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Application of non-linear time series analysis in physical, engineering and financial systems

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Structure of the presentation

- Time series analysis-univariate
- Phase space reconstruction methods
  - Recurrence plots
- Complex networks
  - Visibility Networks
  - Online change point detection
- Bubble detection
- Time series prediction using neural networks

- (applications will be presented)

Time series

- An observed dynamical system
(simulated / experimental / field measurements)
results in observables varying in time

- In physics, engineering, environmental, finance systems, we are aware of system monitoring

- Do time series contain information about the underlying system dynamics that can be useful?

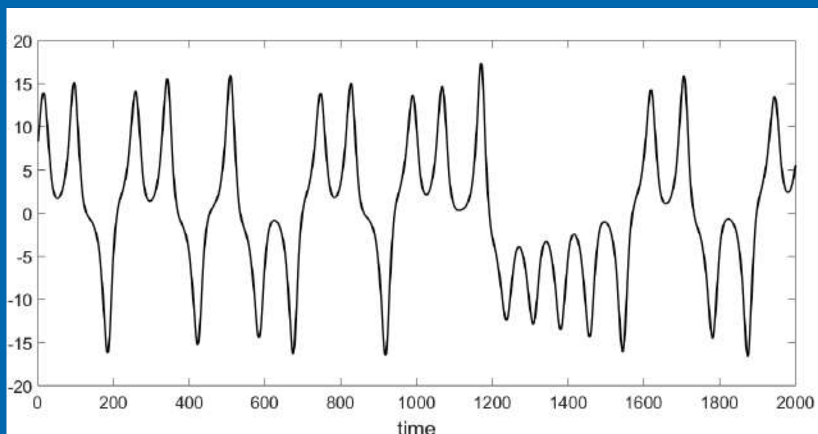
Case 1: Known equations

$$\frac{dx_1}{dt} = \sigma(x_2 - x_1)$$

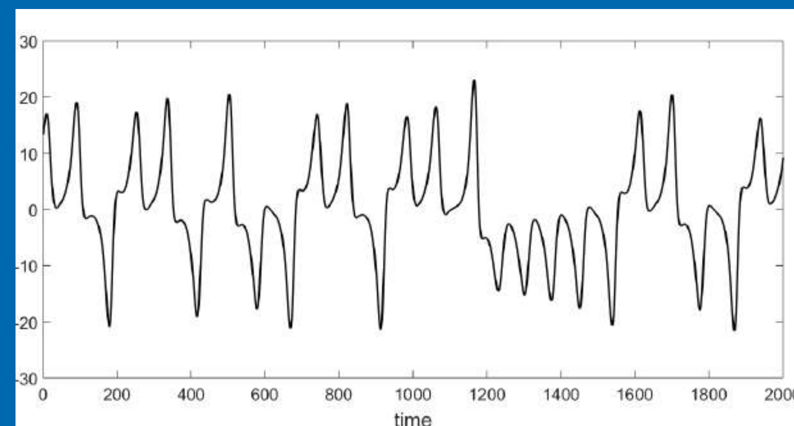
$$\frac{dx_2}{dt} = rx_1 - x_2 - x_1x_3$$

$$\frac{dx_3}{dt} = x_1x_2 - bx_3$$

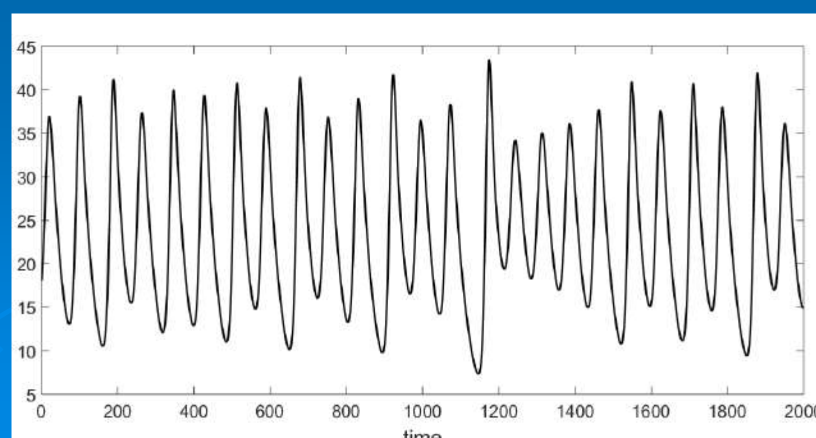
$x_1(t)$



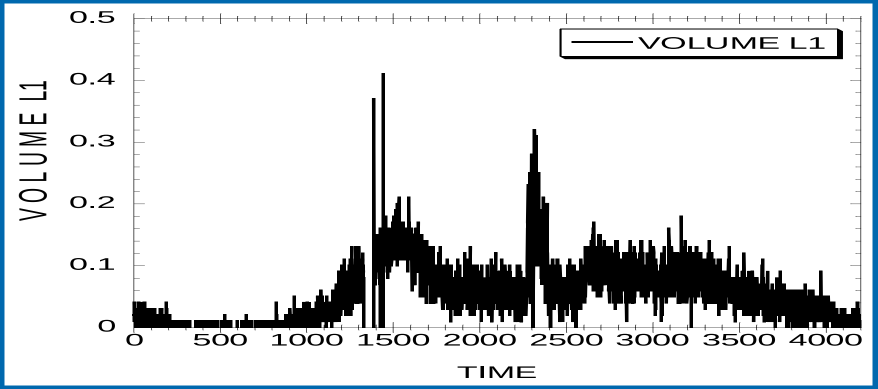
$x_2(t)$

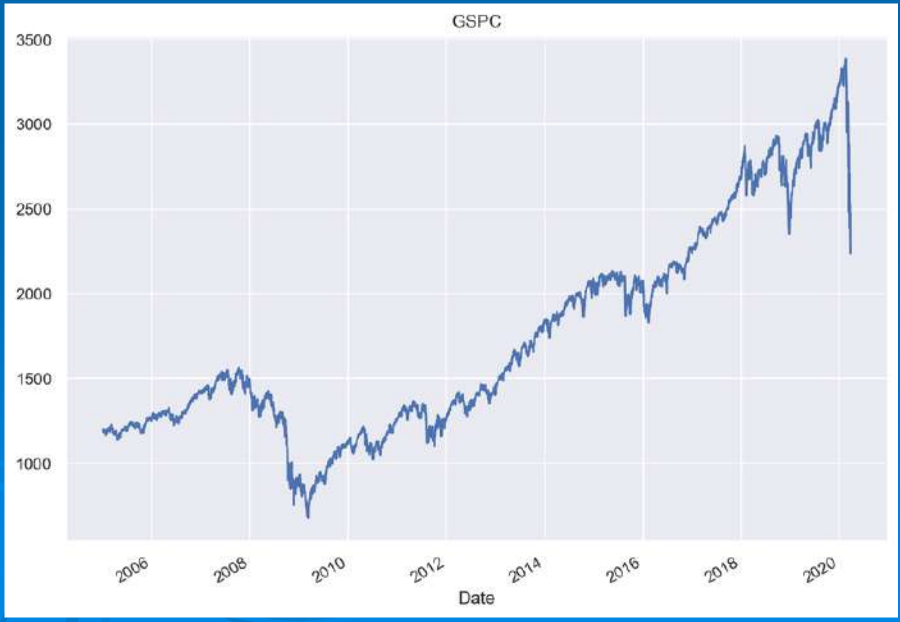


$x_3(t)$



Case 2: We don't know the detailed equations





Unfortunately nature

is in general more complex

and in general non-linear

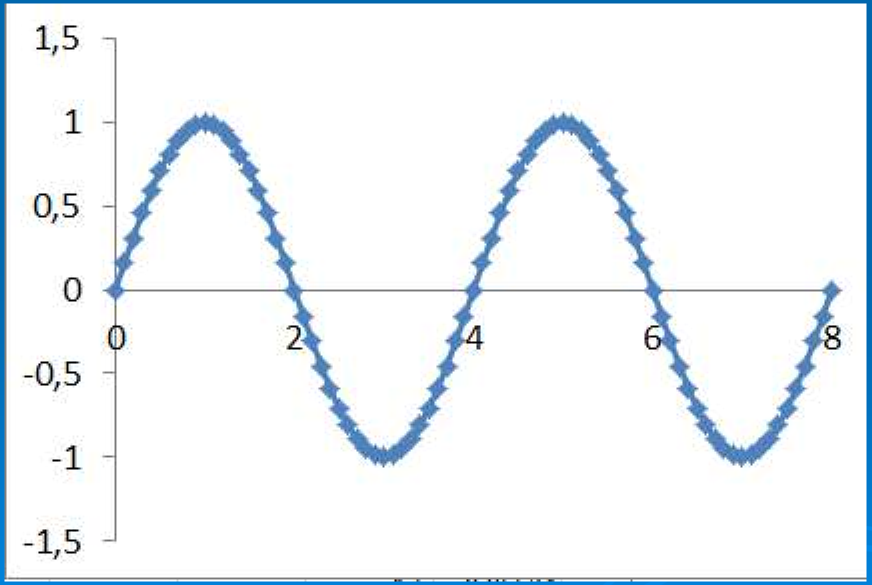
Statistics does not contain all information about the system dynamics

methods from dynamical systems can provide insight such as phase space methods

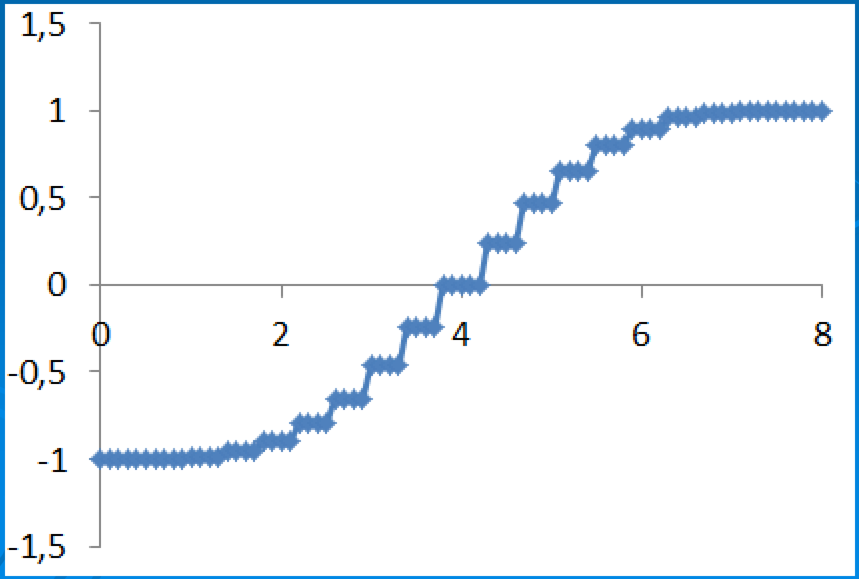
What you already know to do

Statistical analysis mean value, standard deviation, skewness, kyrtosis

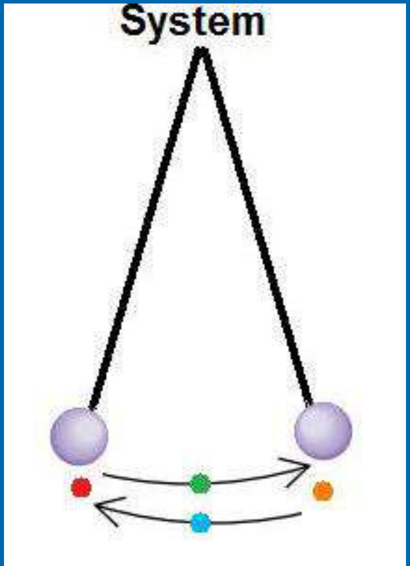
The problem is that statistical analysis
gives the same results for different temporal variations

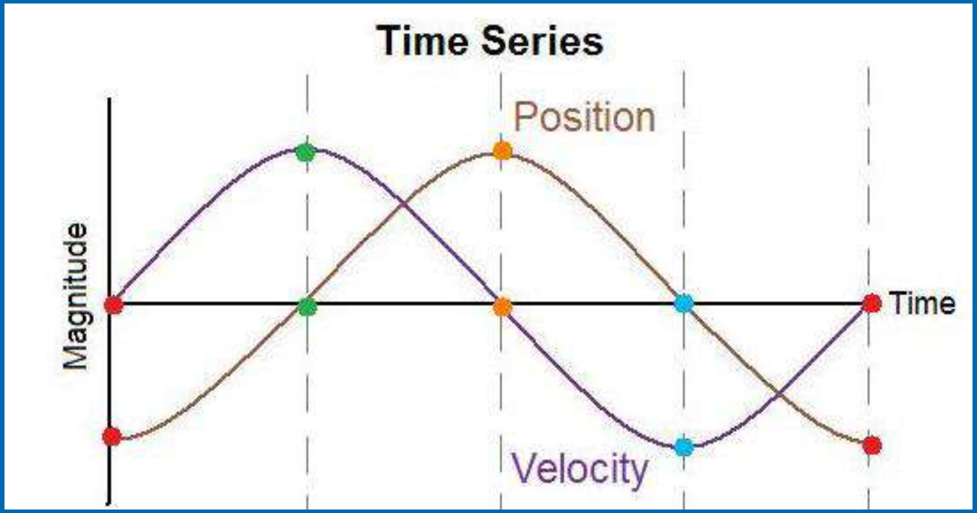


Just order the data →
same statistic but..



