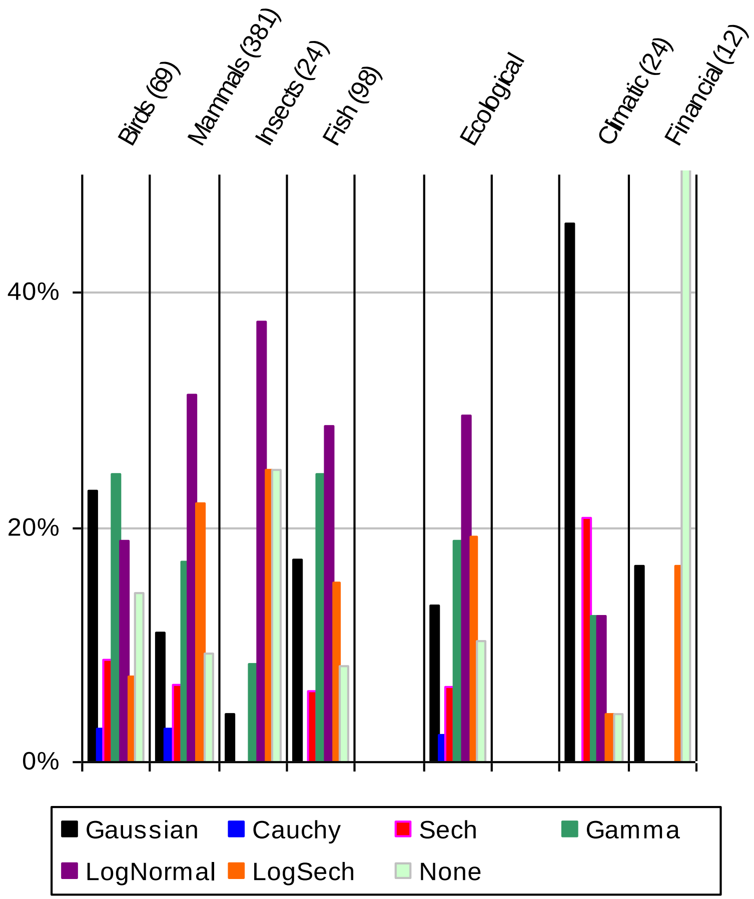
Two questions asked by statisticians:

1. Is the distribution Gaussian?
2. Are the values autocorrelated in time?

KS Tests for Real Ecological Time-Series distribution



1. The lognormal seems to be the "best" model for most ecological series
2. Normal is best for most climate series
3. Levy-stable (heavy-tailed) distributions have many applications in economics



Time series

Much of our data comes as time series.

Two questions asked by statisticians:

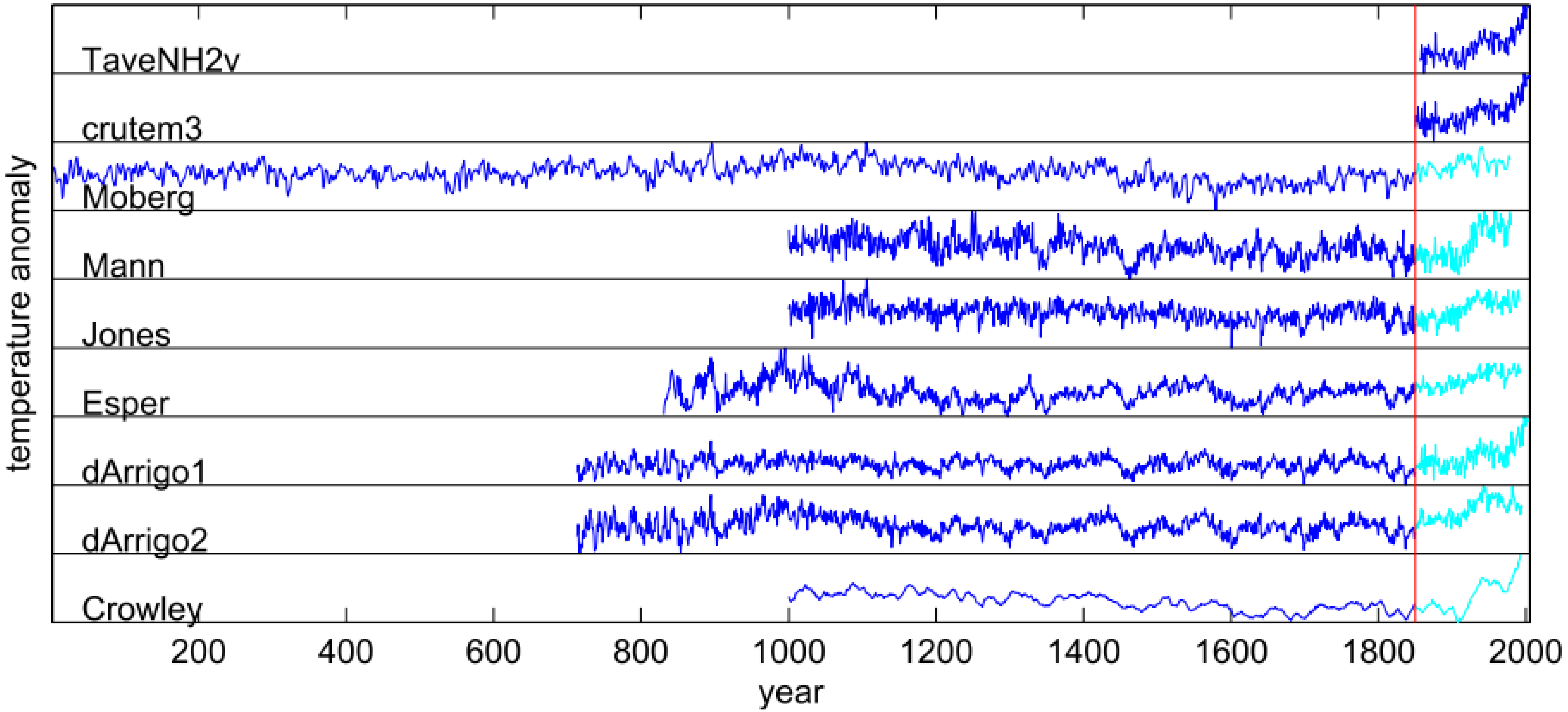
1. Is the distribution Gaussian? ✔ **(more or less !)**
2. Are the values autocorrelated in time?

Time series from Climatic Databases

1. There are many such databases (CRU, GISS, …)
2. Here I use mostly reconstructions of past temperatures
3. Temperatures, rainfall etc.

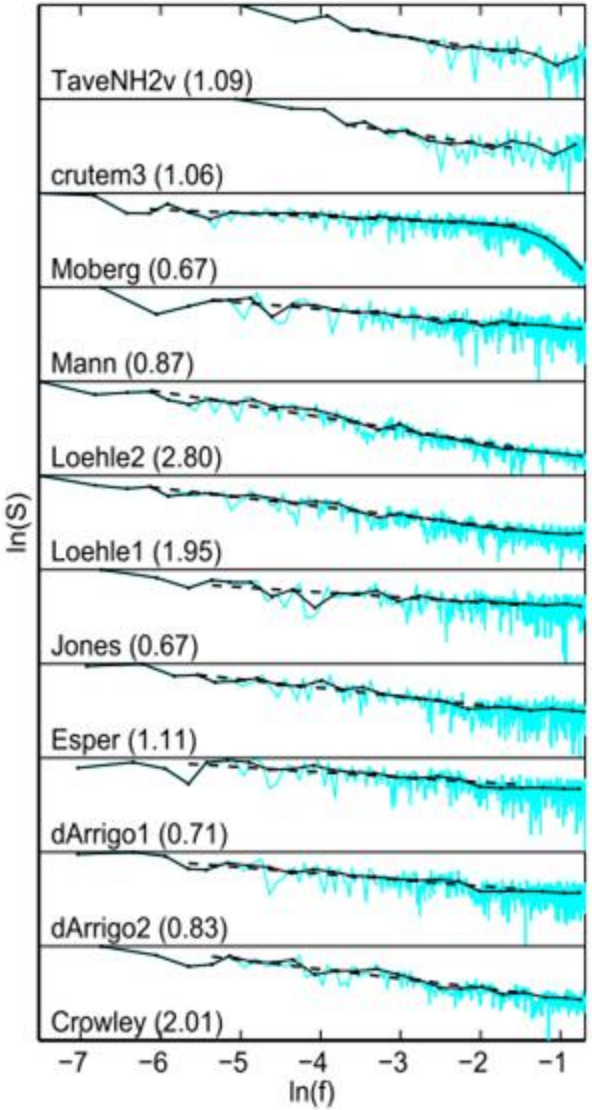
Reconstructions of Earth's Temperature

(Paleo-climate Reconstructions. Reconstructions of temperature using tree-rings, ice-cores etc.)



Halley J.M. (2009) Using models with long-term persistence to interpret the rapid increase of Earth's temperature. *Physica A* (388, 2492-2502)

Power spectra of reconstructed global temperature



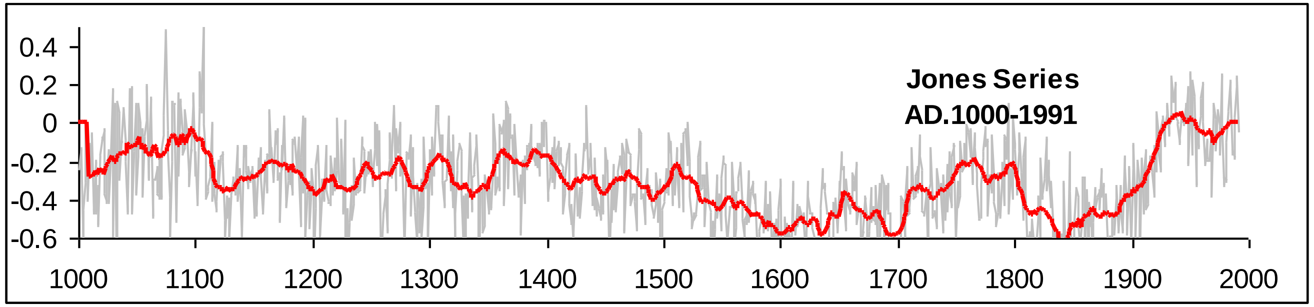
Log-log plot of power spectrum vs frequency for the 9 reconstructed time series and the two modern time series of global temperature, one in each panel.

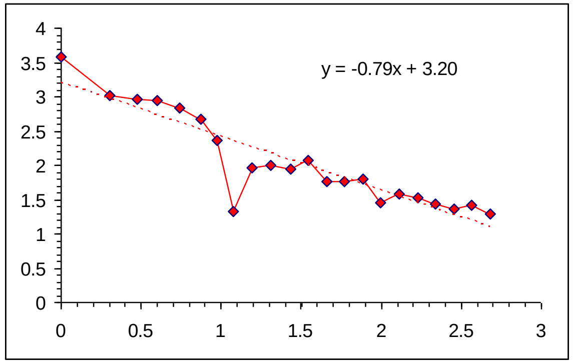
- Raw periodogram
- Averaged periodogram
- linear fit (slope ν is given in parenthesis)

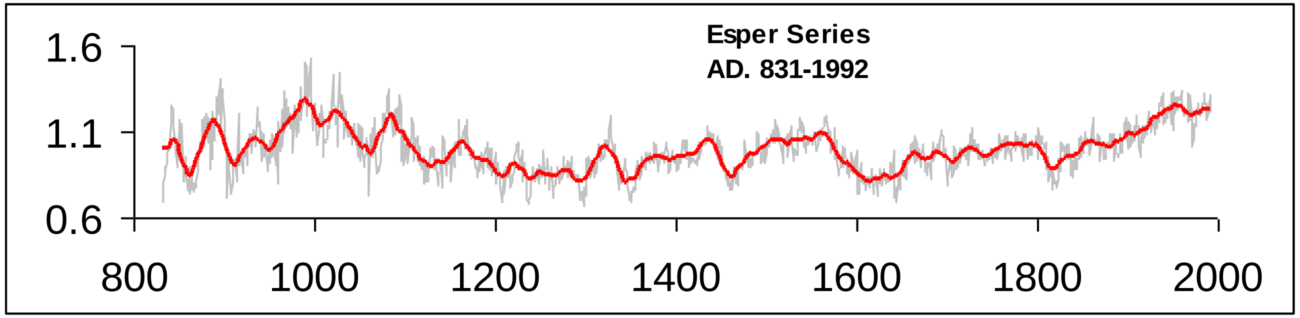
Most of the series fit well the form of $1/f^{\nu}$ –noise:

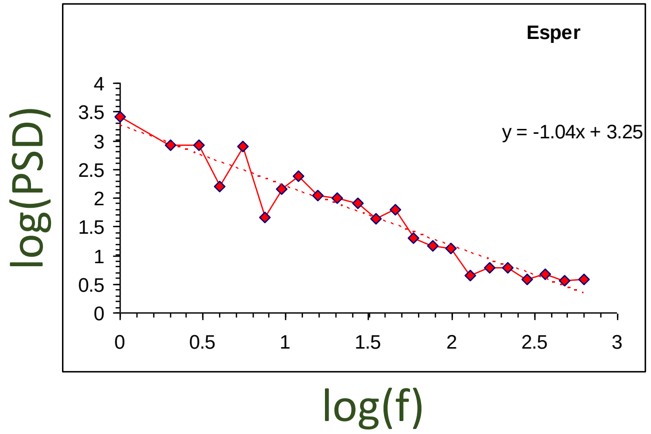
$$S(f) \propto \frac{1}{f^{\nu}}$$

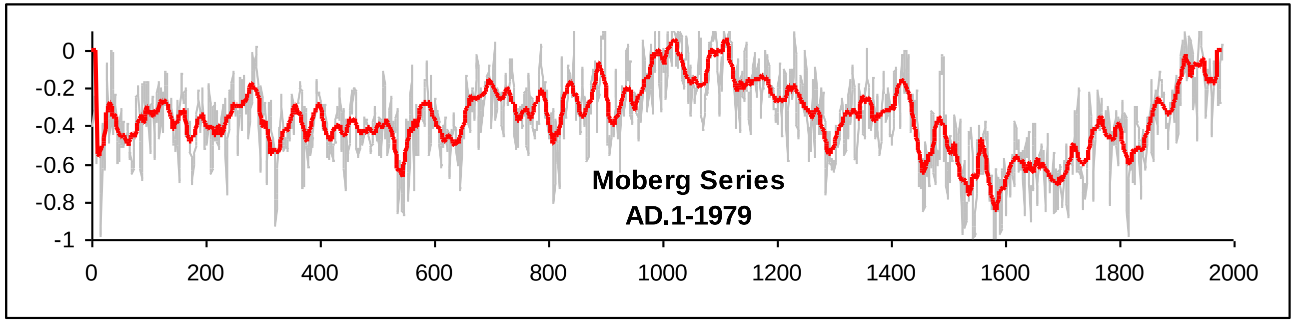
Temperature reconstructions in time and their PSD