Phase Space:
Example (1) - Pendulum
Known Dynamical System → Time Series



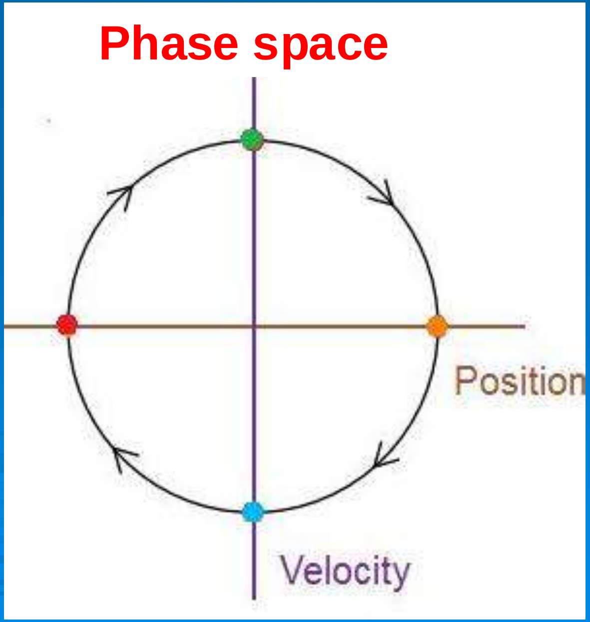


$$\dot{\theta}(t) = \omega(t)$$

$$\dot{\omega}(t) = -\frac{g}{L}\sin\theta(t)$$

From time

To Geometry

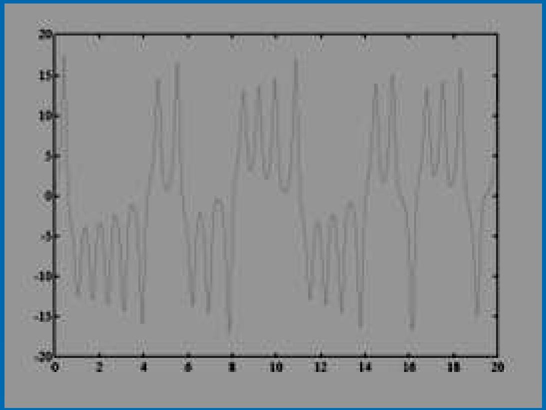


Example (2)

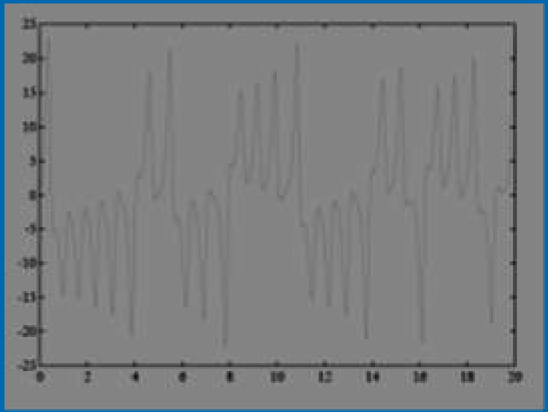
Known dynamical system Lorentz system (chaotic) ⇨ time series

$$\frac{dx_1}{dt} = \sigma(x_2 - x_1),$$
$$\frac{dx_2}{dt} = rx_1 - x_2 - x_1x_3,$$
$$\frac{dx_3}{dt} = x_1x_2 - bx_3,$$

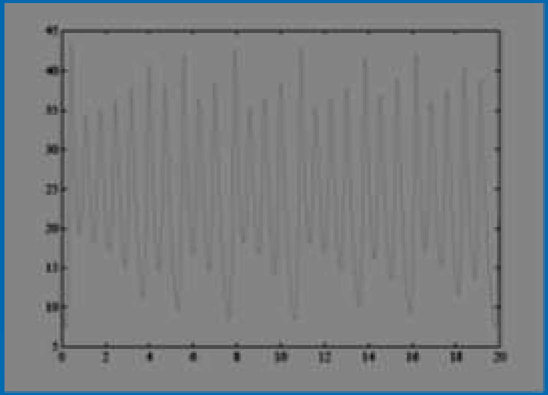
$x_1(t)$



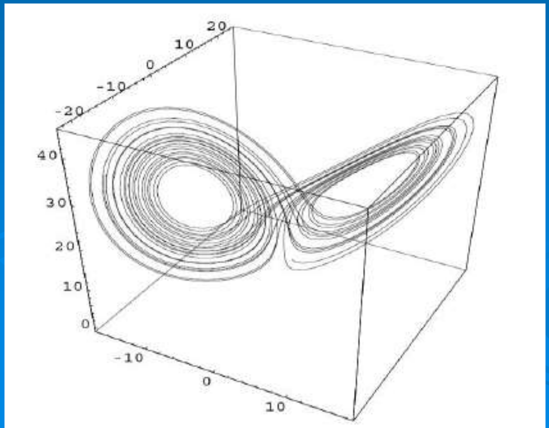
$x_2(t)$



$x_3(t)$



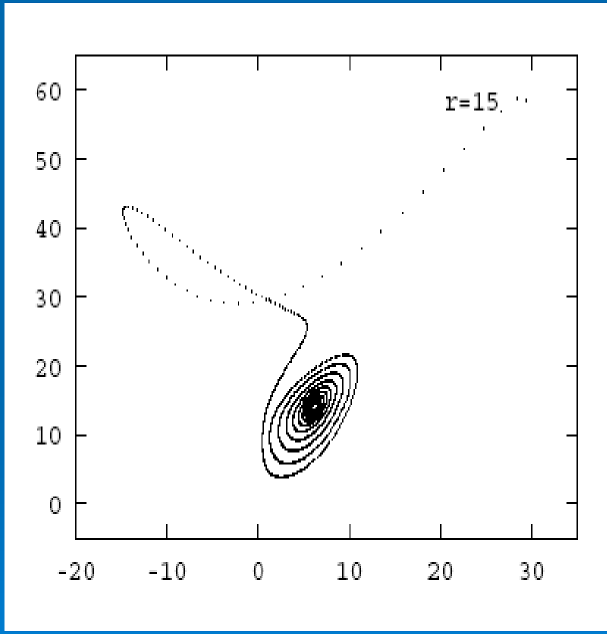
Phase space



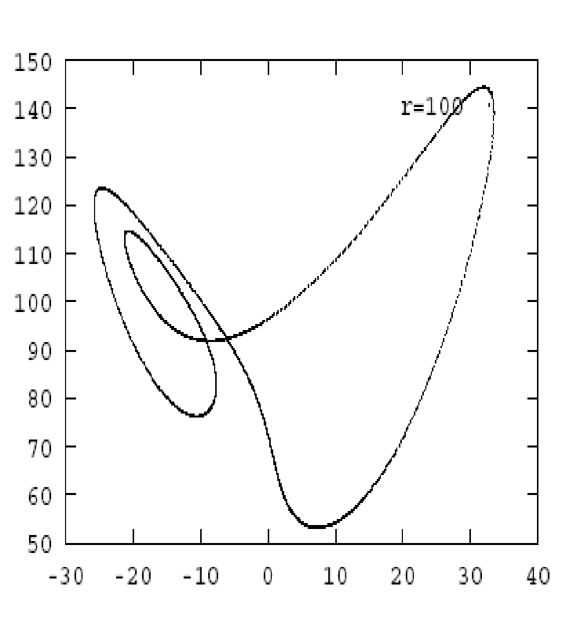
Example Lorentz (cont'd)

Phase space variation due to variation of the r parameter (2-D projections)

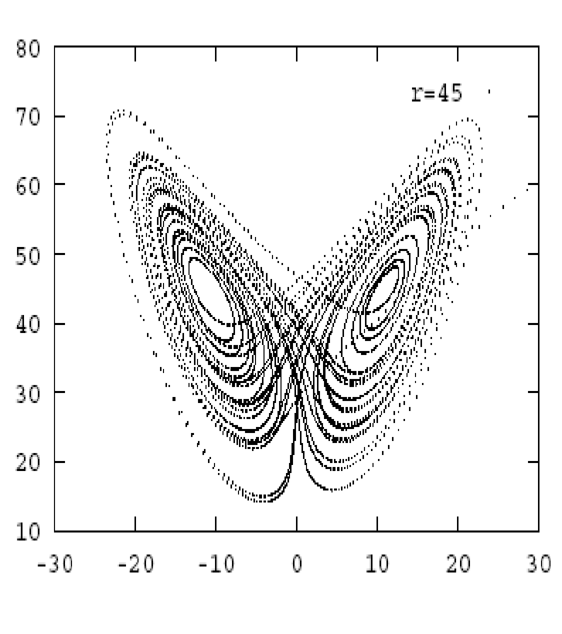
Geometry can help detect transitions!



Point attractor

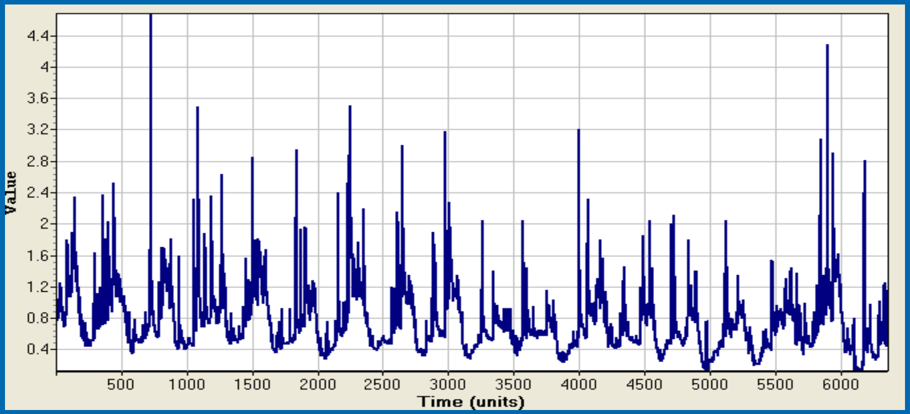


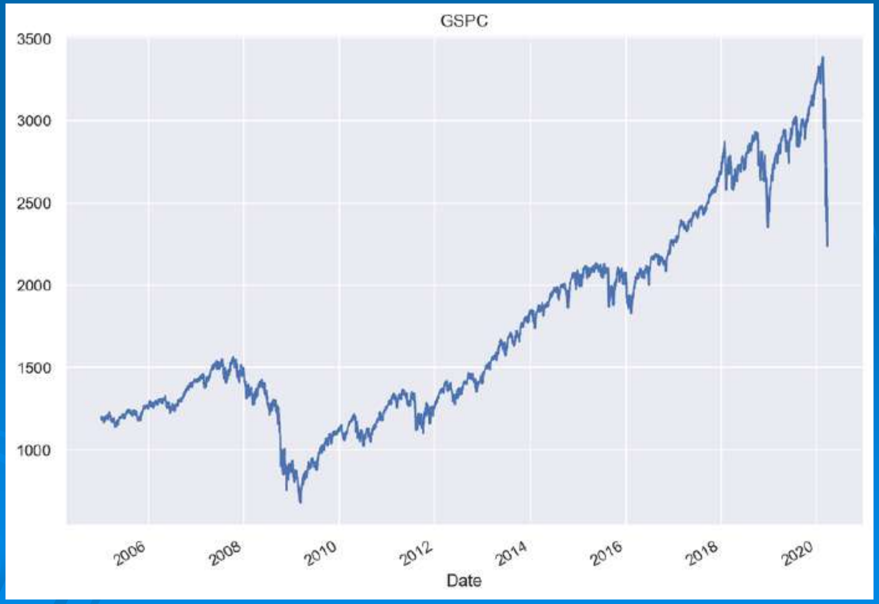
Periodic orbit



Chaotic (strange) attractor

Inverse problem
Known time series – unknown dynamical system





What can be the results of the analysis?

- Elements of system dynamics:

characteristic times/scales of the underlying system

- System Identification, Change of State
- Prediction of future values
- System Control

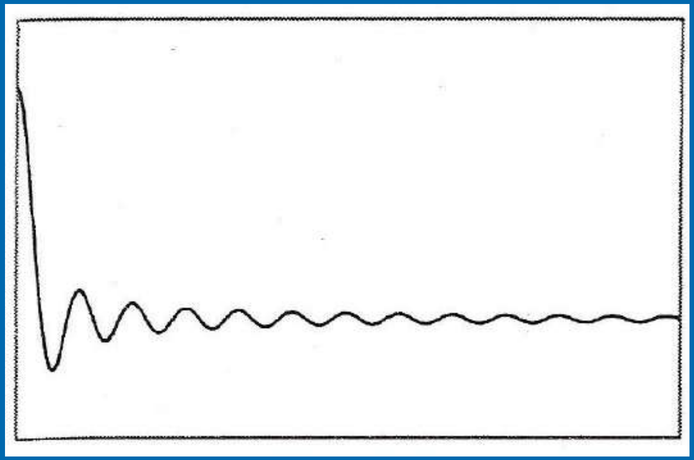
Time and Geometry
in asymptotic behavior of the system in time $\rightarrow \infty$

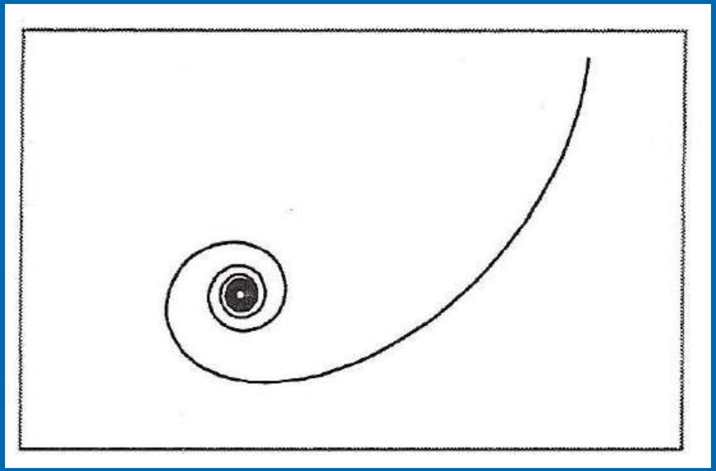
Point attractor

Time series

Phase space

x

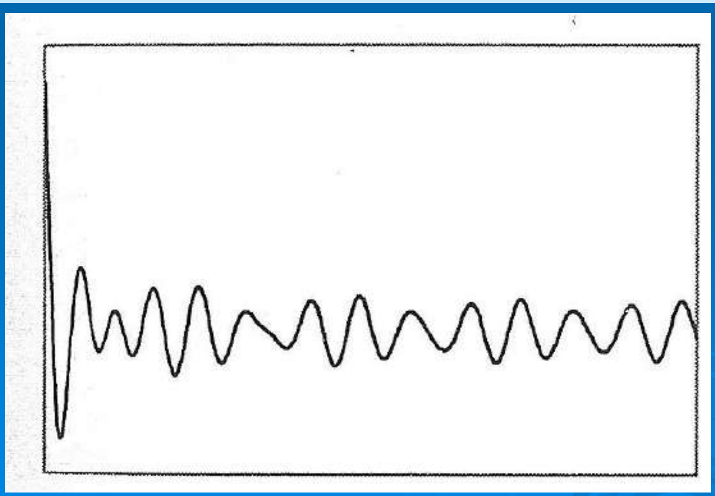


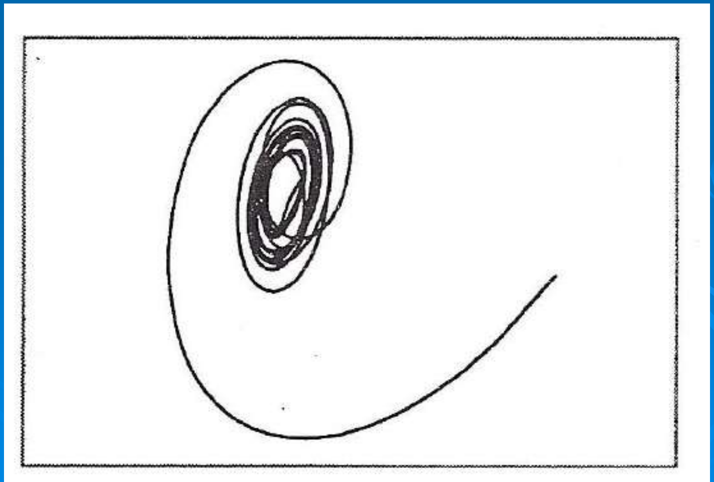


t

Limit cycle attractor

x





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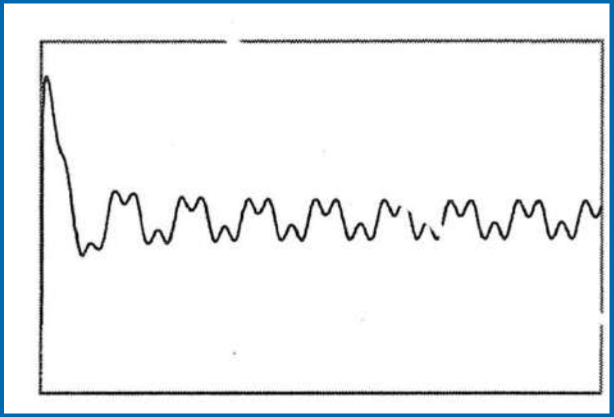
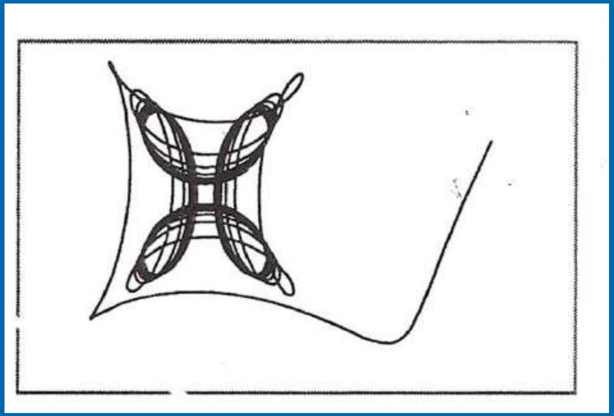
Toroidal attractor

Time series

Atrractor

x





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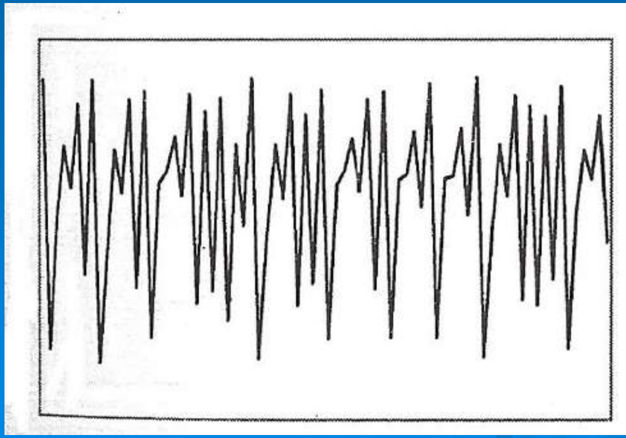
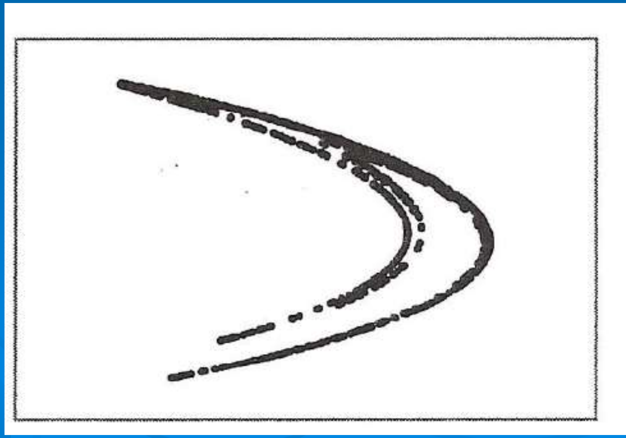
Chaotic or Strange attractor