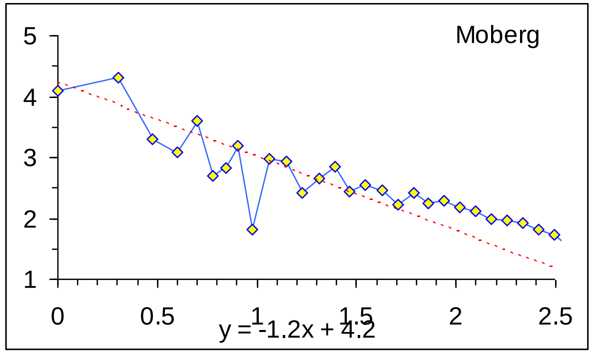












$1/f^{\nu}$-noises

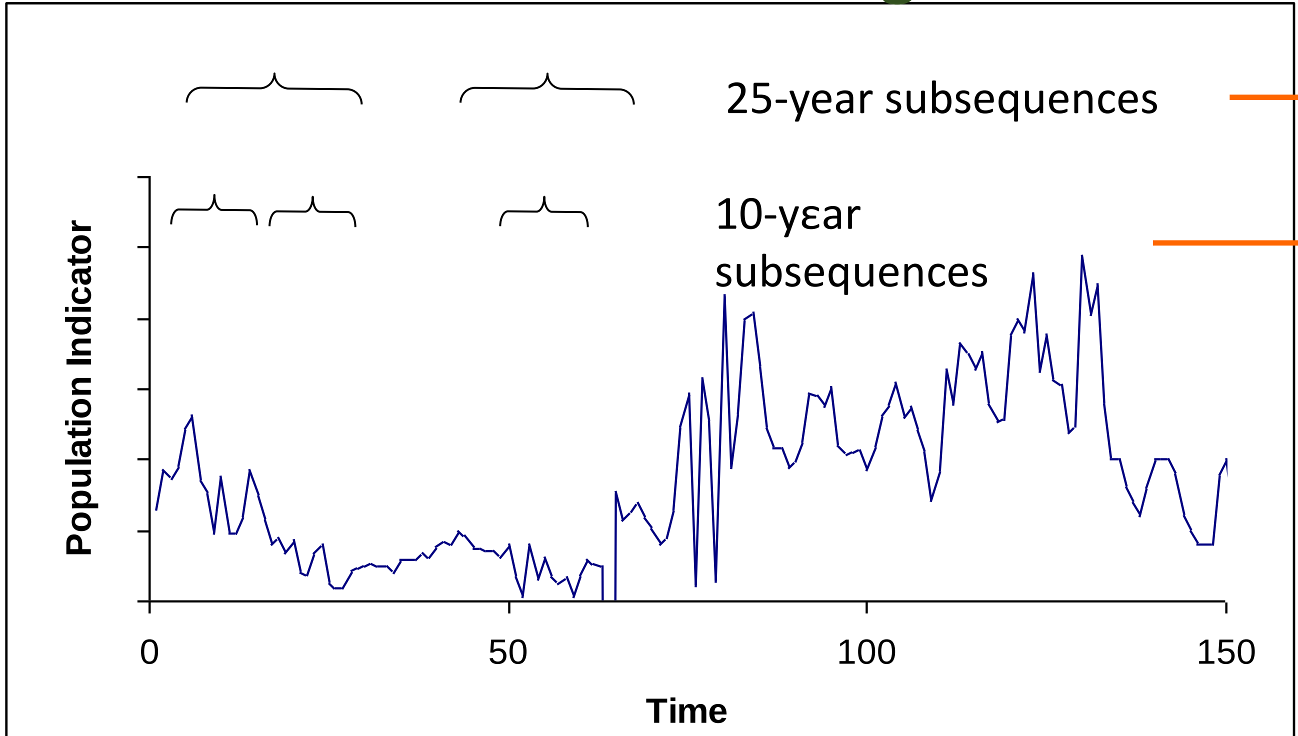
- **$1/f^{\nu}$-noises with ν~1 have been observed in many phenomena.**
  - Electronic circuits
  - Geophysical time series
  - Music
  - Landscape structure
  - DNA base-sequences
  - Ecological abundance
  - …. and many more!

- **"Canonical" members are white (ν=0), brown (ν=2) and pink (ν=1)**

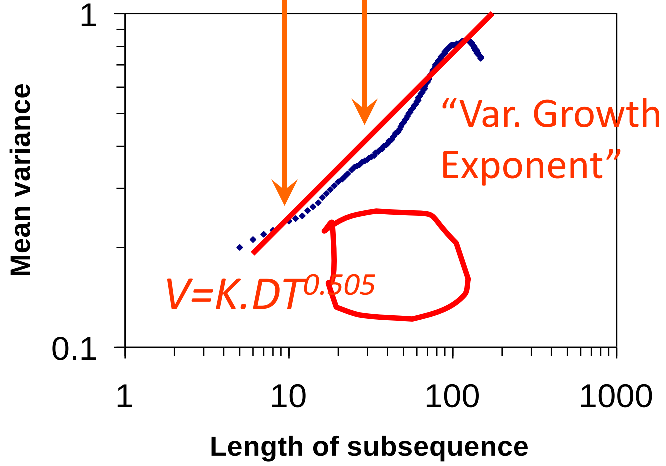
Time series from Ecological Databases

1. There are now many such databases (GBIF, BioTime, iNaturalist, eBird, …)
2. Examples here are from GPDD population sizes

Measuring Variance Growth

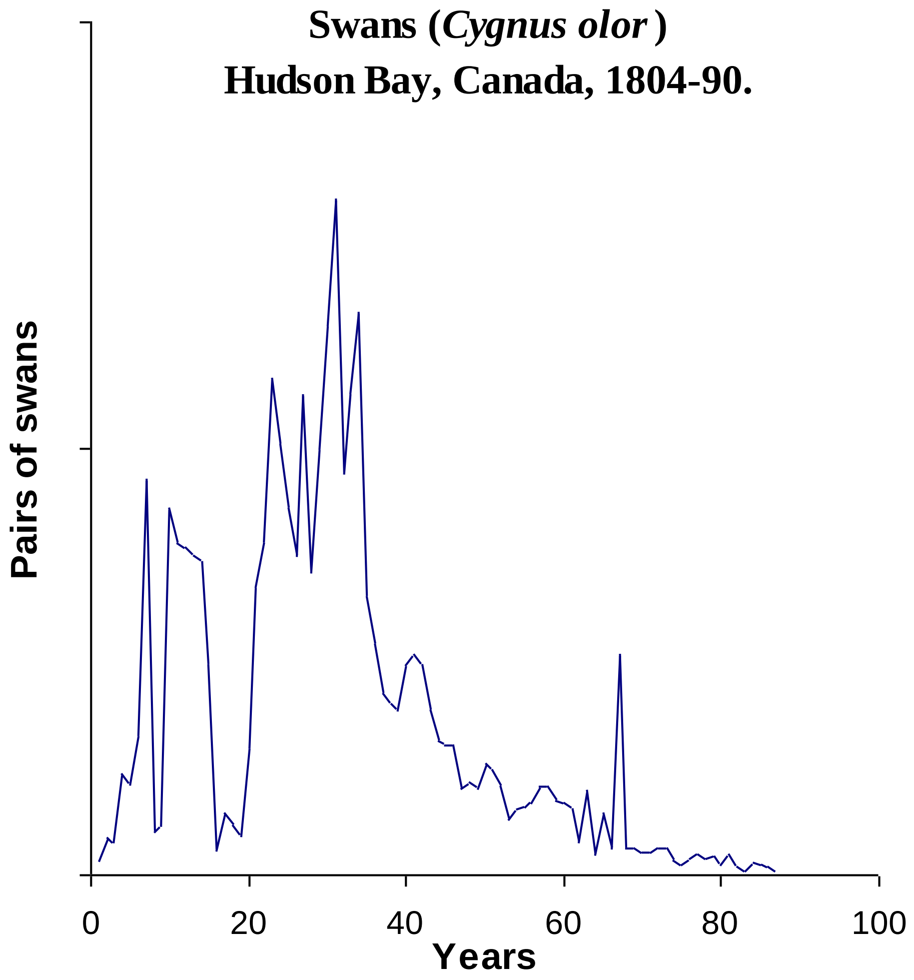


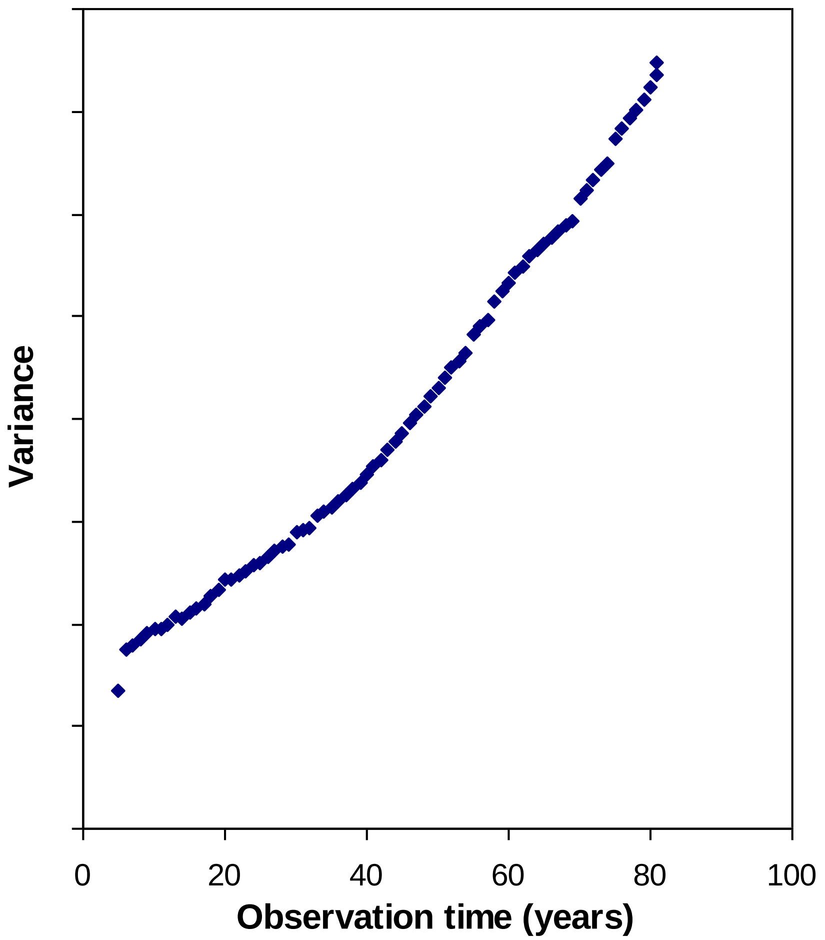
Wolverine (Gulo gulo) Hudson Bay, 1752-1908

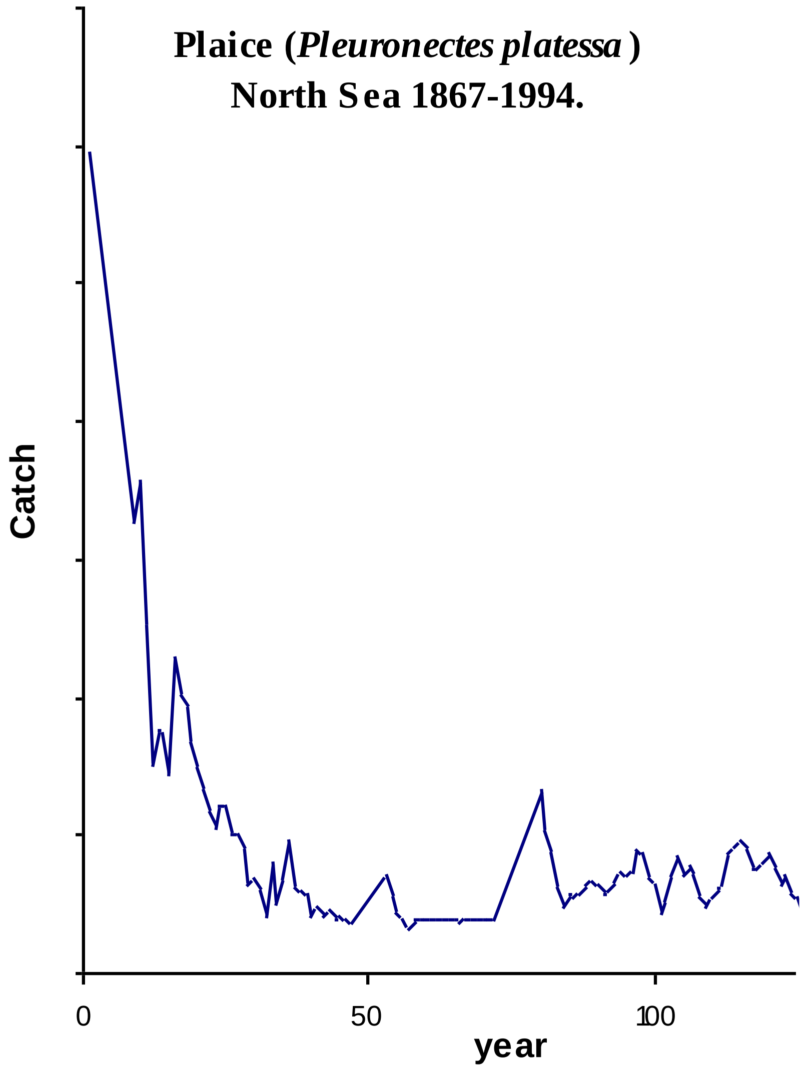


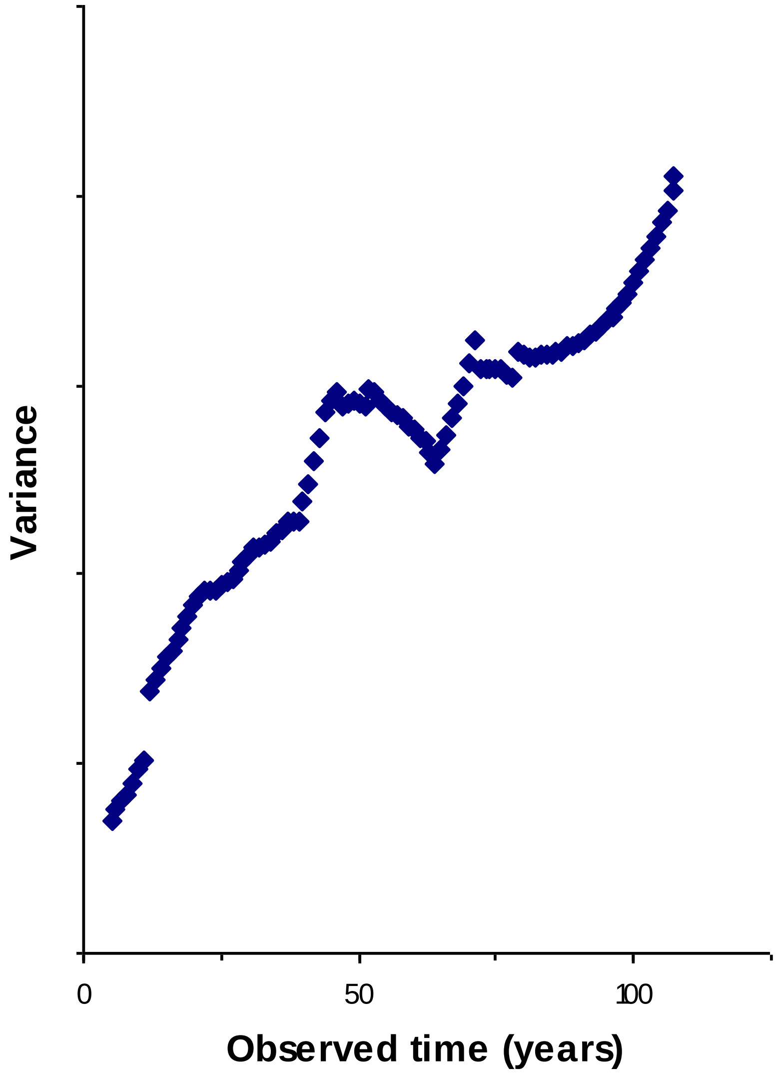
Environmental Noise

Ecological Factors (e.g. other populations) will not be Stationary









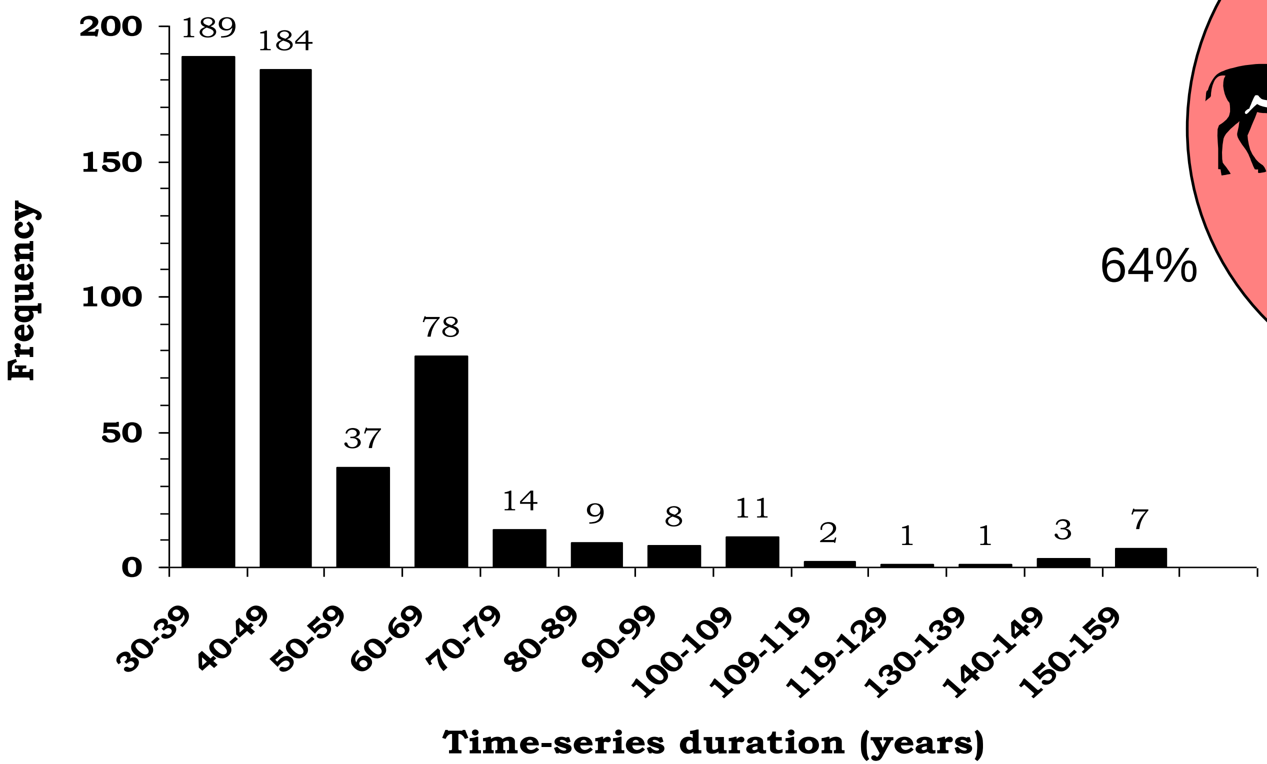
Πηγή : Global population-dynamics database, Imperial College London.
http://cpbnts1.bio.ic.ac.uk/gpdd/
Inchausti & Halley (2002), *Evol.Ecol. Res.*, **4**, p1-16

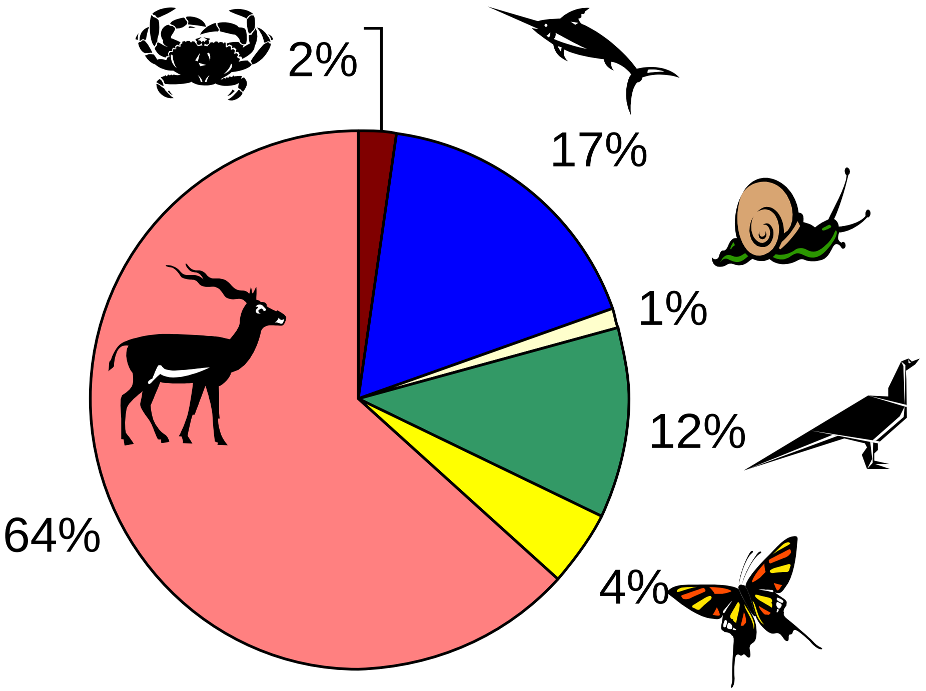
Data: 544 Time-series

•Source : Global population-dynamics database, Imperial College London.

•http://cpbnts1.bio.ic.ac.uk/gpdd/

•Inchausti & Halley (2002), *Evol.Ecol. Res.*, 4, p.1





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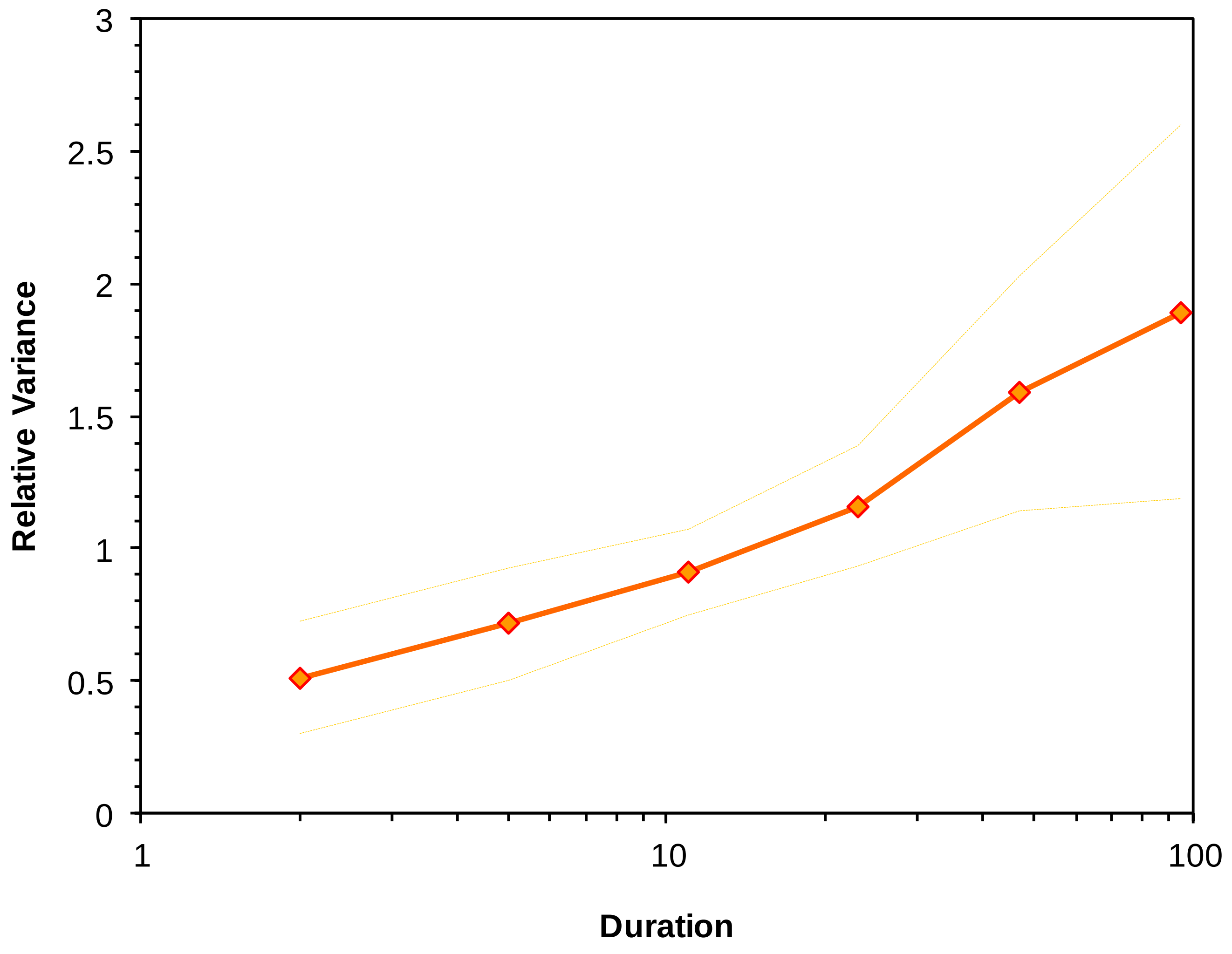
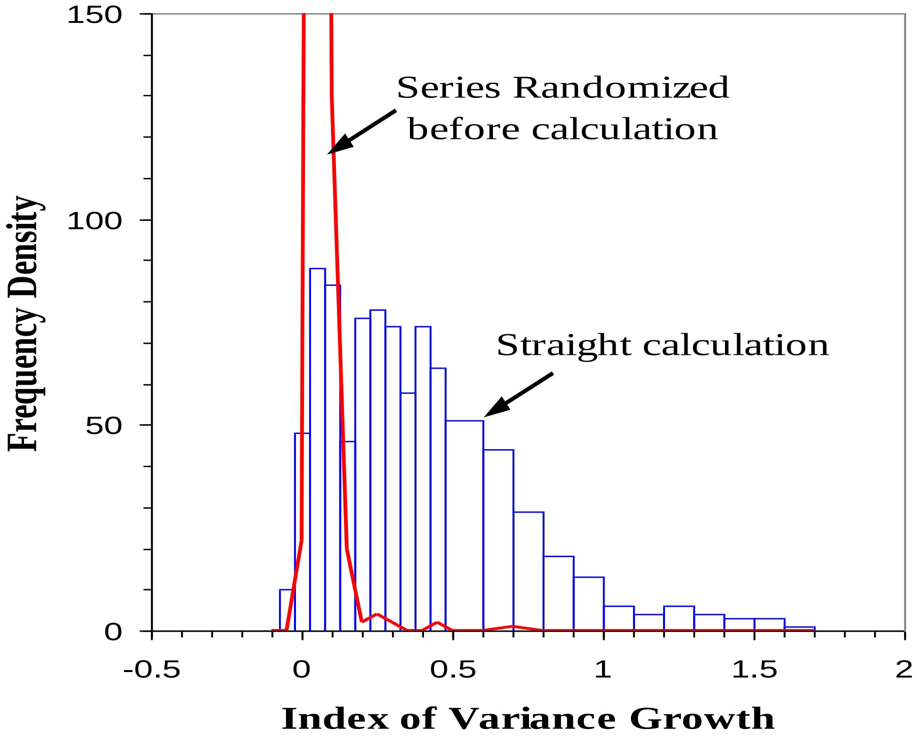
Variability increases with observation time for the 544 time-series
Variance increases approximately linearly with log of observation time