x

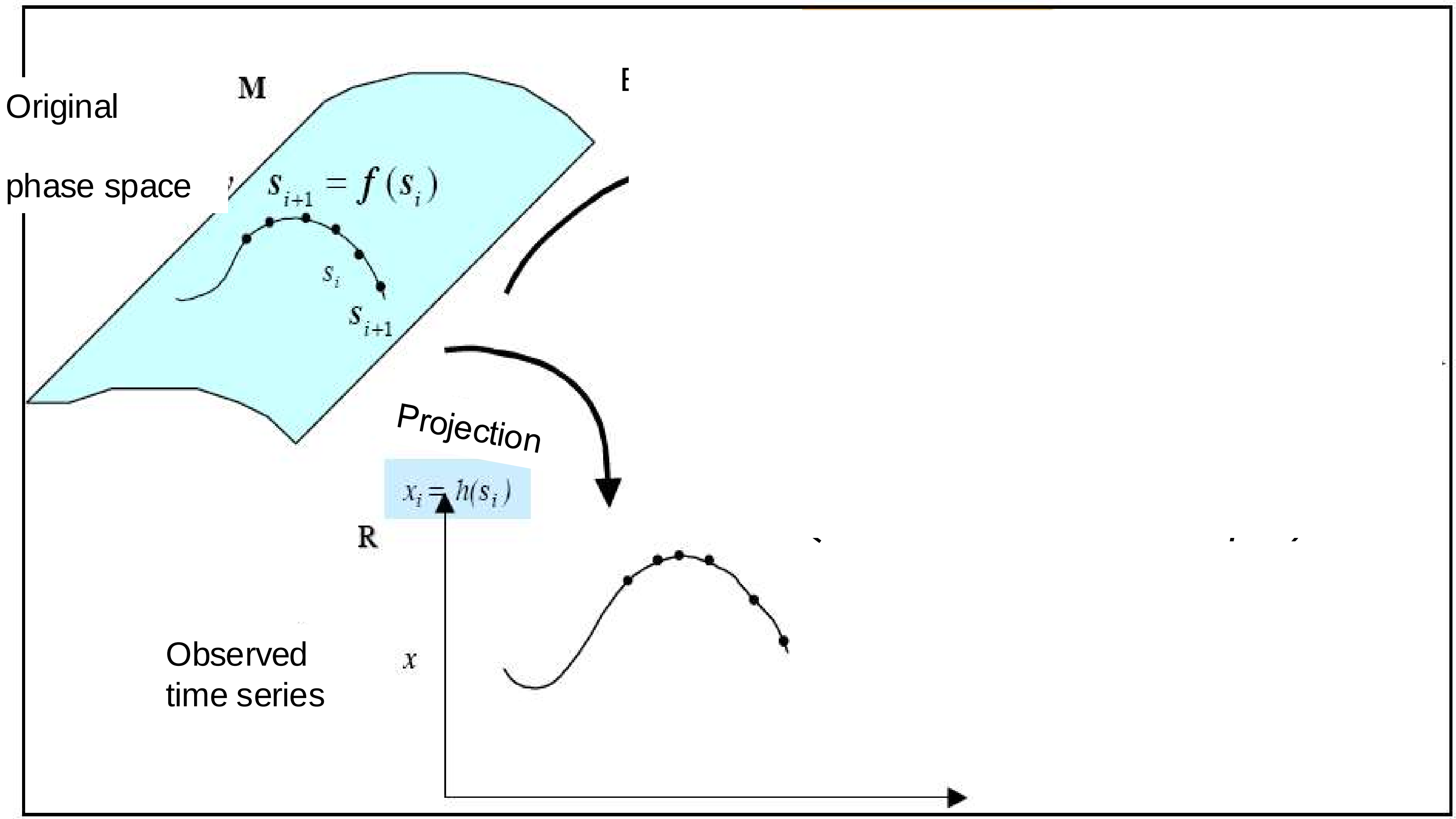




t

Examples of Analysis

Phase space reconstruction

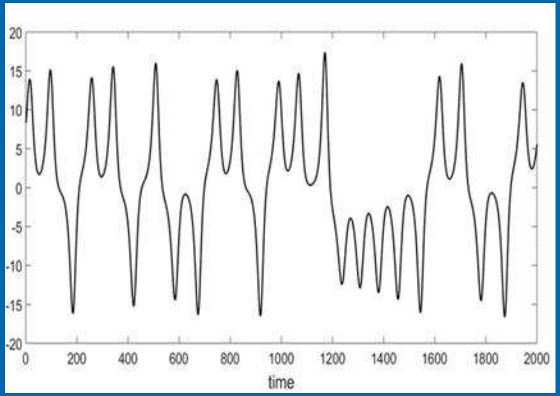


Kugiumtzis 1999

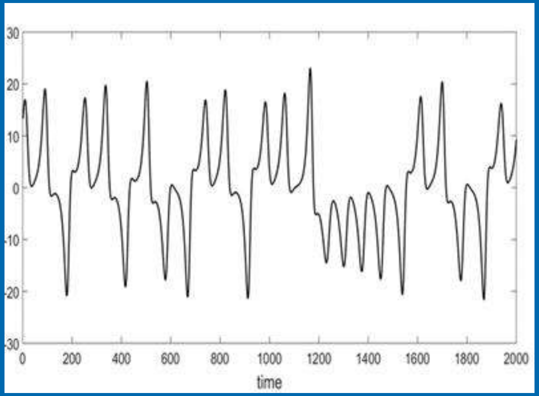
The invariant quantities of the two spaces must remain the same

Knowing just one tme series can we reconstruct the phase space????

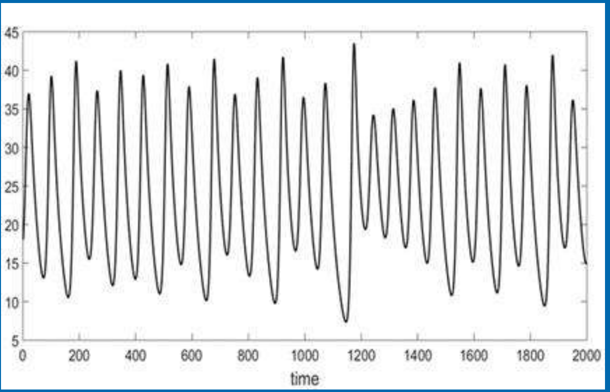
$x_1(t)$



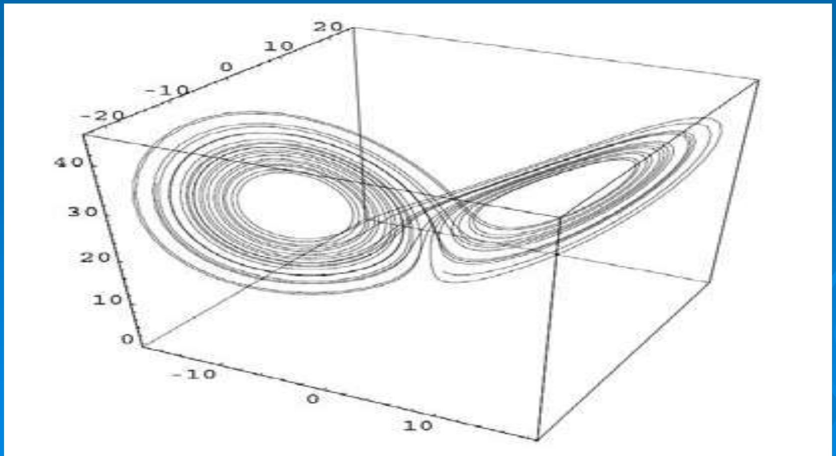
$x_2(t)$



$x_3(t)$



Phase space of the Lorenz system



Phase Space Reconstruction

- We are interested in
  metric (topological) properties such as the dimension of the attractor

- It is not necessary to reconstruct the full phase space since in the majority of cases the system presents an attractor with smaller dimension

- The reconstructed phase space has the following properties
  - Each point is mapped through the dynamics to a unique successive point
  - There is a smooth and nonsingular transformation between the reconstructed space and the original space.

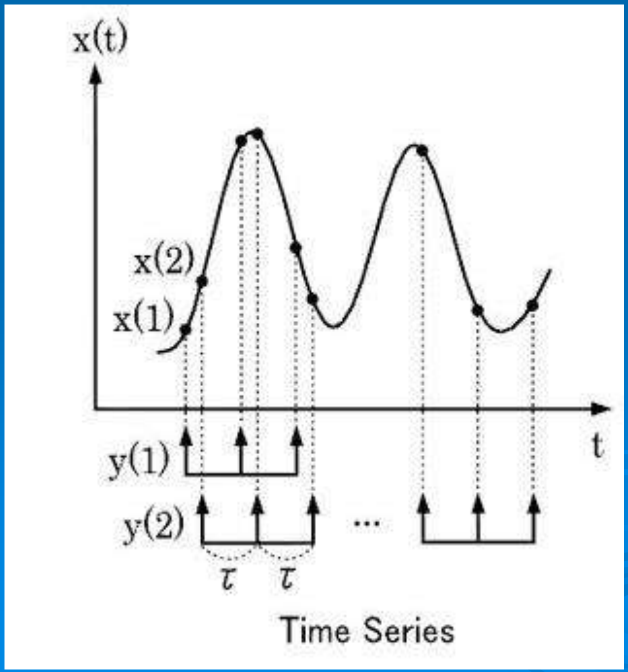
  - This methodology was introduced by Packard et al (1980) and Takens (1981)

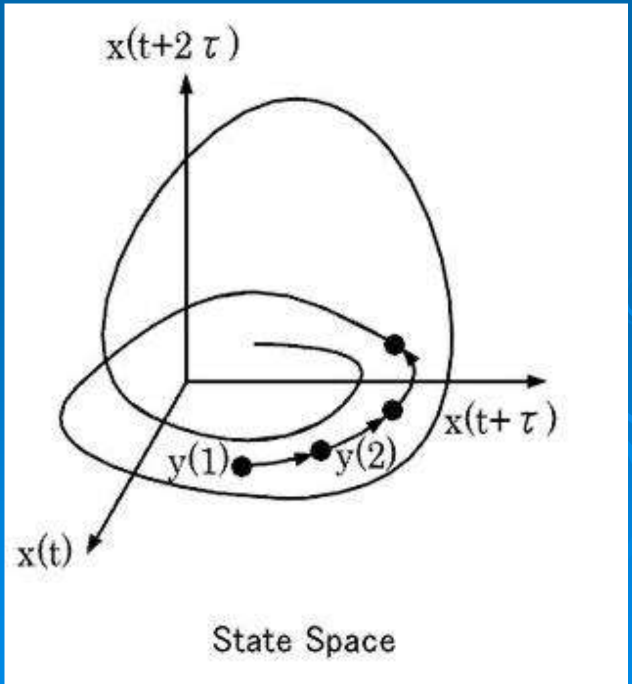
Phase Space Reconstruction: Method of Delays

- The reconstruction of the phase space is done via the construction of a m-dimensional vector states S_i from the time series x_i

$$S_i = \left[x_i, x_{i+\tau}, x_{i+2\tau}, \ldots, x_{i+(m-1)\tau} \right]$$

- Parameters necessary for the reconstruction
  - A) embedding dimension m (in what space I plot my points)
  - B) time delay τ (how close the timeseries points are)





Linear temporal tool: Autocorrelation function

$$R(\tau) = \frac{\frac{1}{N}\sum_{i=1}^{N}\left[x(i\Delta t+\tau)-\bar{x}\right]\left[x(i\Delta t)-\bar{x}\right]}{\frac{1}{N}\sum_{i=1}^{N}\left[x(i\Delta t)-\bar{x}\right]^2}$$

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x(i\Delta t)$$

Nonlinear temporal Tools: Average Mutual information

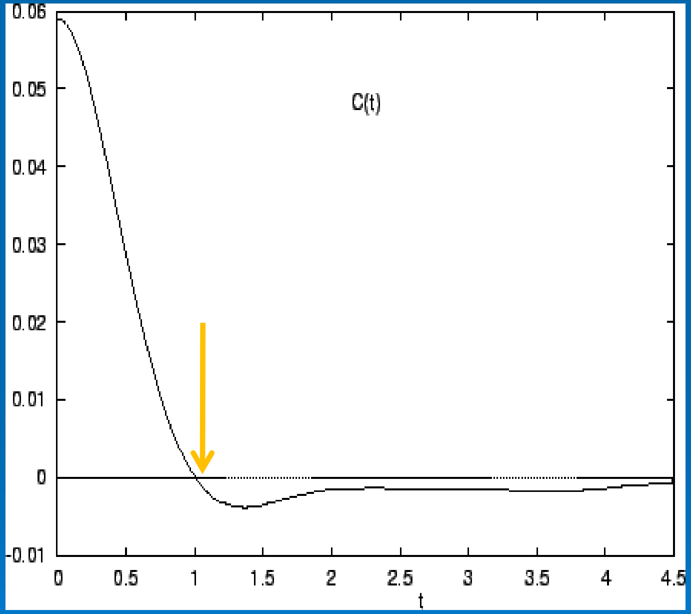
$$I = -\sum_{ij} p_{ij}(\tau)\ln\left(p_{ij}(\tau)/p_i p_j\right)$$

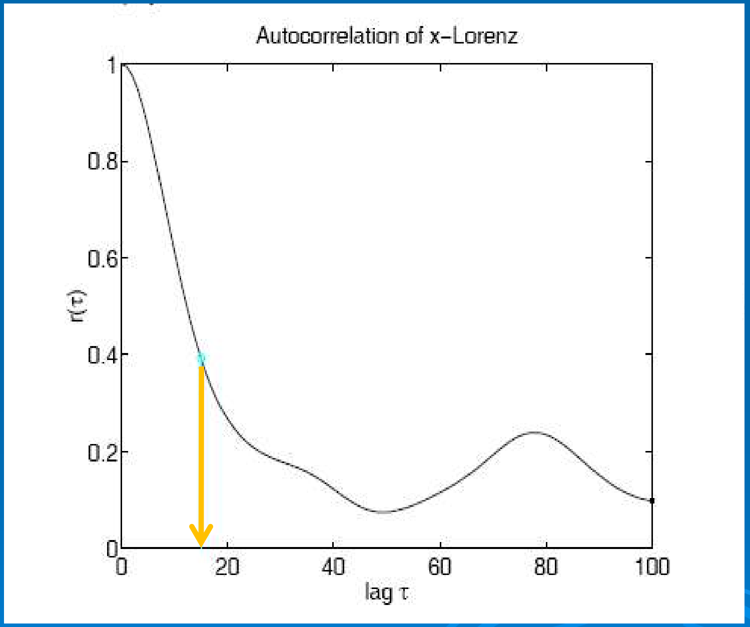
where for some partition on the real numbers, pi is the probability to find a value in the i-th interval, and pij(t) is the joint probability that an observation falls into the i-th interval and time t later falls into the j-th.