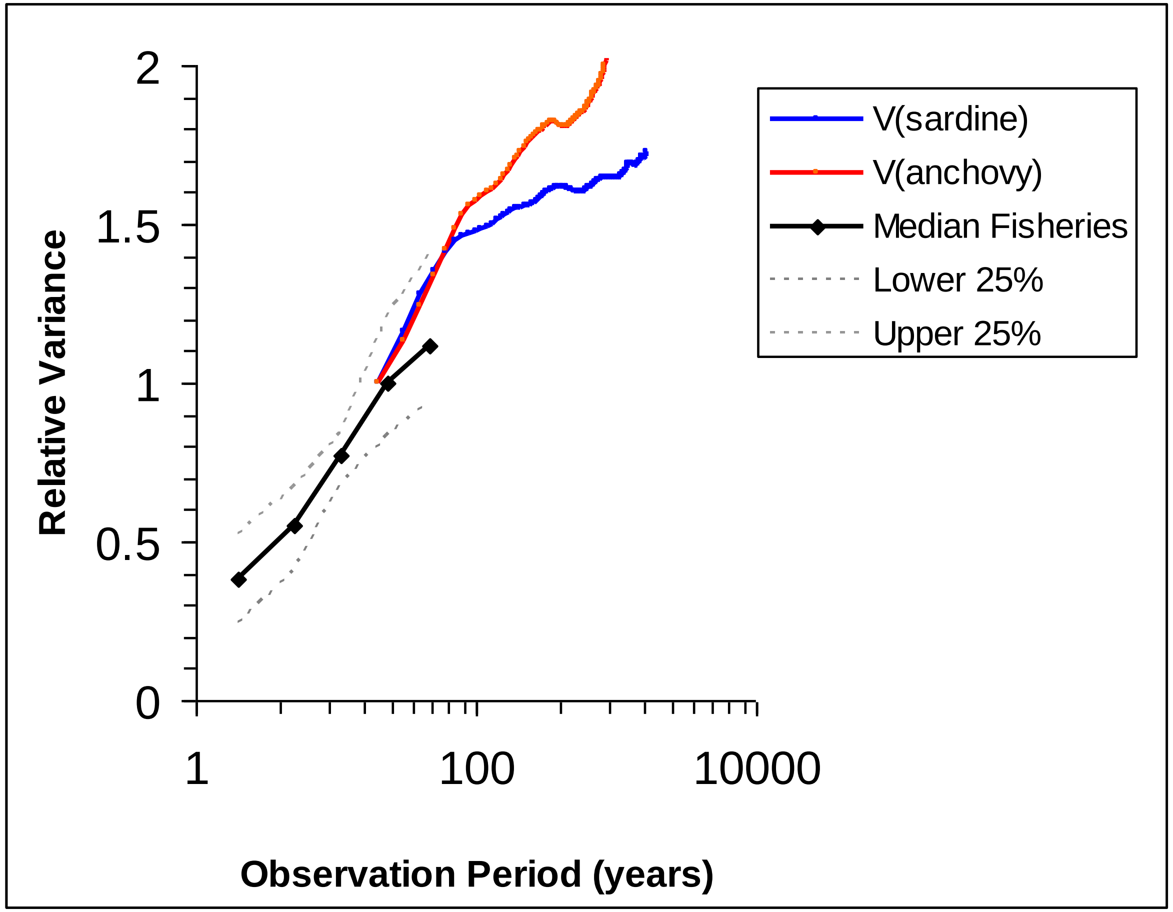
Variance Growth = Memory of the Past.

If we randomize observations (i.e. destroy all memory of past)
H-exponents collapse to zero.



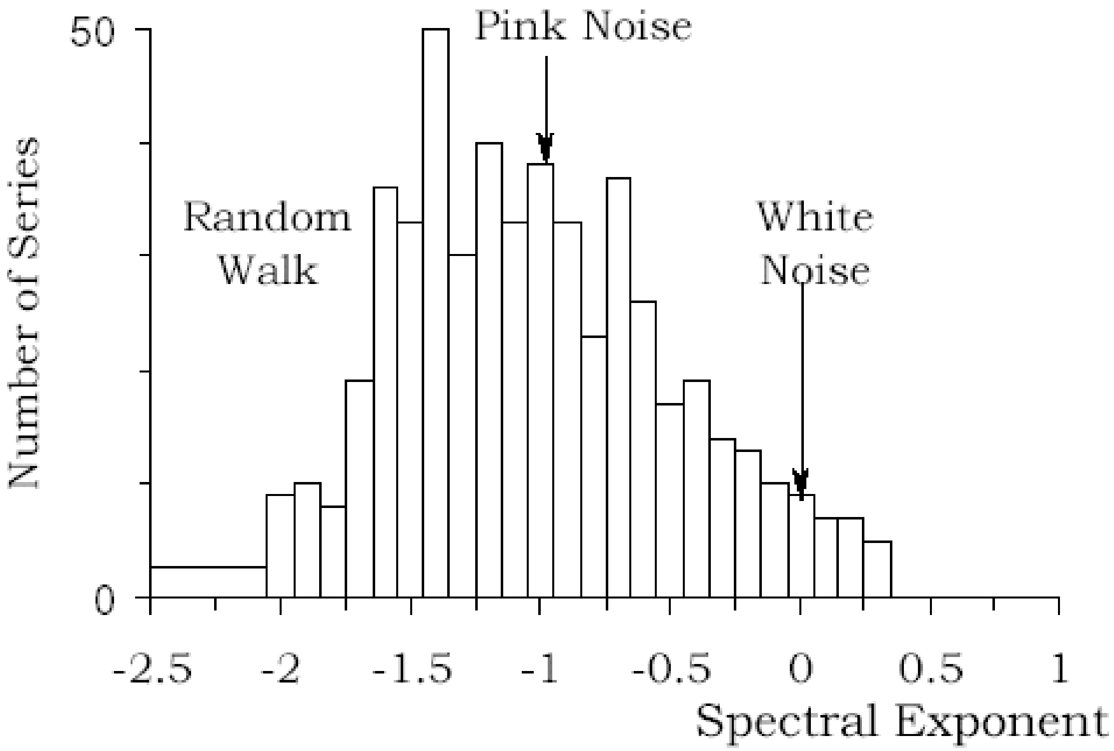
Fisheries Landings

Source: FAO 1950-1996 (NE, NW, EC, WC, SE and SW Atlantic and Pacific, E and W Indian, Mediterranean-Black Seas).



Power Spectra of Ecological Populations

Fitting lines to power spectra (1/f noise models) give as large spread of results. But the median is close to pink noise (*v*=1).



Environmental Variability and temporal Autocorrelation (Climatic & Ecological)

$\Rightarrow$ Autocorrelated in time (span multiple timescales)

$\Rightarrow$ More time means more variance (nonstationarity)

$\Rightarrow$ Power-spectral density is typically $1/f^{\nu}$

$\Rightarrow$ Power-law autocorrelation (typically)

$\Rightarrow$ Has fractal properties

$\Rightarrow$Long memory of the past (“long-term persistence”)

Models with autocorrelation

1. White noise (string of iid RVs) has none
2. Random walk (Brown noise)
3. Pink noise
4. Autoregressive models (OU, AR-1, ARMA, ARIMA, ARFIMA…)
5. Fractional Brownian motion, fractional Gaussian noise…
6. $1/f^{\nu}$-noise family

Models : Stochastic Processes

$$\{X(t), \quad t \in \mathbb{R}\}$$

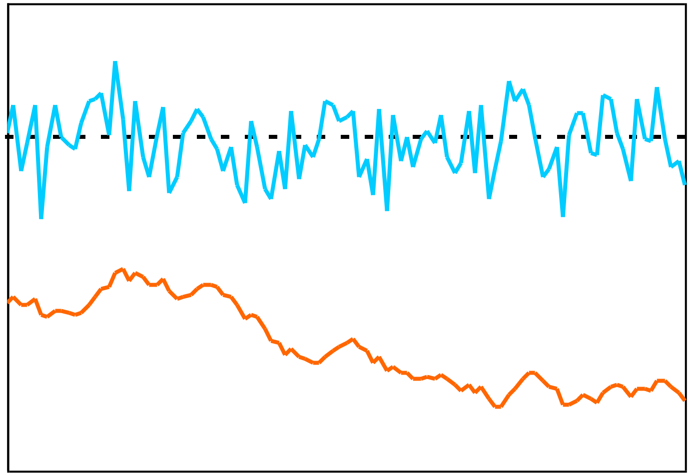
Brownian Motion (Wiener process, brown noise)

(i) $B(0) = 0$ and continuous

(ii) $P[B(t+h) - B(t) \leq x] \sim \frac{1}{\sqrt{2\pi h}} \int_{-\infty}^{x} e^{-\frac{u^2}{2h}} du$

(iii) Independent increments

Gaussian increments



Χρόνος

White Noise

(i) $W(s) \sim N(0, \sigma^2)$

(ii) $\langle W(s)W(t)\rangle = \delta(t-s)$

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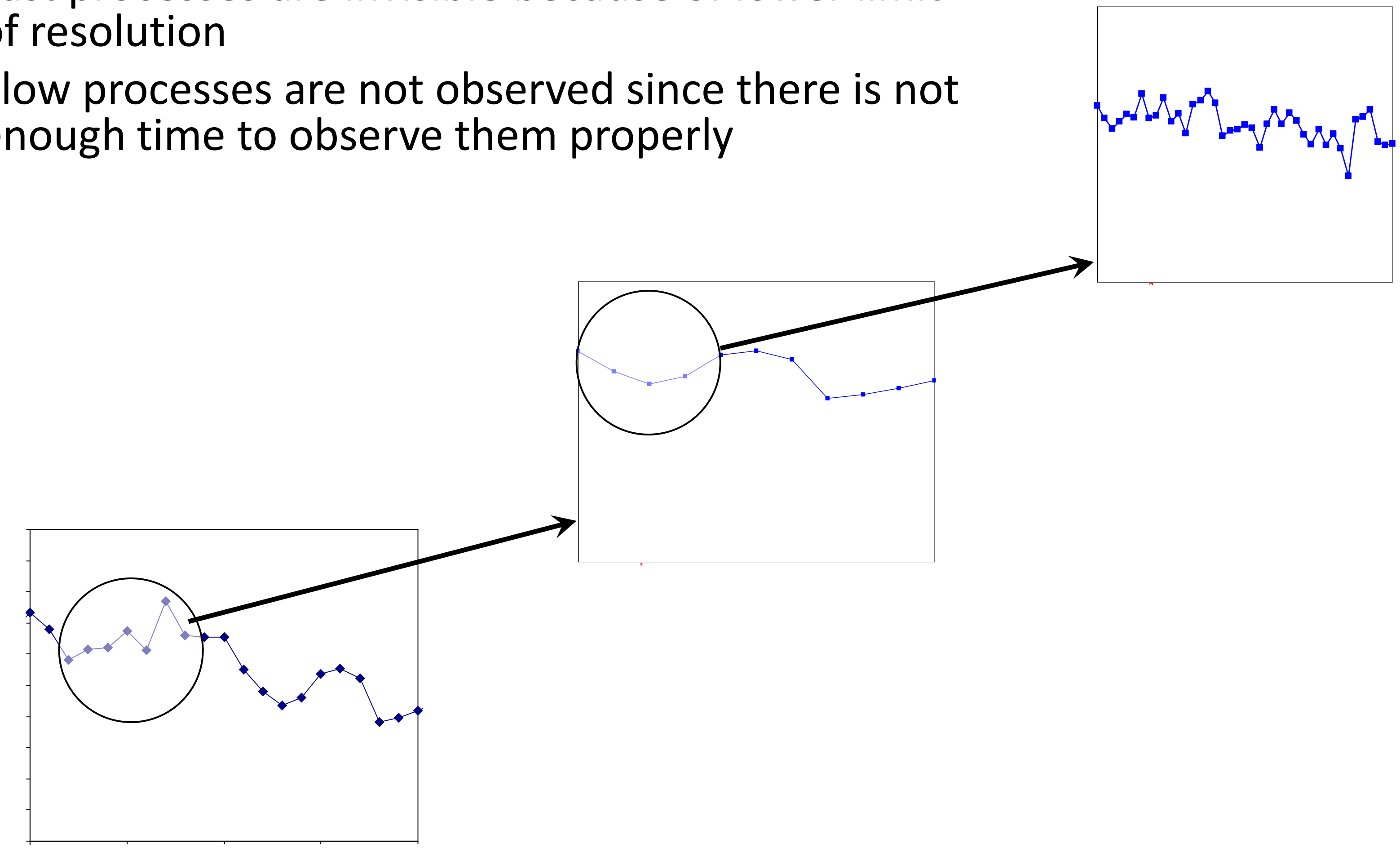
$1/f^{\nu}$-noises

- **$1/f^{\nu}$-noises with ν~1 have been observed in many phenomena.**
  - Electronic circuits
  - Geophysical time series
  - Music
  - Landscape structure
  - DNA base-sequences
  - Ecological abundance
  - …. and many more!

- **"Canonical" members are white (ν=0), brown (ν=2) and pink (ν=1)**

Self-affinity of 1/f noises

- Every time-series represents a process on many time-scales. Given any finite "window"
  - Fast processes are invisible because of lower limit of resolution
  - Slow processes are not observed since there is not enough time to observe them properly



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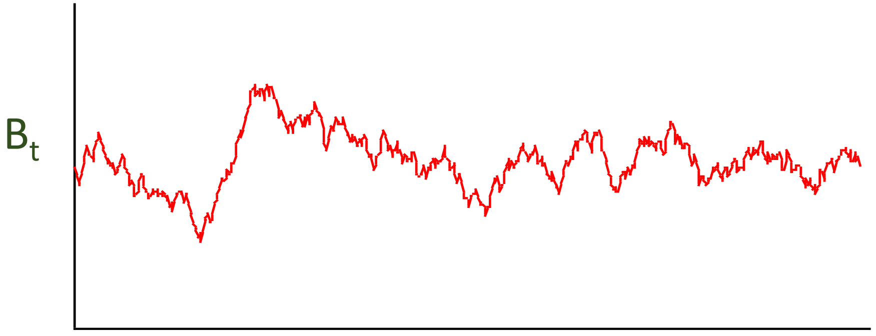
Fractal Dimensions



Brownian motion:

$$D_B = \lim\left[\frac{\ln(N_\delta(B))}{-\ln(\delta)}\right]$$

Fractional Brownian motion for index α (2>α>1):

$(i) \quad B(0) = 0$ and continuous

$$(ii) \quad P[B(t+h) - B(t) \le x] \sim \frac{1}{\sqrt{2\pi h^{2\alpha}}} \int_{-\infty}^{x} e^{-\frac{u^2}{2h^{2\alpha}}} du$$

Gaussian increments

Dimensions (box and Hausdorff) :

$$D_B = D_H = 2 - \alpha$$

1/*f*-noises (1/*f* $^{\nu}$-noises):

$$S_X(\omega) \propto \frac{1}{\omega^\nu} \qquad \omega \ge 0, 2 > \nu \ge 0,$$

$$D_B = D_H = \frac{5-\nu}{2} \quad \forall 2 > \nu > 1$$

Ornstein-Uhlenbeck Process (Langevin Equation)

$$\frac{dA}{dt} = -\frac{A}{\tau_c} + W(t), \qquad W(t) \sim N(0, V), \quad A(0) = 0 \quad A, t, W \in \mathbb{R} \quad V > 0$$

The autocorrelation function: $R_A(s) = \langle A(t) A(t+s) \rangle$

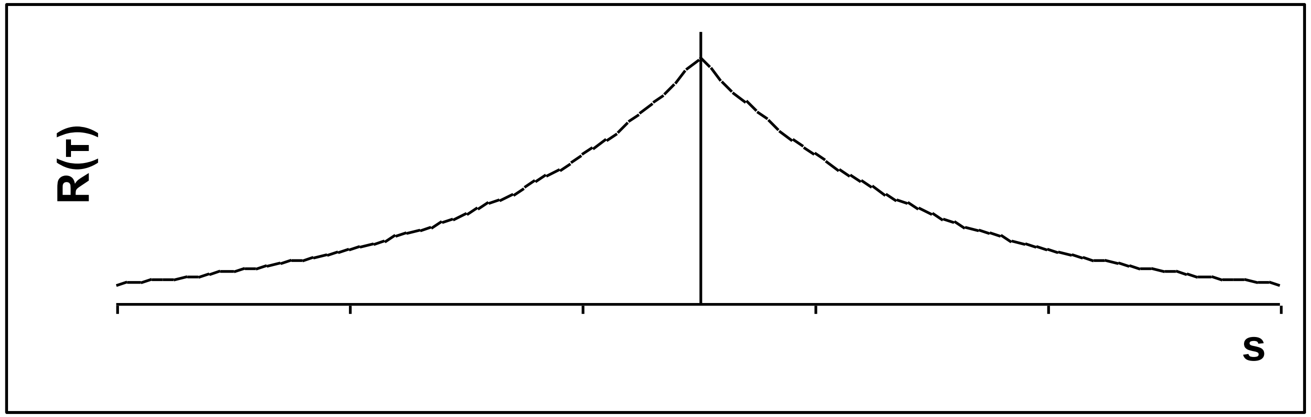
For the OU process this is: $R_A(s) = V \exp\left(-\frac{|s|}{\tau}\right)$

If variance V=1 then:

$$\langle A(t) \rangle = 0,$$

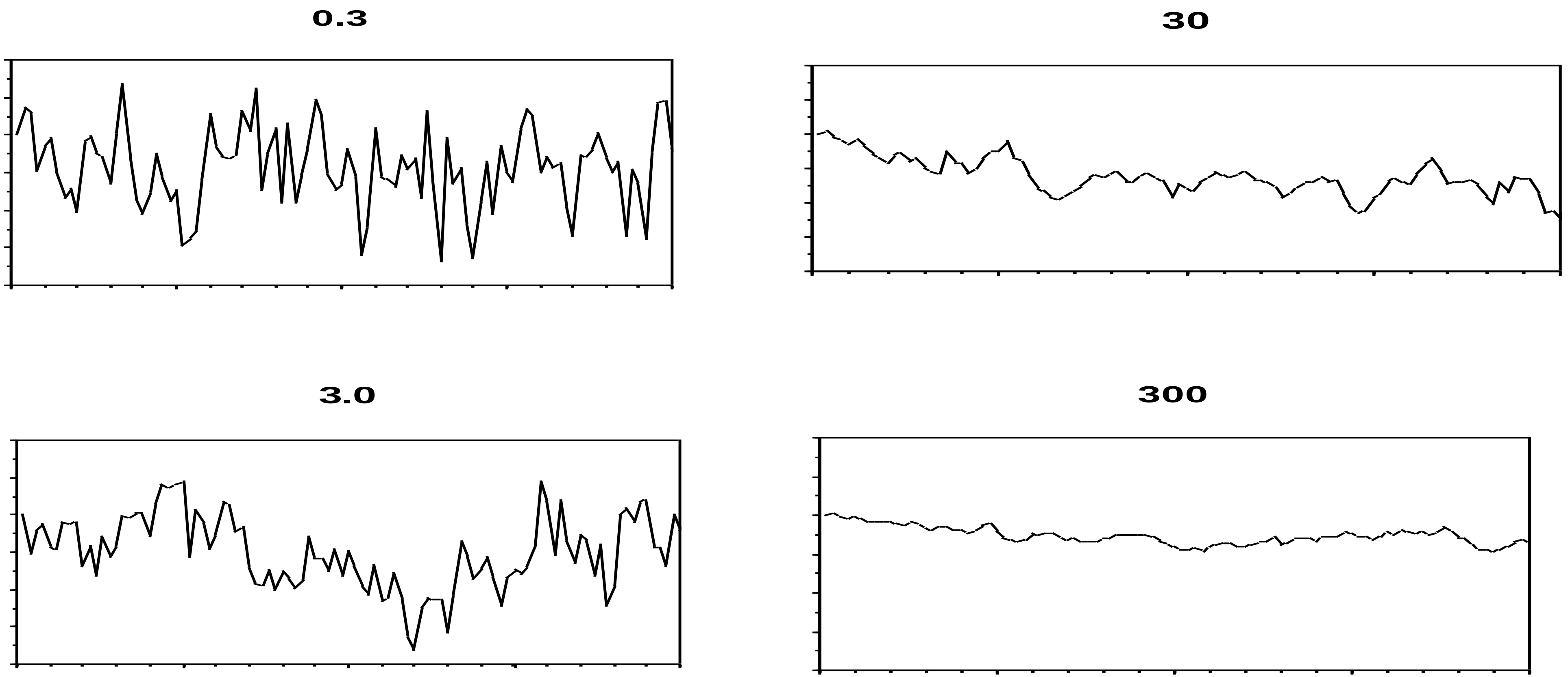
$$\langle A(t)^2 \rangle = 1,$$

$$R_A(s) = \exp\left(-|s|/\tau\right)$$



"Unit" OU process

Four OU processes
for timescales τ = 0.3, 3.0, 30 and 300



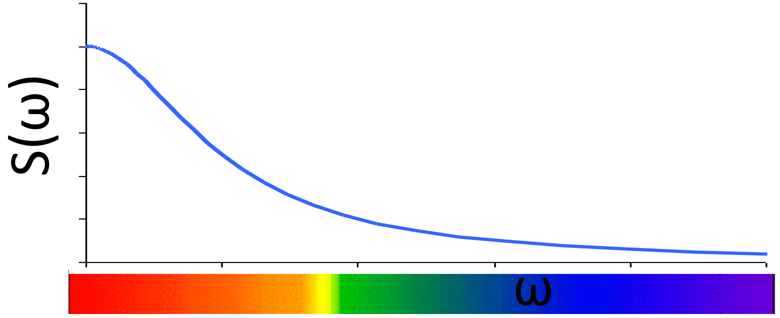
The Spectrum of the OU Stochastic Process

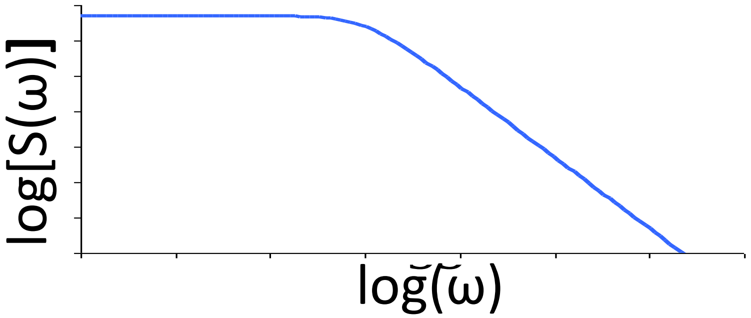
Weiner-Khinchin theorem: "power spectrum" is Fourier Transform of autocorrelation:

$$S(\omega) = \int_{-\infty}^{+\infty} R(\tau) e^{-i\omega\tau} d\tau$$

For the OU process this spectrum is:

$$\Rightarrow S_A(\omega) = \frac{2\tau/\pi}{1+(\tau\omega)^2}, \quad \omega \in [0,\infty)$$





Note angular frequency here is $\omega = 2\pi f$

The per-octave Spectrum of OU process

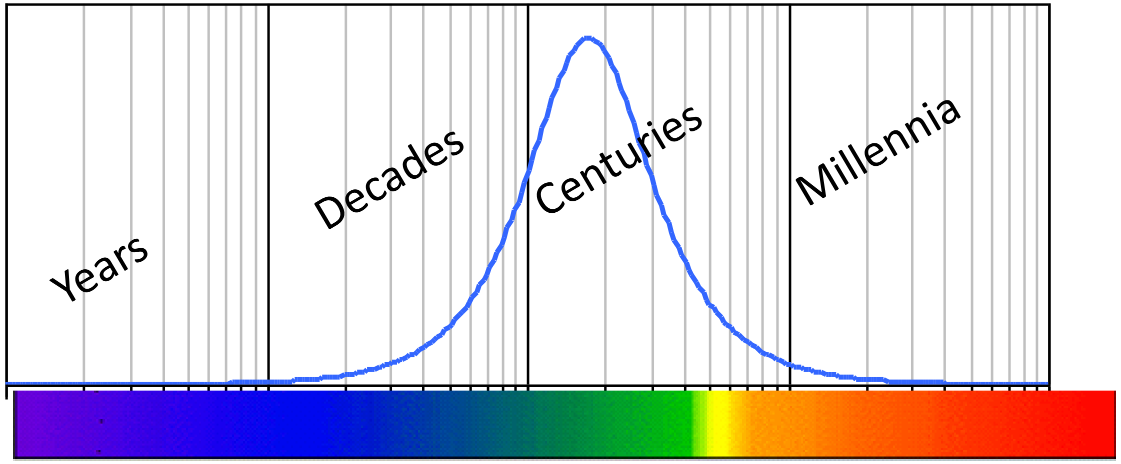
$$S_A(\omega) = \frac{2\tau / \pi}{1 + (\tau\omega)^2}, \quad \omega \in [0, \infty), \quad \tau > 0$$

The per-octave spectrum of time-scales is found by the transforming the spectrum as a PDF, using the change of variable φ=ln(ω) and θ=ln(τ):

Using these changes of variables and : $S(\omega)d\omega = Q(\phi)d\phi$

$$Q_A(\phi) = \text{sech}[\phi + \theta]/\pi \qquad \phi, \theta \in \mathbb{R}$$

“per-octave” (or “per-decade”) spectrum of the OU process.



Decomposition of the Spectrum

Define:

$$P(t) = \sum_{k=-\infty}^{+\infty} c_k A_k(t), \qquad c_k \geq 0$$

$A_k(t)$ are independent unit OU processes with time-constants τ_k such that:

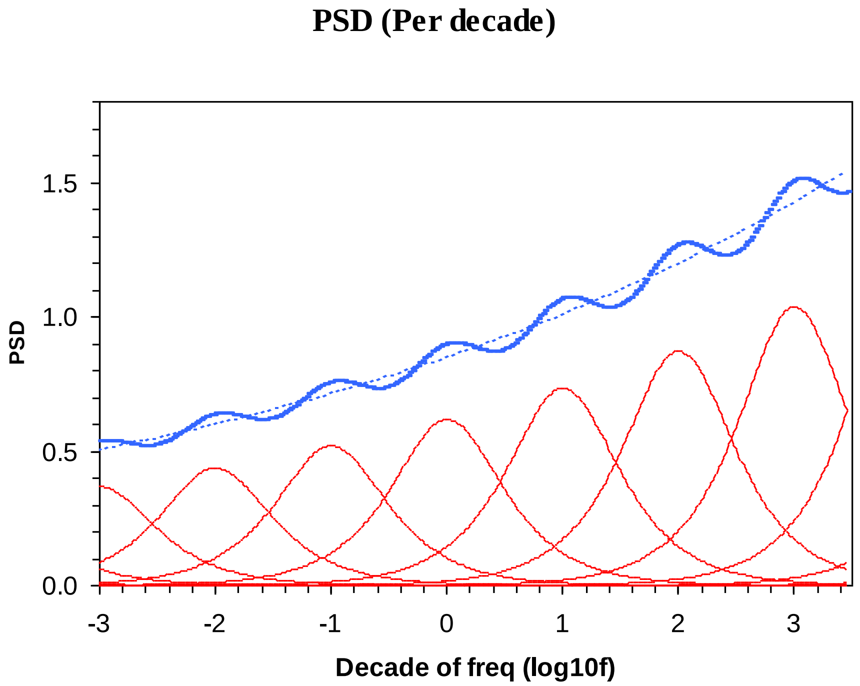
$$\tau_k = \exp[k\Delta\varphi]$$

We can show that:

$$Q_P(\varphi) = \sum_{k=-\infty}^{+\infty} c_k^2 Q_{Ak}(\varphi)$$

$$= \frac{1}{\pi} \sum_{k=-\infty}^{+\infty} c_k^2 \operatorname{sech}[\varphi + \theta_k]$$

$$= \frac{1}{\pi} \sum_{k=-\infty}^{+\infty} b_k \operatorname{sech}[\varphi + k\Delta\varphi]$$



The OU processes can be used to create spectra of more complex long-range processes, including fractal noises. Also, $1/f^{\nu}$-noises can be interpreted as a superposition of OU processes.