- AMI takes into account nonlinear correlations between successive measurements
- If I(τ) has a marked minimum at $t=t_d$, then it is argued that td is a reasonable choice of time delay for embedding in phase space reconstruction

Choice of time delay: Autocorrelation function (AF)

- We choose the time τ for which AF is zero for the first time.
- If AF does not fall to zero quite fast we chose as τ the time for which it falls to 1/e (approximately 40%) of the value for τ=0.





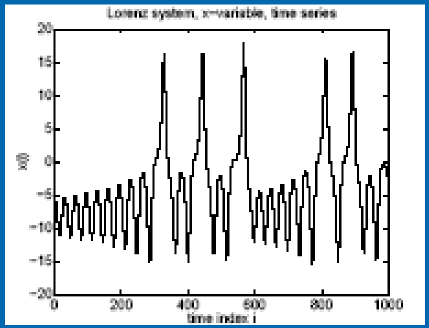
- Remember: AF takes into account only linear correlations between successive points of the time series.

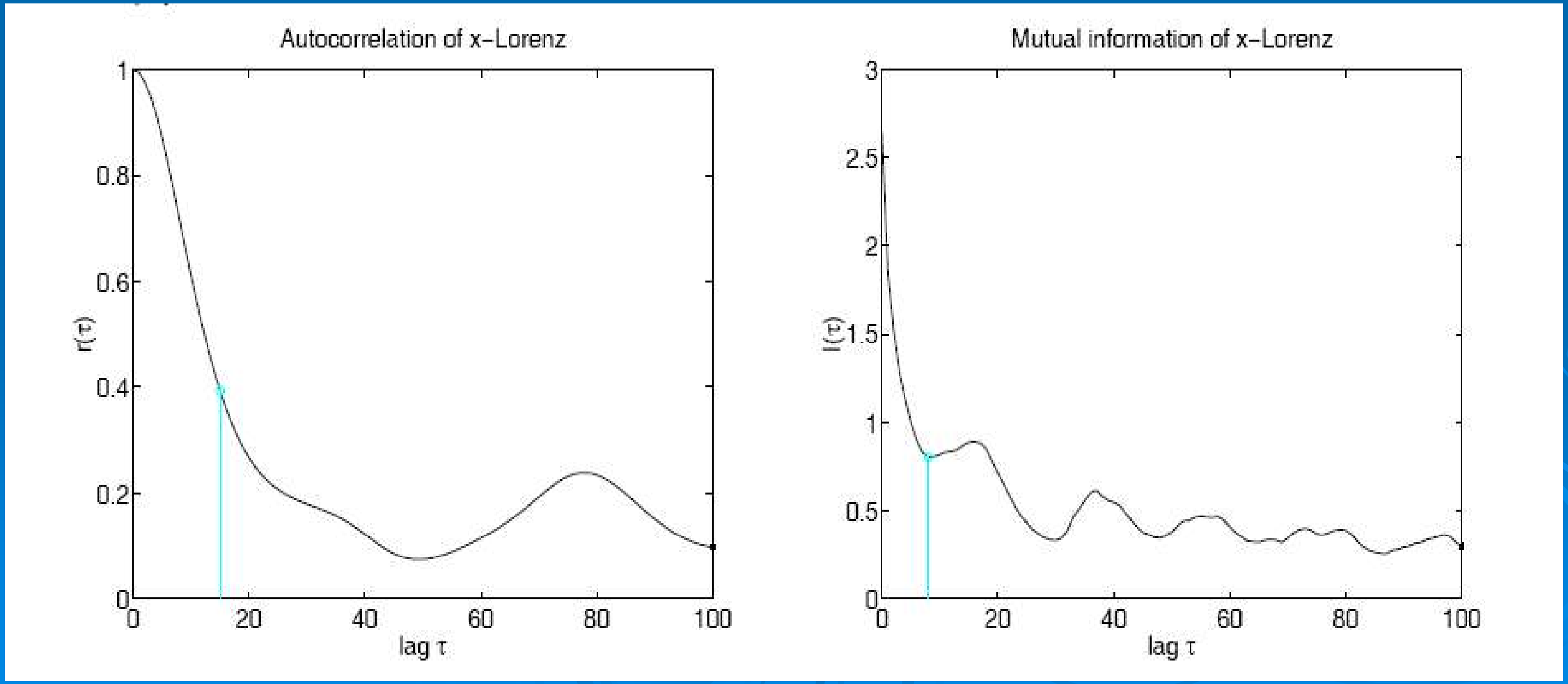
Example of choice of τ

- Lorentz system (only one component)

$$\dot{s}_1 = -a(s_1 - s_2)$$
$$\dot{s}_2 = -s_1 s_3 + b s_1 - s_2$$
$$\dot{s}_3 = s_1 s_2 - c s_3$$

a=10, b=28, c=8/3





Choice of embedding dimension m

Takens' theorem sets the condition $m \geq 2D+1$ where D is the attractor dimension.

Other researchers set a more loose condition: $m \geq D$

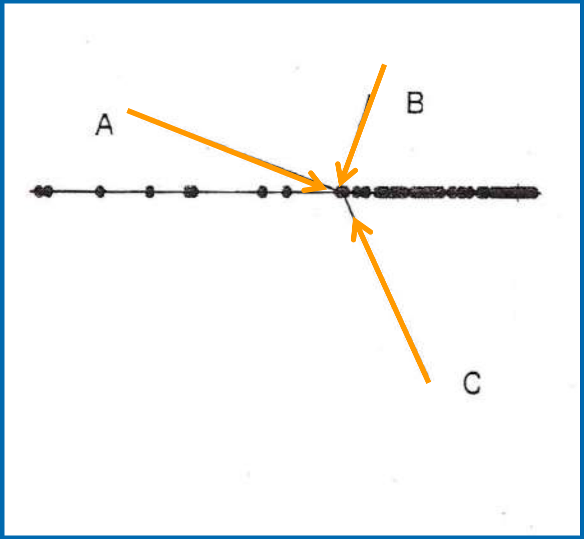
If we choose small embedding dimension m the attractor will be "squeezed" and will present self crossing, thus not being equivalent to the original attractor.

If the embedding dimension is larger than what is necessary the corresponding calculations will be unnecessarily more complex and time consuming.

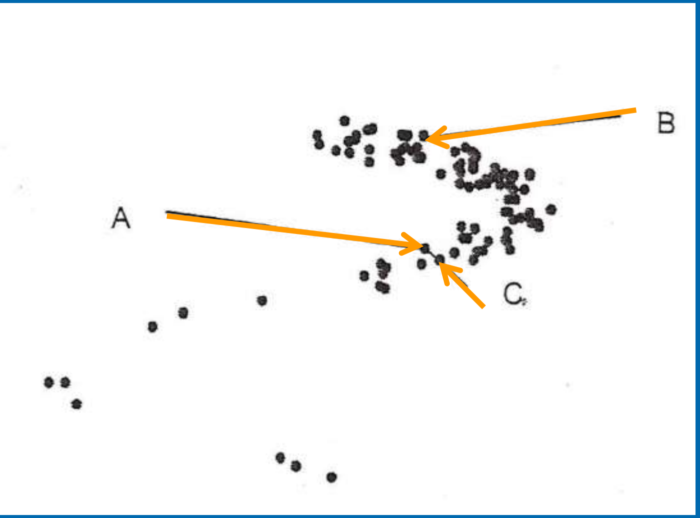
Method of False Nearest Neighbors

The principle of the method (Abarbanel 1993)

Embedding dimension m



Embedding dimension m+1

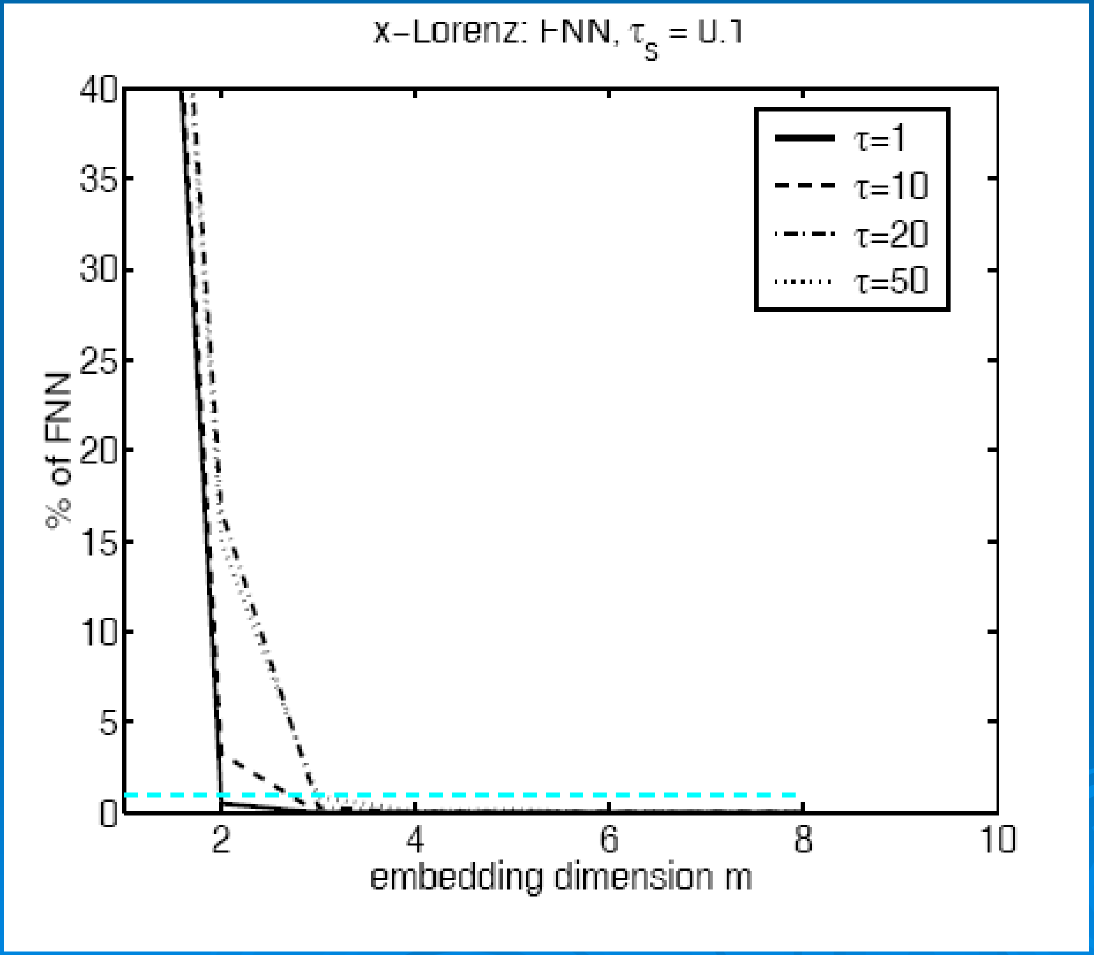


At small embedding dimension the attractor's points are quite close and points A, B and C seem to be neighbors.