Construction of Fractal Noises

Most fractal noises have a $1/f^{\nu}$-spectrum

$$S_X(\omega) \propto \frac{1}{\omega^{\nu}} \qquad \omega \geq 0,\ 2 > \nu \geq 0,$$

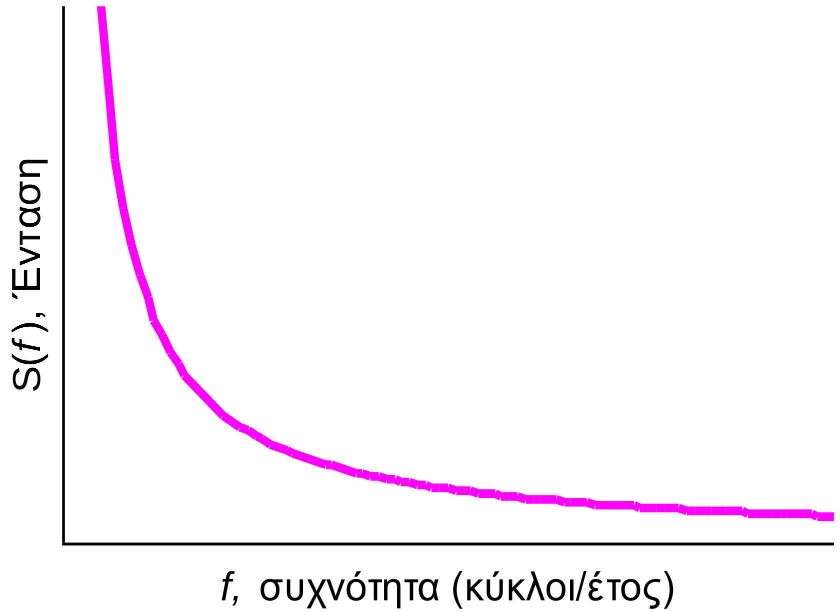
We require the representation , in terms of unit OU processes, of a $1/f^{\alpha}$-spectrum.
The per-octave spectrum is:

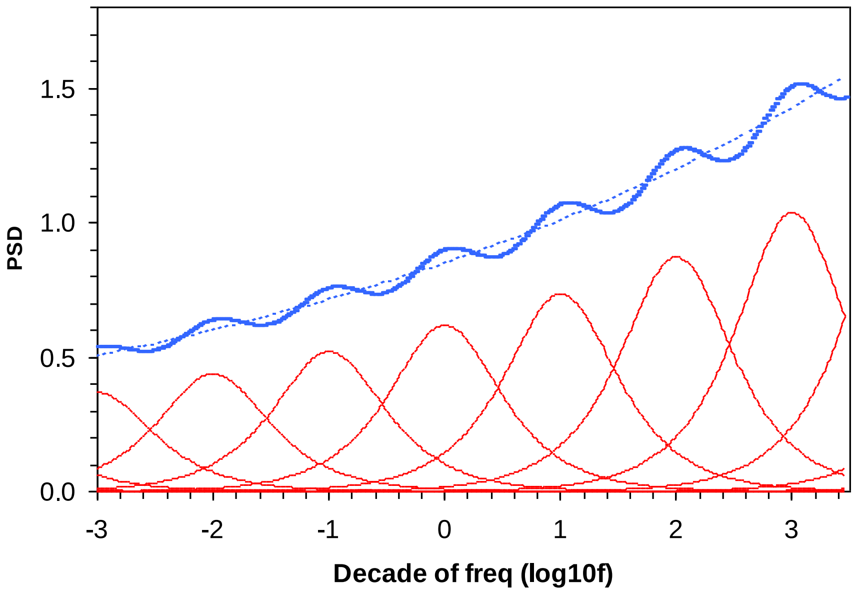
$$Q_X(\omega) = Be^{(1-\nu)\varphi}$$

This is normally done by simply giving the constants b_k weightings proportional to Q_X at $k\Delta\varphi$. That is:

$$b_k \propto e^{(1-\nu)k\Delta\varphi}$$

This is not based on rigorous analysis.



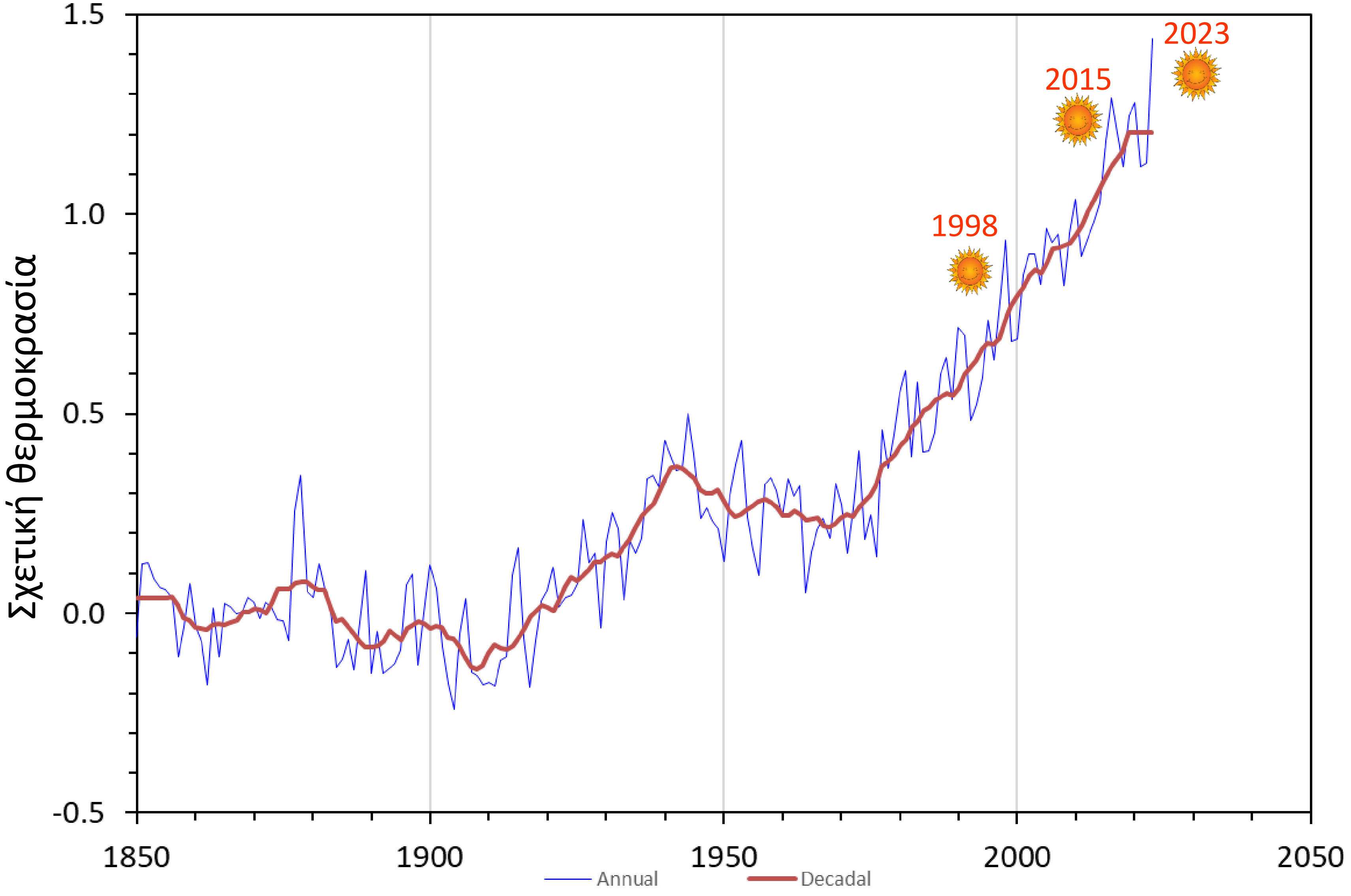


Does it matter which model we use?

Our model of environmental variability, especially its autocorrelation structure, matters in many ways. For example :

1. Predictions of extinction
2. Statistical interpretation of spatial patterns (how much correlation?)
3. Interpretation of statistical trends

The attribution problem



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Linear Regression Analysis

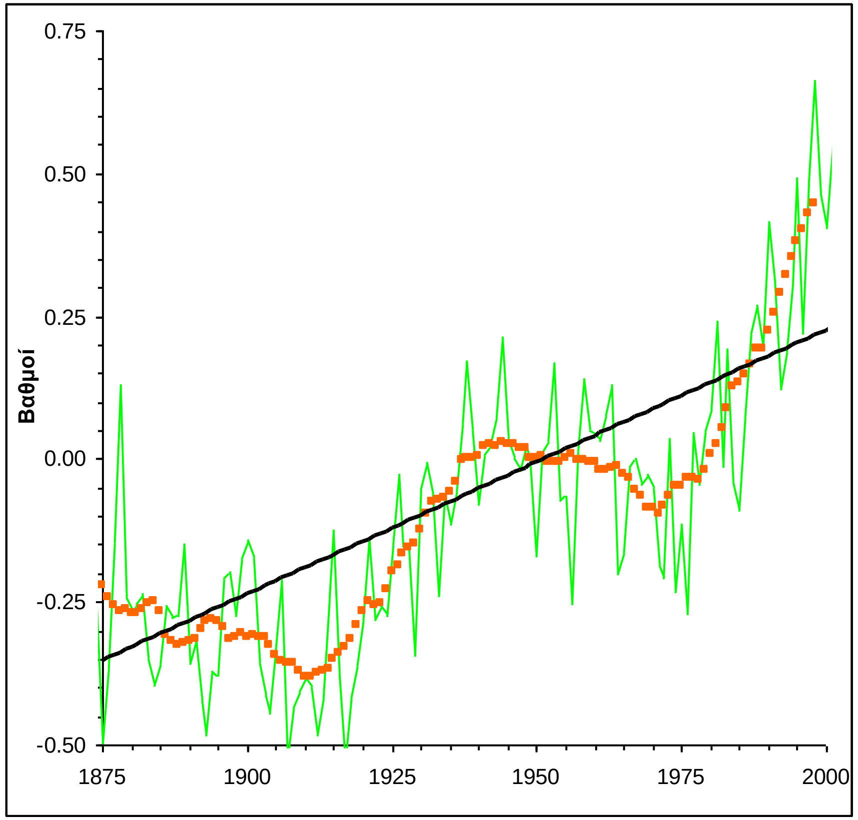
What is the probability of a slope of 0.61 degrees/century happening naturally?

$$Y(t) = \beta t + Y_0 + \varepsilon_t, \quad \varepsilon_t \sim N(0,\sigma^2)$$

$$\langle \varepsilon_t \varepsilon_{t+\Delta t} \rangle = 0$$

Linear trend (us)

“noise” (i.e. natural processes only)