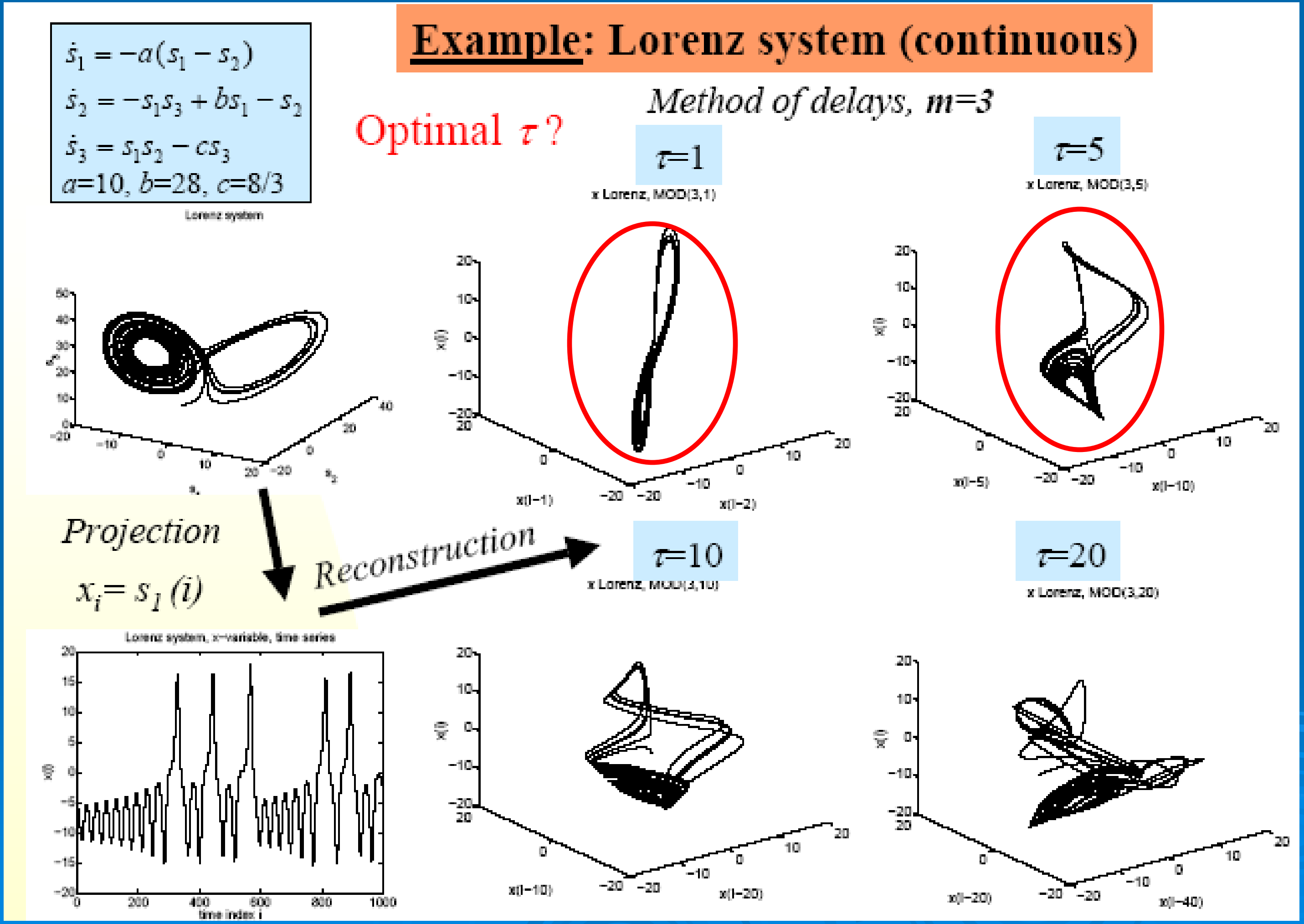
Increasing the embedding dimension by +1 point C remains neighbor of point A
However point B moves away from the neighborhood of point A.
Thus point B is a false nearest neighbor of point A. In contrast point C is a true neighbor of point A.

Method of False Nearest Neighbors

Application to the Lorenz system



Choice of reconstruction parameters: time delay and embedding dimension



Kugiumtzis 1999

SYSTEM-ATTRACTOR DIMENSION

- We suppose that the system possesses a low dimensional attractor i.e. for large times the system converges towards an attractor
- It is not necessary to know the dimension (d) of the full phase space but only the dimension of the subspace (D) that contains the attractor

- We are going to see how we can estimate the dimensionality of the system through the time series analysis.
- Notice: In the case of chaotic systems the dimension D of the attractor in general is not an integer since the attractor is a fractal object..

The basic principle of the methods

We embed the time series with increasing embedding dimension
With the aim to see if we have any convergence
Of the object that is formed (hopefully the attractor).

Correlation Dimension (1)

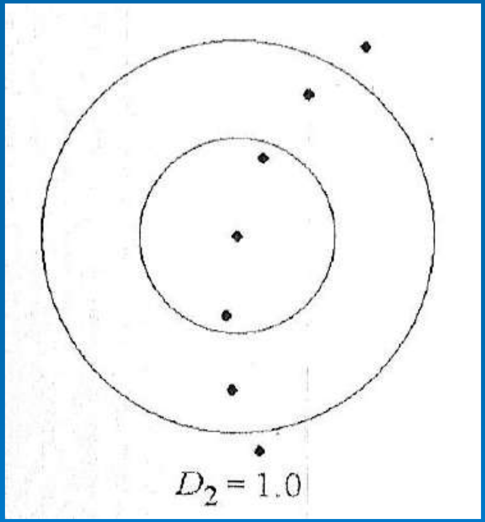
- An attractor is a set of points x_i and let $P(|x_i-x_j|<r)$ the probability of two points of the attractor to be within a distance smaller than r.
- If C(r) is the number of points that are located within a hypersphere of radius r centered on point x_i, then the average time <N(r)> approximates well the above probability.
- Then one expect to have a scaling law

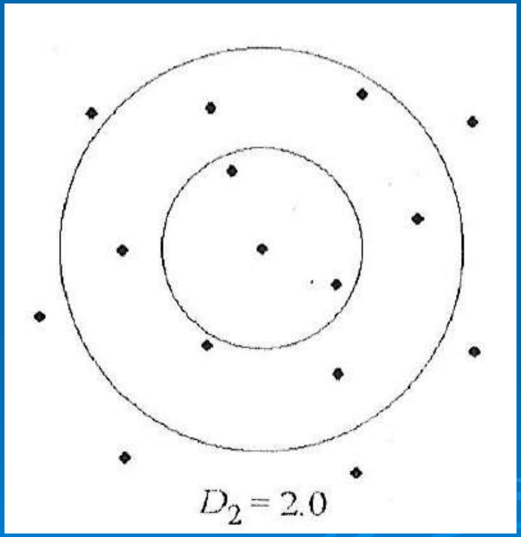
$$C(r) \propto r^{\nu}$$

- i.e. in fact the probability that two points of the attractor lie within a distance r is proportian to the power of a distance, with a constant exponent v. This relation is valid for $r \to 0$.

Correlation dimension (2)

- Suppose that the object is formed from N points in an embedding space (lets suppose 2).
- This can be seen also as a graph of pairs (2-d reconstructed vectors) (X_n, X_{n-1}) i.e. delay τ=1).
- Lets consider a points as an origin an design a circle of radius r around this point and measure the number of points within the circle. We apply the process for circles of various sizes for all the points.





$N \propto r^1$

$N \propto r^2$

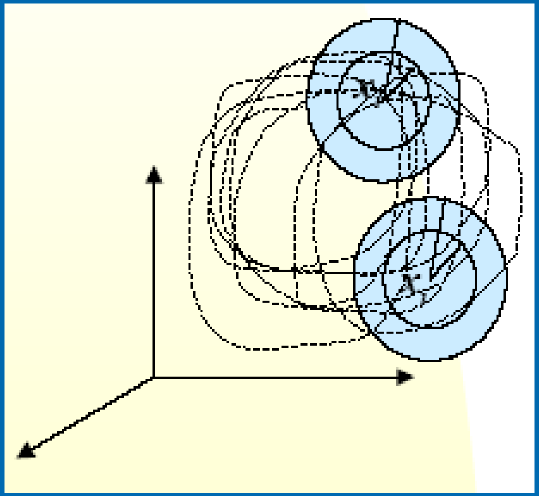
Correlation Sum (1)

- <C(r)> for a set of points x_i i=1,..N as it is the reconstructed trajectory from the time series can be estimated from the correlation sum C(r).

$$C(r) = \frac{2}{N(N-1)} \sum_{i=1}^{N} \sum_{j=i+1}^{N} \Theta(r - \| \underline{x}_i - \underline{x}_j \|)$$

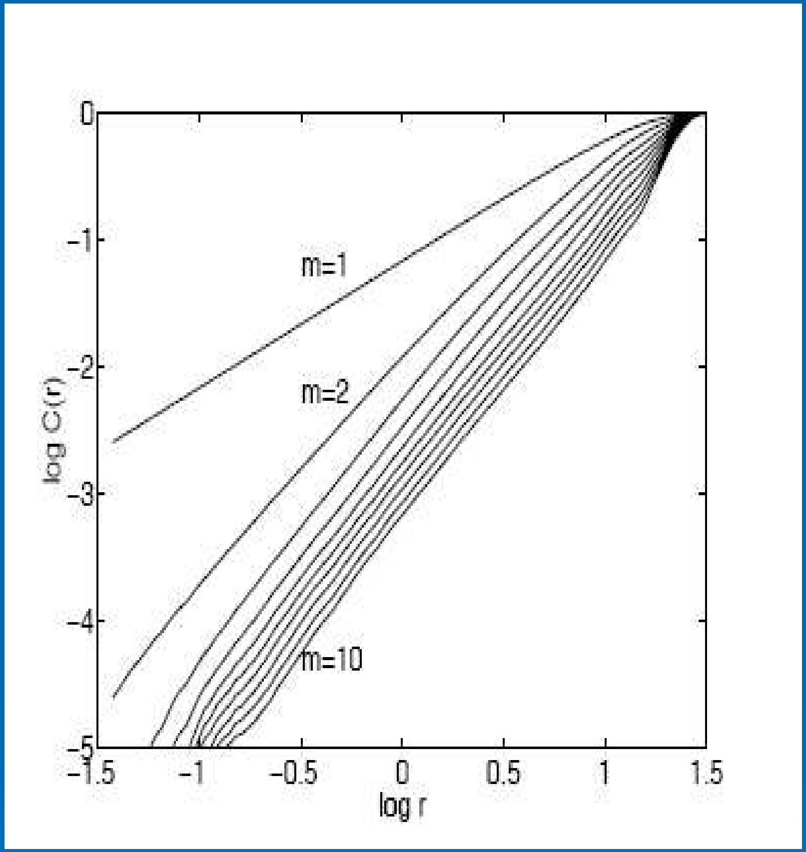
Where Θ(x) is the Heaviside function

$$\Theta(x) = \begin{cases} 0 & x \leq 0 \\ 1 & x > 0 \end{cases}$$



- C(r) counts all the possible pairs of points (x_i, x_j) of the reconstructed trajectory, that are located within a distance r, for a given embedding dimension.
- i.e. in fact C(r,m).