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Regression Analysis

The regression equation is
Instr. T = - 9.04 + 0.00464 Year (AD

Predictor        Coef        Stdev     t-ratio        p
Constant      -9.0437       0.6384      -14.17    0.000
Year (AD    0.0046351    0.0003306       14.02    0.000
```

Very significant trend, given the model.

But the model excludes autocorrelation !

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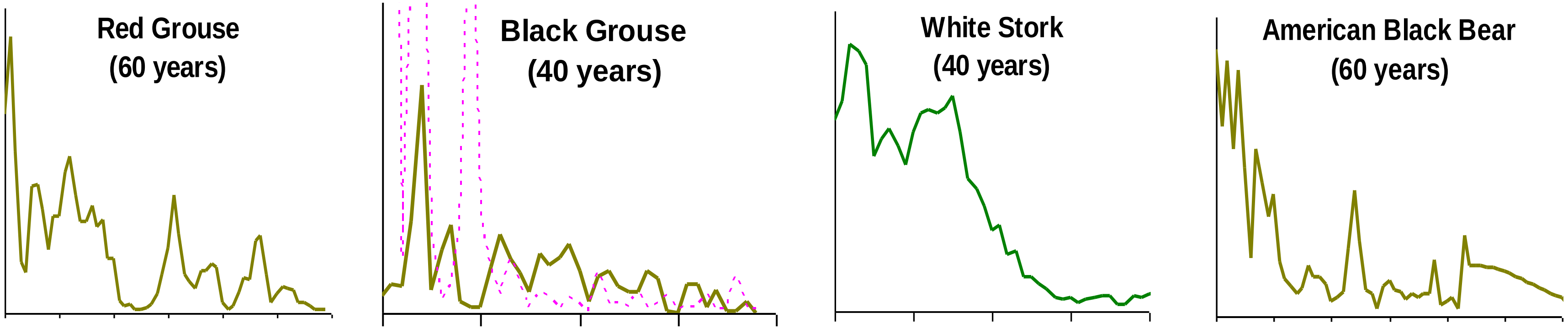
Estimations of significance

Series	Total variance	Av. spectral exponent, *v*	Min – Max of spectral exponent	*p*-value (crutem3nh)
Jones	0.052	0.77	0.63 - 0.89	$<10^{-5}$
Moberg	0.048	1.00	0.83 - 1.24	4.2×10^{-4}
Esper	0.019	0.96	0.76 - 1.23	2.0×10^{-5}
Mann	0.017	0.85	0.72 - 1.01	$<10^{-5}$
d'Arrigo1	0.057	0.59	0.39 - 0.8	$<10^{-5}$
d'Arrigo2	0.065	0.78	0.59 - 1.0	2.4×10^{-4}
Crowley	0.012	1.55	1.24 - 2.0	$<10^{-5}$

Natural variability at best a ~4/10,000 chance of explaining current global warming

Detecting a Population Decline

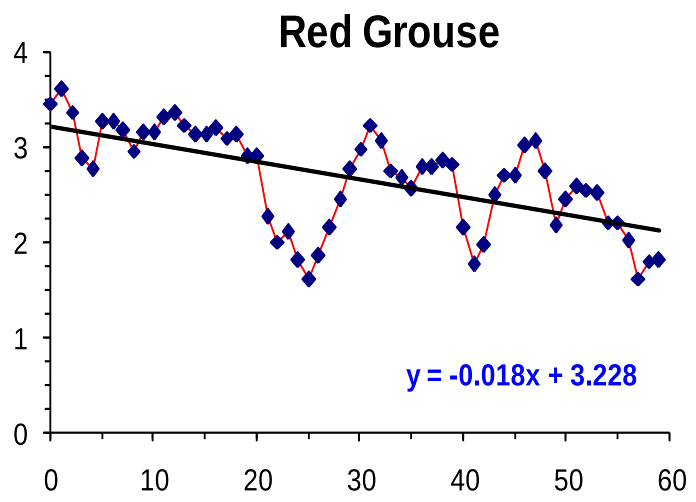
"Real" vs. "natural" declines

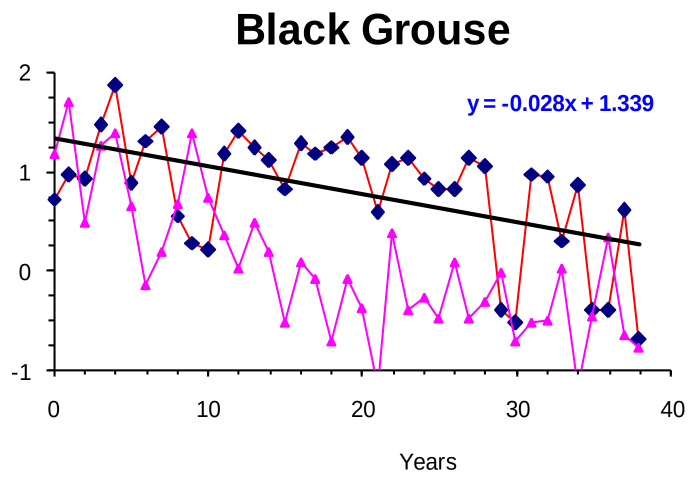


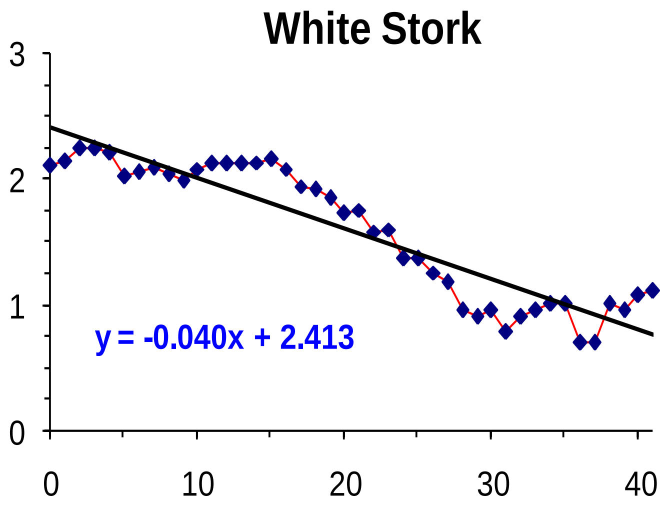
Ecological time series contain long-term components anyway, so how do we identify an "unnatural" decline?

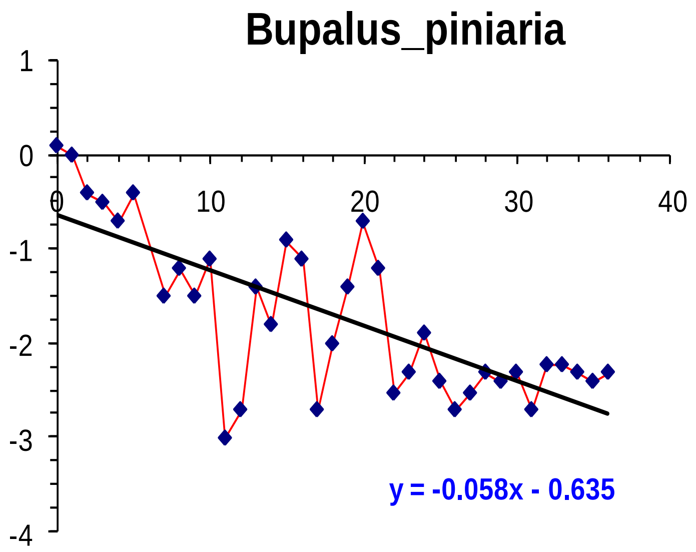
(i.e. One that has consequences for "management" and requires intervention?)

Examples (with linear regression)

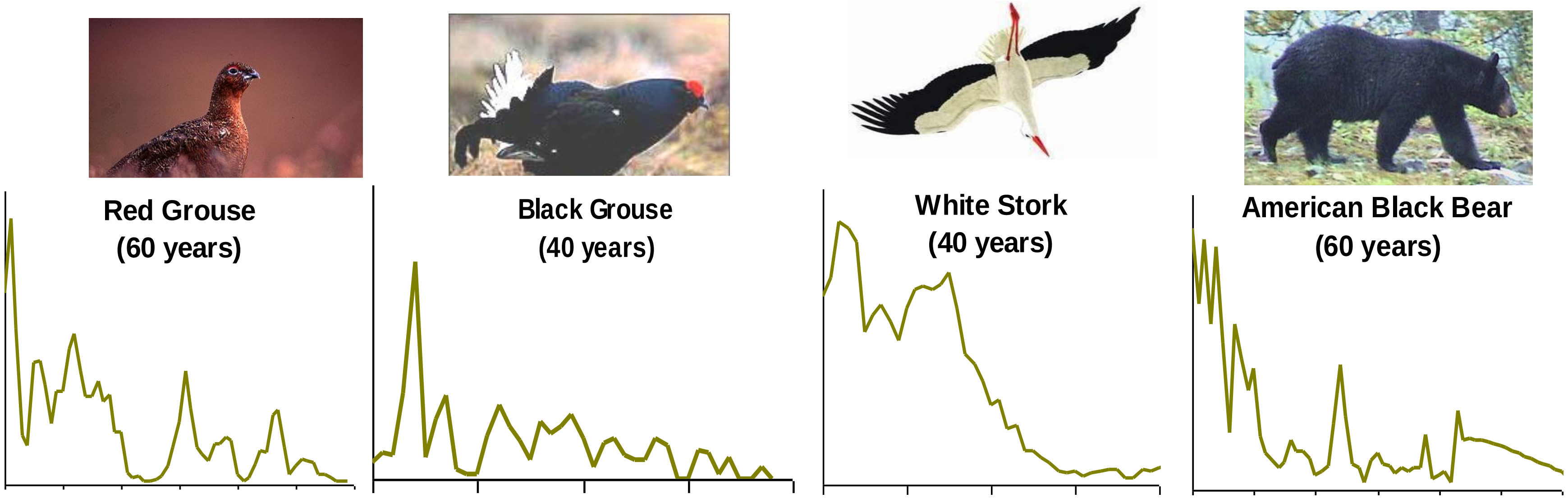








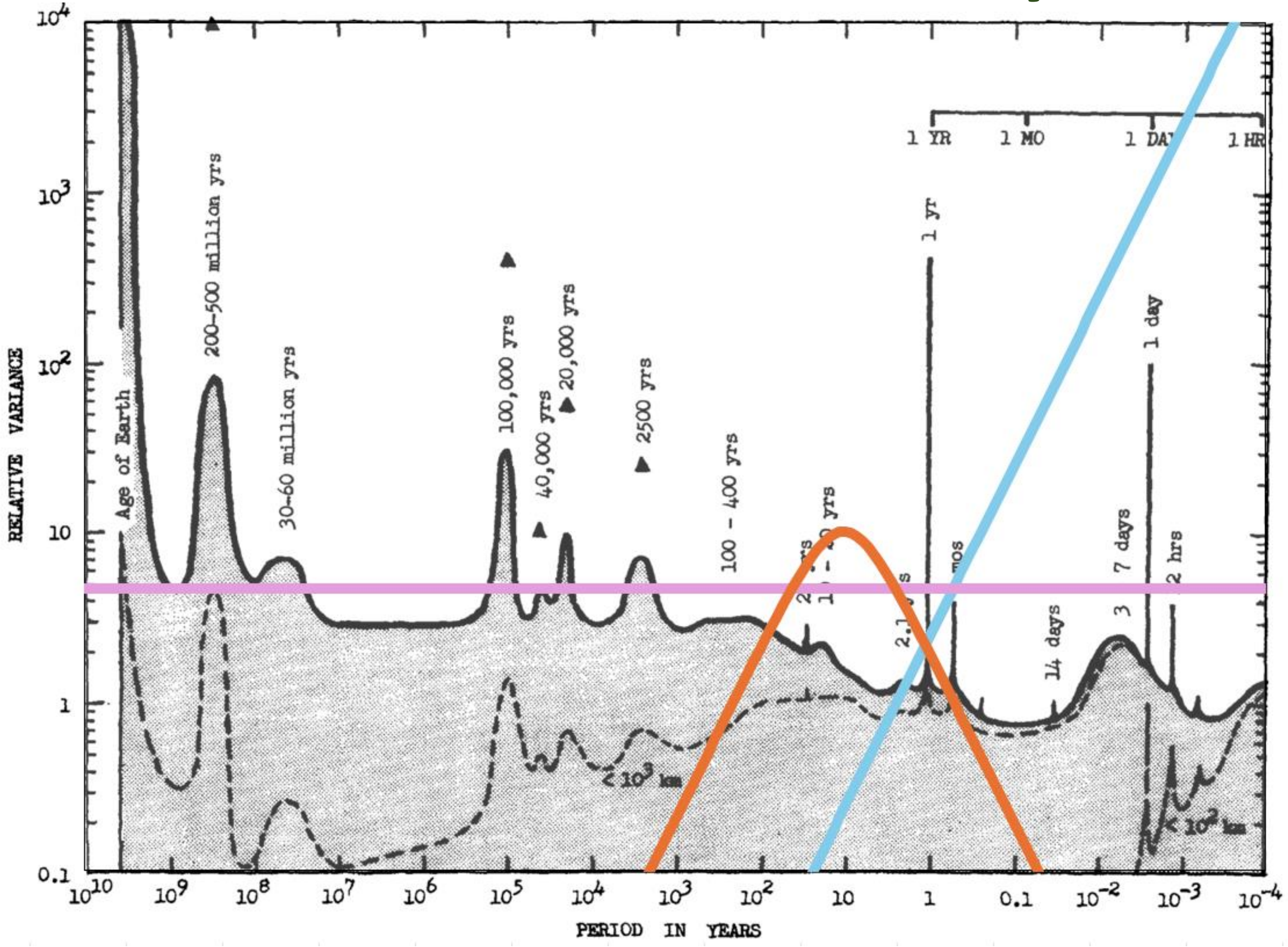
When is an Observed Population Decline Significant?



	Red Grouse	Black grouse	White stork	American black bear
N (points)	60	39	42	63
p-value (standard)	0.00%	0.10%	0.00%	0.00%
p-value (1/*f*)	1.52%	4.14%	0.10%	2.50%

Halley, J. M. (2006) “When is an observed population decrease significant?” 3rd Okazaki Conference on "Biology of extinction", Okazaki, Japan.

Environmental Variability is Multi-scale



Mitchell's depiction of the variability of the climate system over many decades of time. Superimposed are the variance conserving spectra of three stochastic processes used to model the Environment: white noise (blue), pink noise (pink), first-order autoregressive (orange).

Mitchell, J. M. An overview of climatic variability and its causal mechanisms. *Quat. Res.* **6**, 481–493 (1976)

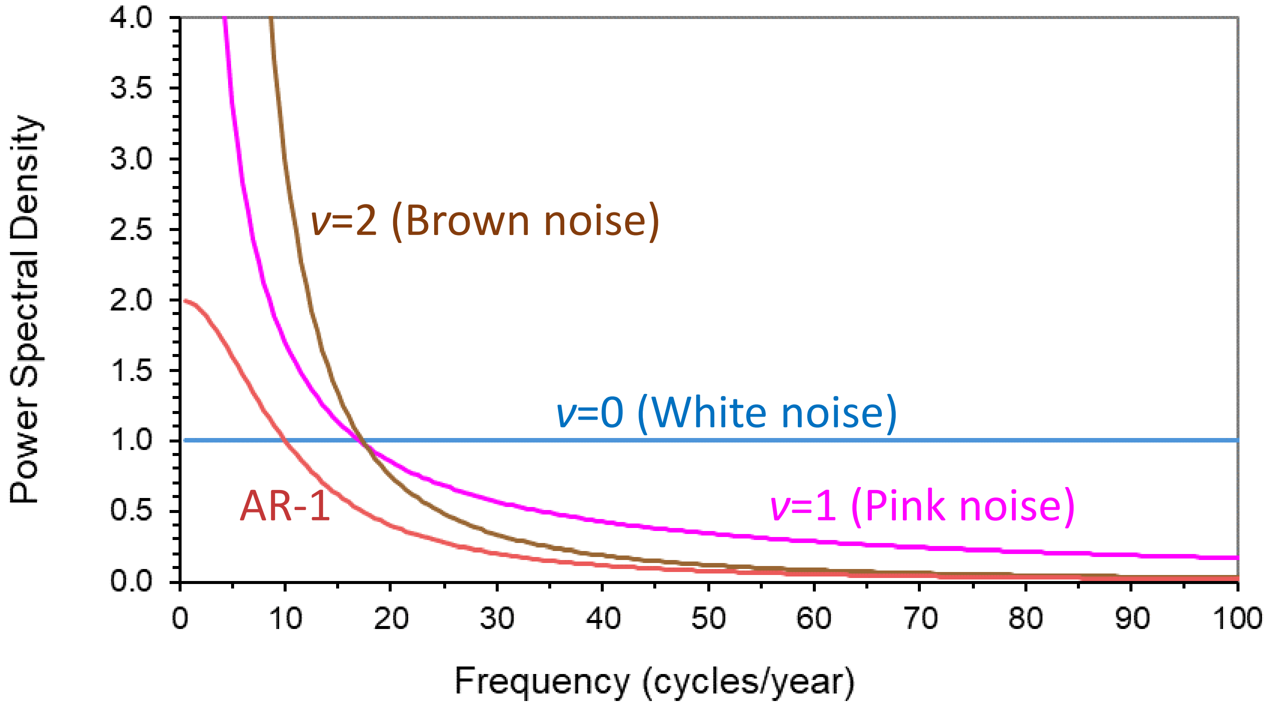
1/*f*- noises and Power Spectral Density - 1

$1/f^{v}$-noise family

$$S(f) \propto \frac{1}{f^{v}}$$

Includes white noise (*v*=0) and Brown noise (*v*=2) as special cases.

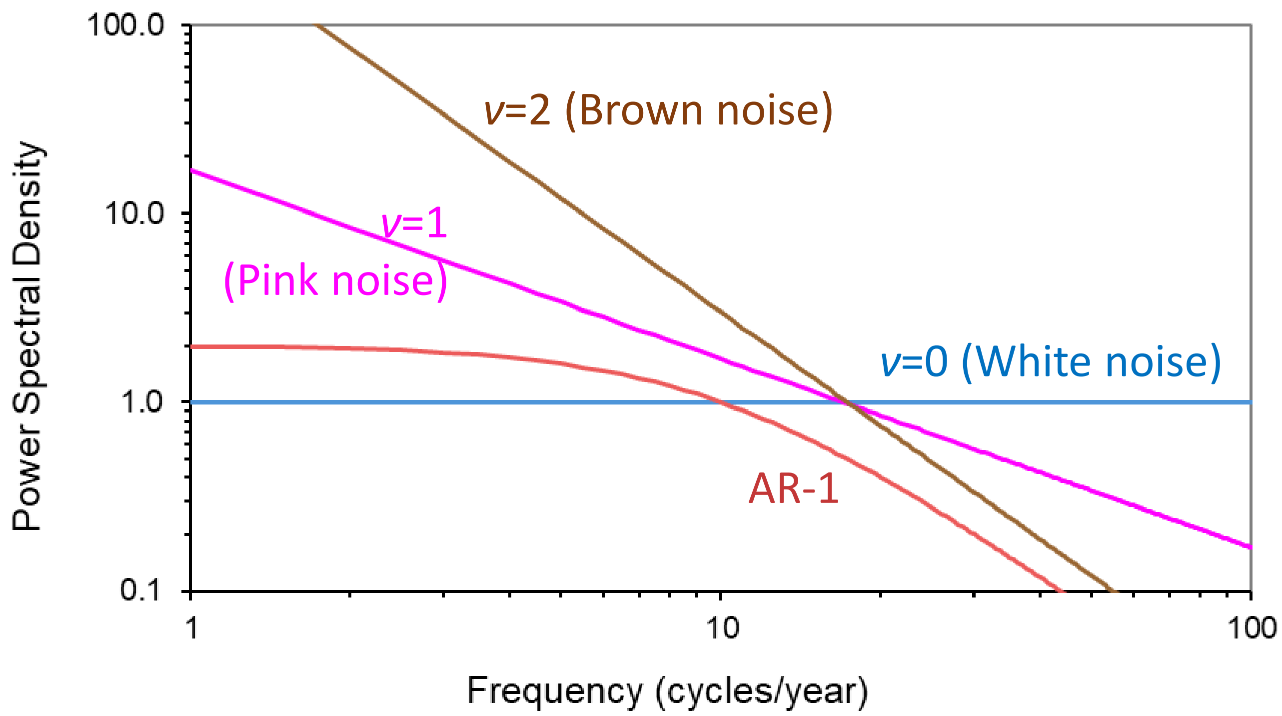
Note that the PSD is a histogram. For example, the variability between 20-30 cycles/year is the **area under the curve** for each process.



1/*f*- noises and Power Spectral Density - 2

Usually, the spectra are drawn on logarithmic axes in order to reveal their power-law character.

Note: These are no longer histograms.

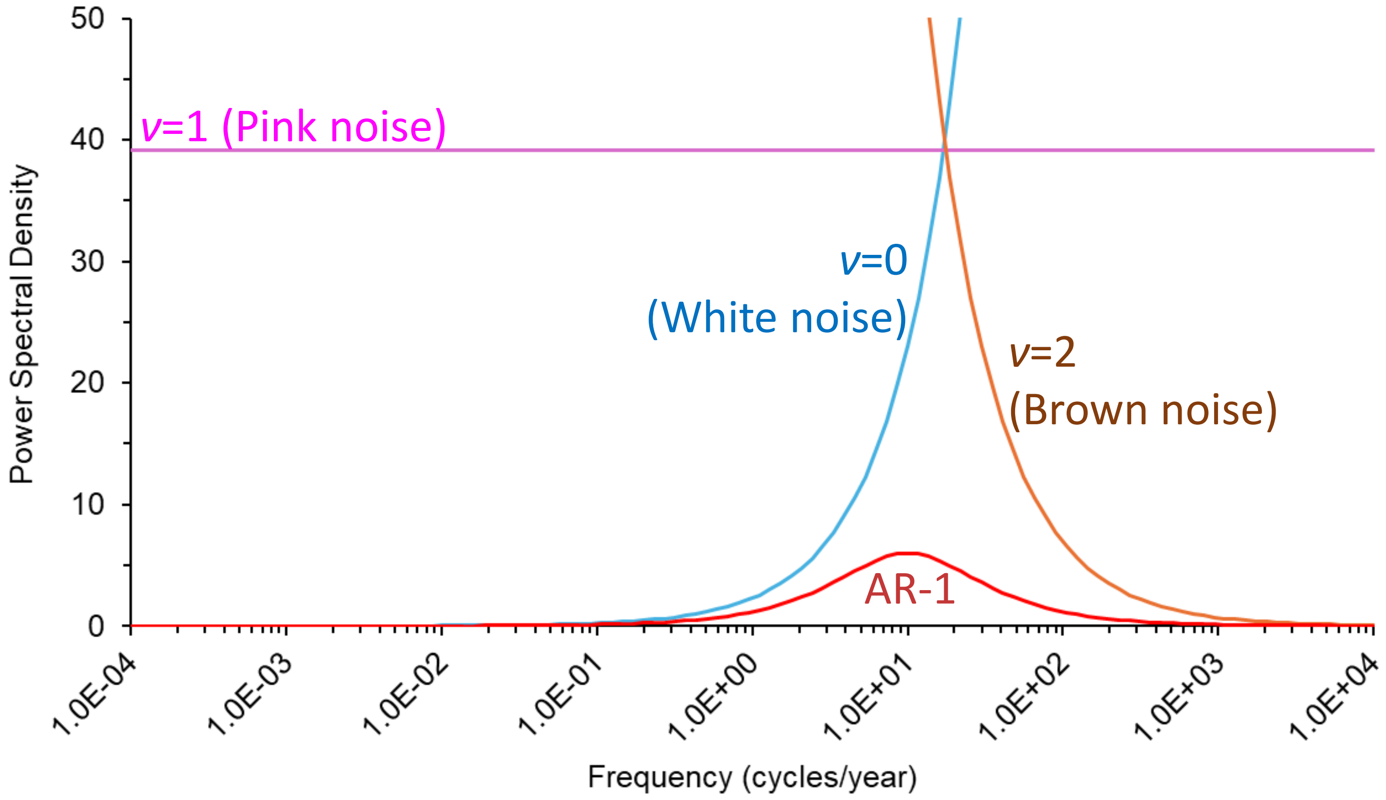


1/*f*- noises and Power Spectral Density - 3

To obtain the histogram ("variance conserving" PSD) on a logarithmic axis of frequency, note the following:

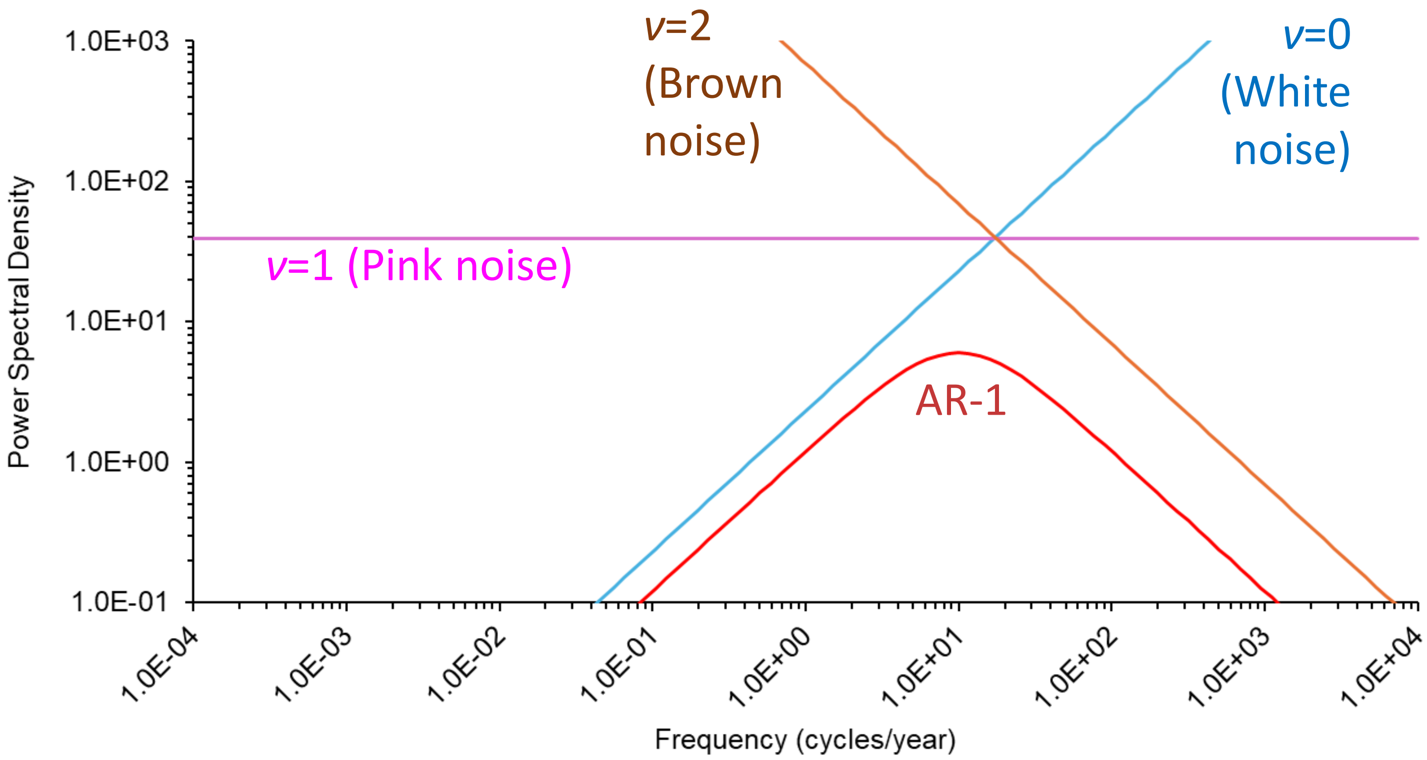
If $f = e^{\varphi}$, then $df/d\varphi = e^{\varphi}$
$S'(\varphi)d\varphi = S(f)df = e^{-(v-1)}\, d\varphi$

Again this "per decade" PSD is a histogram. Variability between 10^{-1} and 10^{+1} cycles/year is the **area under the curve** for each process.



1/*f*- noises and Power Spectral Density - 4

The vertical axis can again be again transformed to obtain a picture of the relative contributions



- White noise contains a surplus of rapid time-scales
- Brownian motion contains a surplus of long-duration time-scales
- Pink noise contains equal amounts of all scales

Which models fit and which are used?

1. White noise (string of iid RVs)
2. Random walk
3. Pink noise
4. Autoregressive (AR) models
5. Other $1/f^{\nu}$-noise

Which models fit and which are used?

1. White noise	**Very often**
2. Autoregressive (AR) models	**Often**
3. Random walk	**Sometimes**
4. Pink noise	Almost **NEVER**

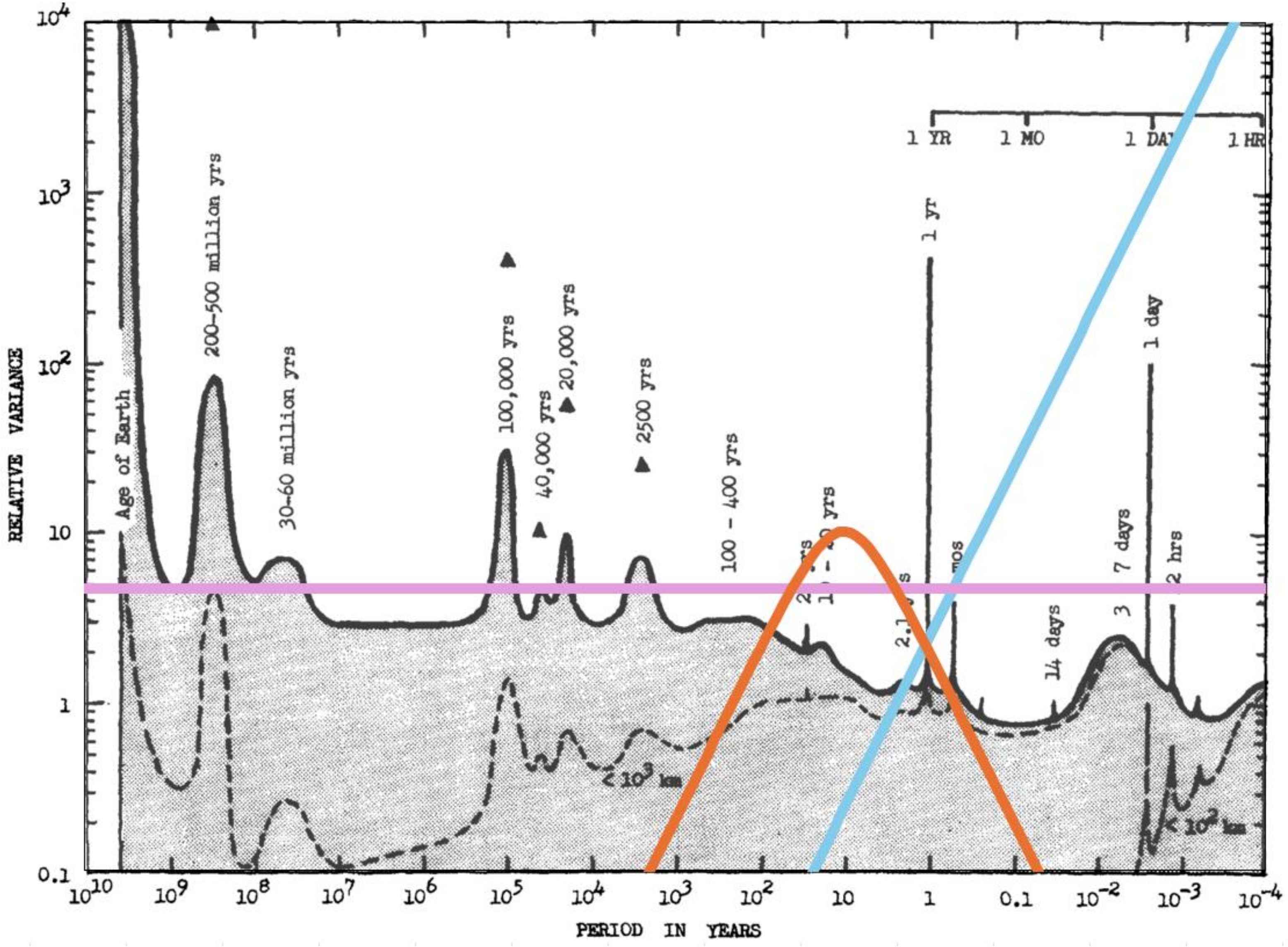
Which models fit and which are used?

1. White noise
2. Autoregressive (AR) models
3. Random walk
4. Pink noise

Why is Pink Noise not used?

1. “Common and interesting phenomenon” (i.e. not fundamental)
2. “Difficult to understand”
3. Incomplete mathematical basis (e.g. for fractal dimension)
4. Lack of software for statistical tests and simulation of pink noise

How to understand Environmental Variability!



Going back to Mitchell's "educated guess" of the variability of the climate we should begin with the "weakest" assumption

A flat spectrum!