Correlation Sum (2)



$$C(r) = \frac{2}{N(N-1)} \sum_{i=1}^{N} \sum_{j=i+1}^{N} \Theta(r - \| \underline{x}_i - \underline{x}_j \|)$$

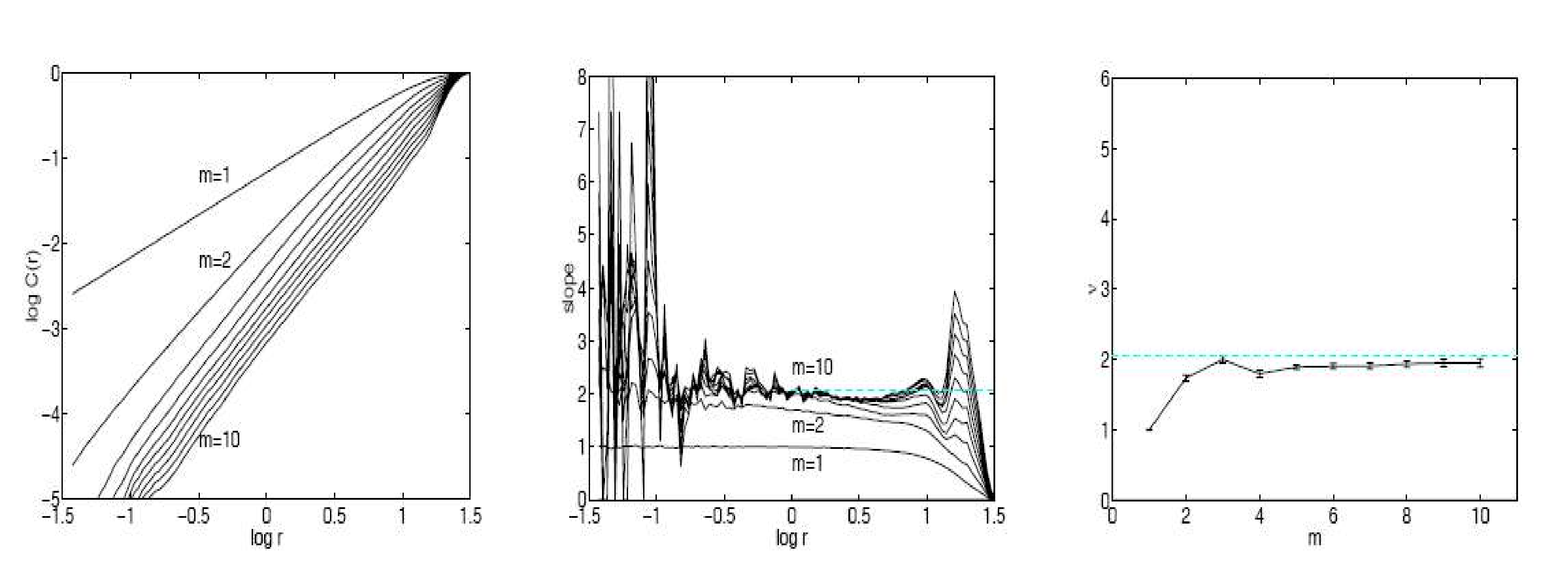
$$\nu = \lim_{r \to 0} \lim_{N \to \infty} \frac{d \log C(r)}{\log r}$$

- the two limits (N→∞ και r→0) cannot be satisfied for a real time series since it has a finite length and the data are of finite precision.
- In practice we expect that the graph LogC(r) vs. logr results in a straight line for a given width of relatively small values of r. This region is called scaling region.
- In general v is denoted as D_2.

Example of estimation of correlation dimension

Lorentz system in the chaotic region

τ_s=0.1s, τ=2τ_s



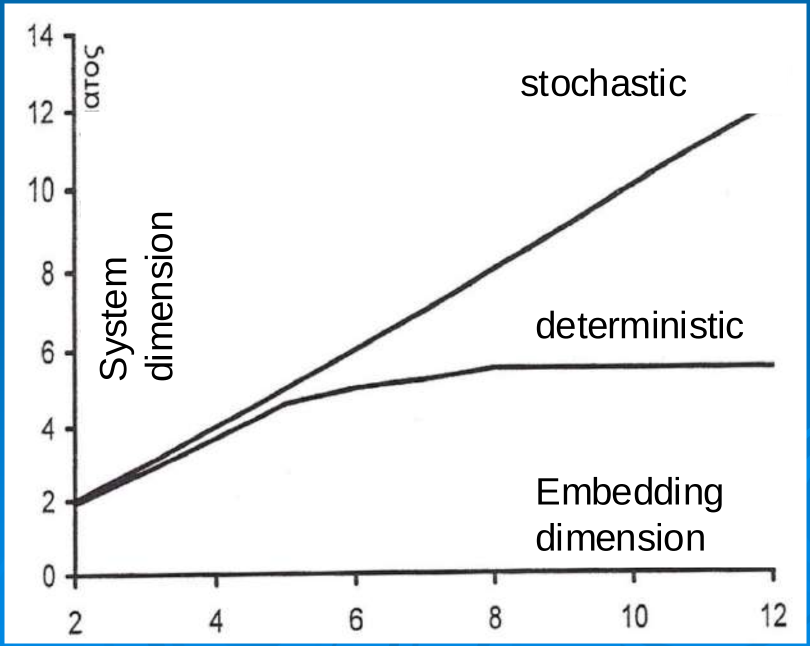
Scaling region: -0.5< logr <1

D_2 close to 2

Kugiumtzis 1999

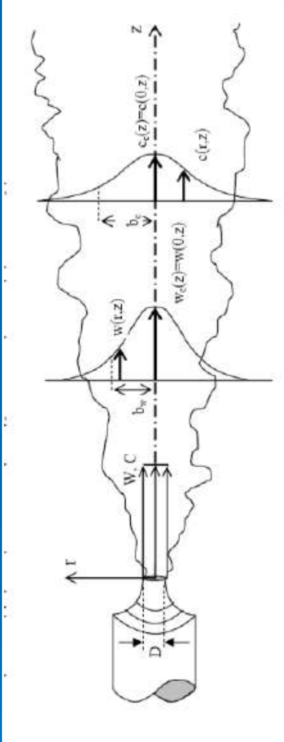
EMBEDDING vs ATTRACTOR DIMESNION

- When we have a white noise (stochastic) process then the dimension of the object formed is practically the same as the embedding dimension, since the system visits with the same probability all the available (or nearly all) points of the phase space.
- In the case of a deterministic system if there is an attractor the dimension will converge toward a value D smaller than the dimension d of the phase space



Turbulence Fluid Complexity

Large Number of Environmental and Engineering Application of Turbulent jet flow

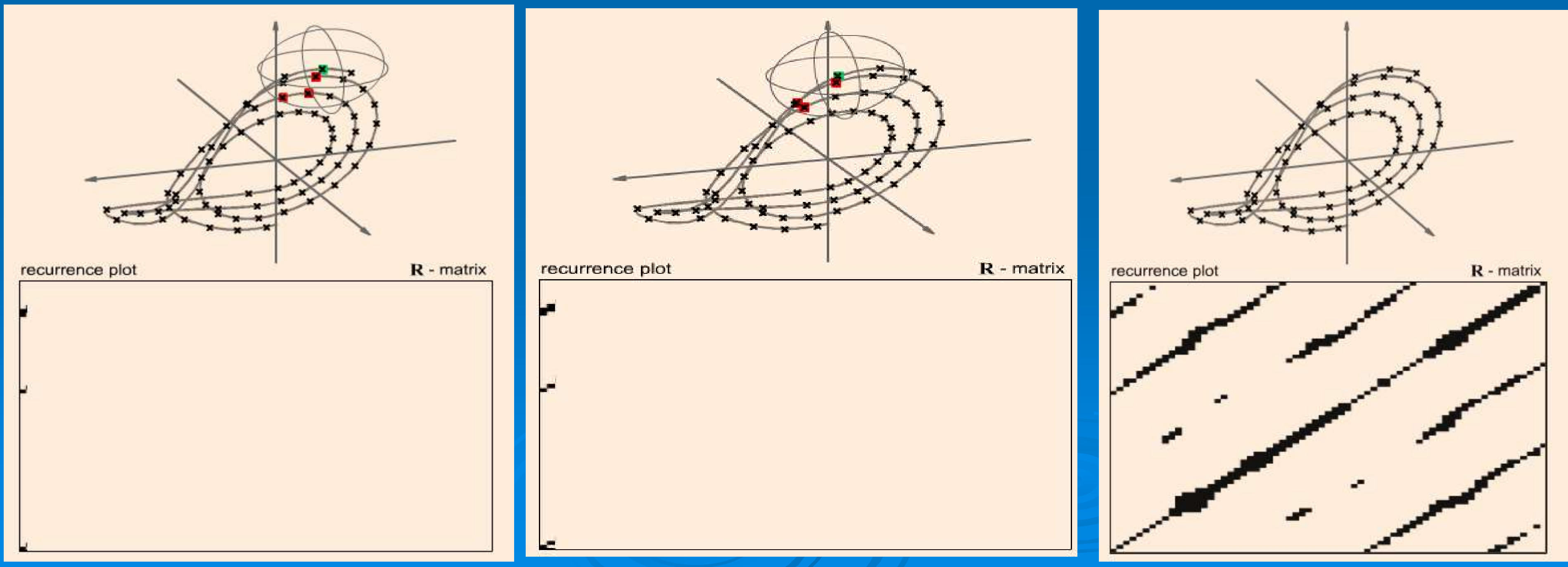


Recurrence Plots

- step 1: Phase Space Reconstruction
- step 2: Calculate the distances between the state vectors
- step 3: Set a cut off value ε for the distance

which are within a distance ε are defined as recurrent points

$$\mathrm{R}_{i,j}^{m,\varepsilon_i} = \Theta(\varepsilon_i - \|\vec{x}_i - \vec{x}_j\|), \quad x_i \in R^m, \quad i,j = 1 \ldots N$$



Several characteristic structures can appear in a R.P.

- Single isolated points
  (homogeneity)

- Diagonal lines
  (Trajectory visits the same region of the phase space at different times. Maybe deterministic process)

- Horizontal, Vertical lines/clusters
  (The state is trapped for some time)

- Periodic patterns
  (periodicities)

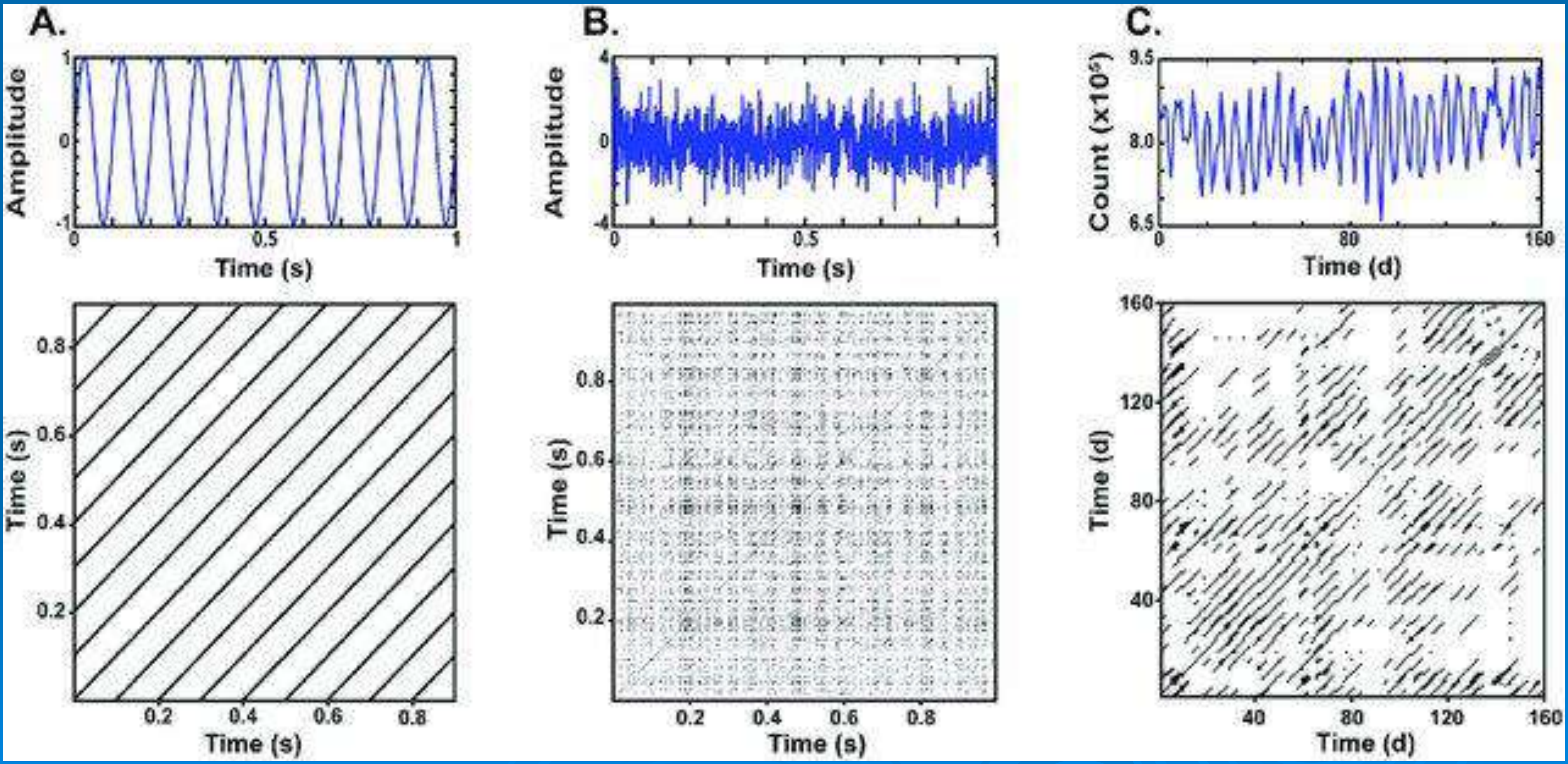
- White bands
  (abrupt changes in dynamics)

Constructing Recurrence Plots

Periodic signal

White noise

Signal with thrend and periods



Recurrence Quantification Analysis

- Zbillut and Weber (1992)
- %recurrence (ratio of the number of recurrence points (pixels) to the total number of points (pixels) of the plot

$$\%REC = \frac{\sum_{i,j=1}^{N} R_{i,j}}{N^2}$$

$$R_{i,j} = \begin{cases} 1 & ,(i,j) \text{ recurrent} \\ 0 & ,\text{otherwise} \end{cases}$$

- %determinism (number of recurrent points forming diagonal lines to the whole number of recurrent points)

$$\%DET = \frac{\sum_{l=l_{\min}}^{N} lP(l)}{\sum_{l=1}^{N} lP(l)}$$

- MaxLine (longest diagonal line segment)

$$L_{\max} = \max(\{l_i; i = 1,\ldots,N_l\})$$

- Trapping Time (average length of the vertical/horizontal structures)

$$TT = \frac{\sum_{\upsilon=\upsilon_{\min}}^{N} \upsilon P(\upsilon)}{\sum_{\upsilon=\upsilon_{\min}}^{N} P(\upsilon)}$$

Applications of Recurrence plots

Case 1

Identify spatial variations
(spatiotemporal phenomena)

Case 2

Identify transitions
in the system evolution

Recurrence Quantification Analysis of MHD turbulent channel flow-Simulations

- Recurrence Plots (RPs) and Recurrence Quantification Analysis (RQA) of time series of velocities in low-Reynolds-number magneto hydrodynamic turbulent channel flow.

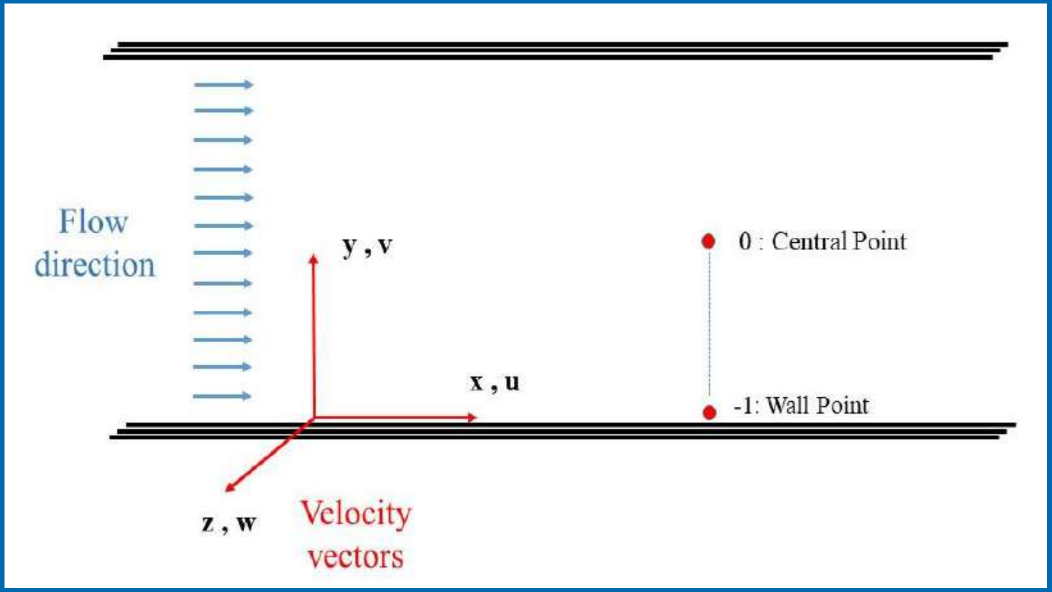
- Flow simulated using a fully spectral code with Fourier and Chebyshev decomposition in the periodic and wall bounded directions, respectively. Direct numerical simulations, for $Re_\tau = 180$, based on the friction velocity and the channel half-height, for the hydrodynamic flow with a streamwise magnetic field.

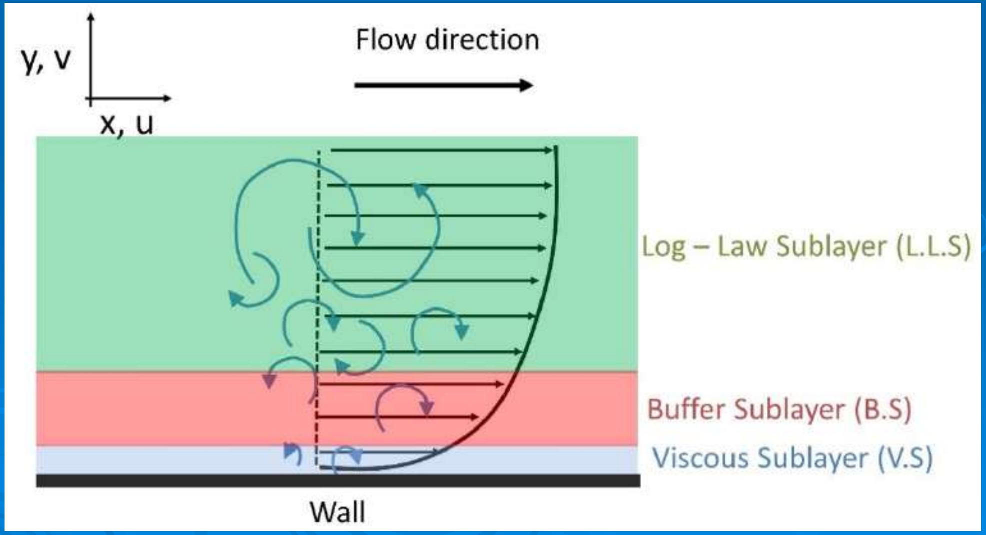
- The velocity time series (with and without magnetic field B) of the flow were recorded at several positions in the wall-normal direction and analyzed along all directions using RPs and RQA.

- Several distinct system regimes were identified and the effect of the magnetic field was localized.

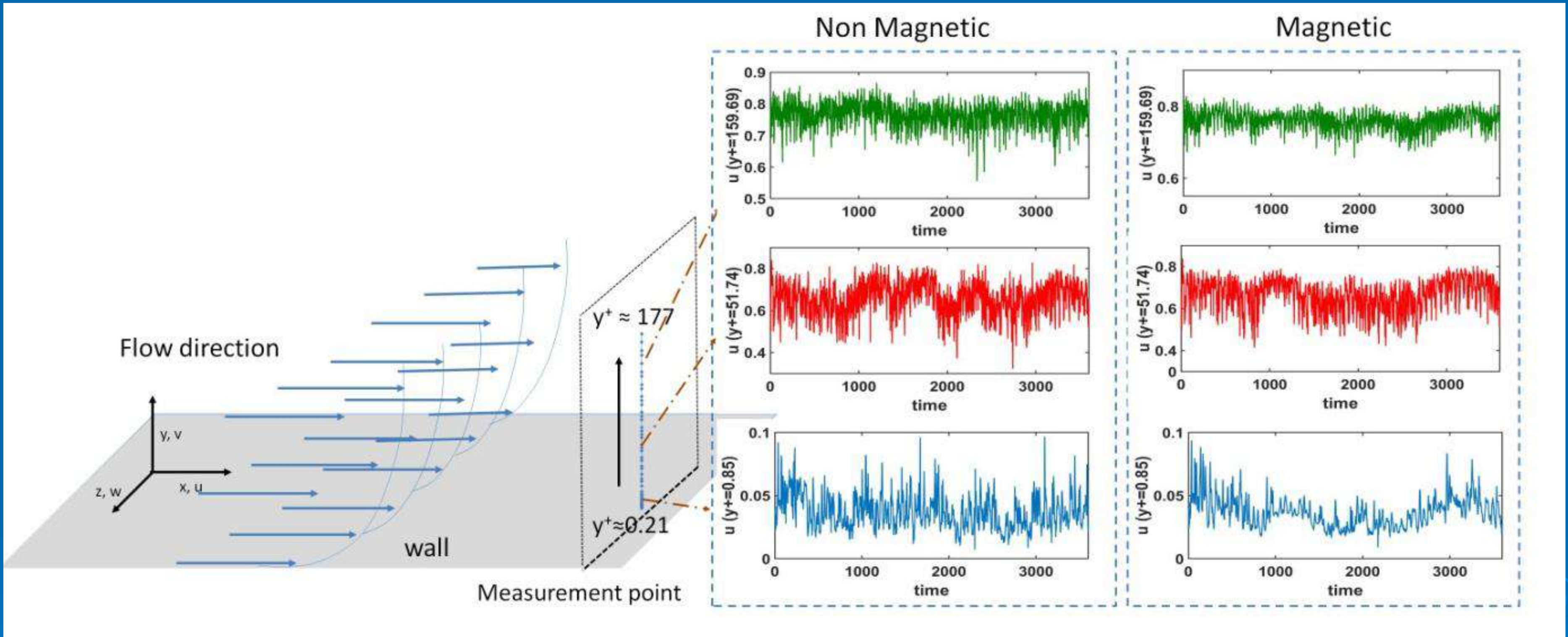
A.D. Fragkou, T.E. Karakasidis, I.E. Sarris, Physica A (2019)

Turbulent channel flow-Simulations

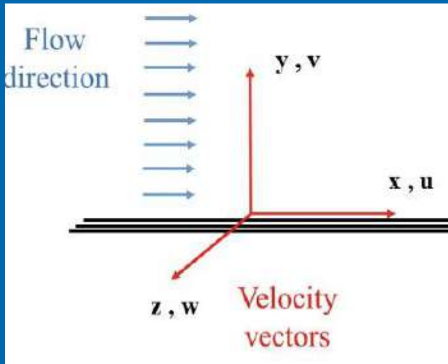




Turbulent channel flow-Simulations



Fragkou, A. D., Karakasidis, T. E., & Sarris, I. E. (2019). Recurrence quantification analysis of MHD turbulent channel flow. *Physica A*, *531*, 121741.



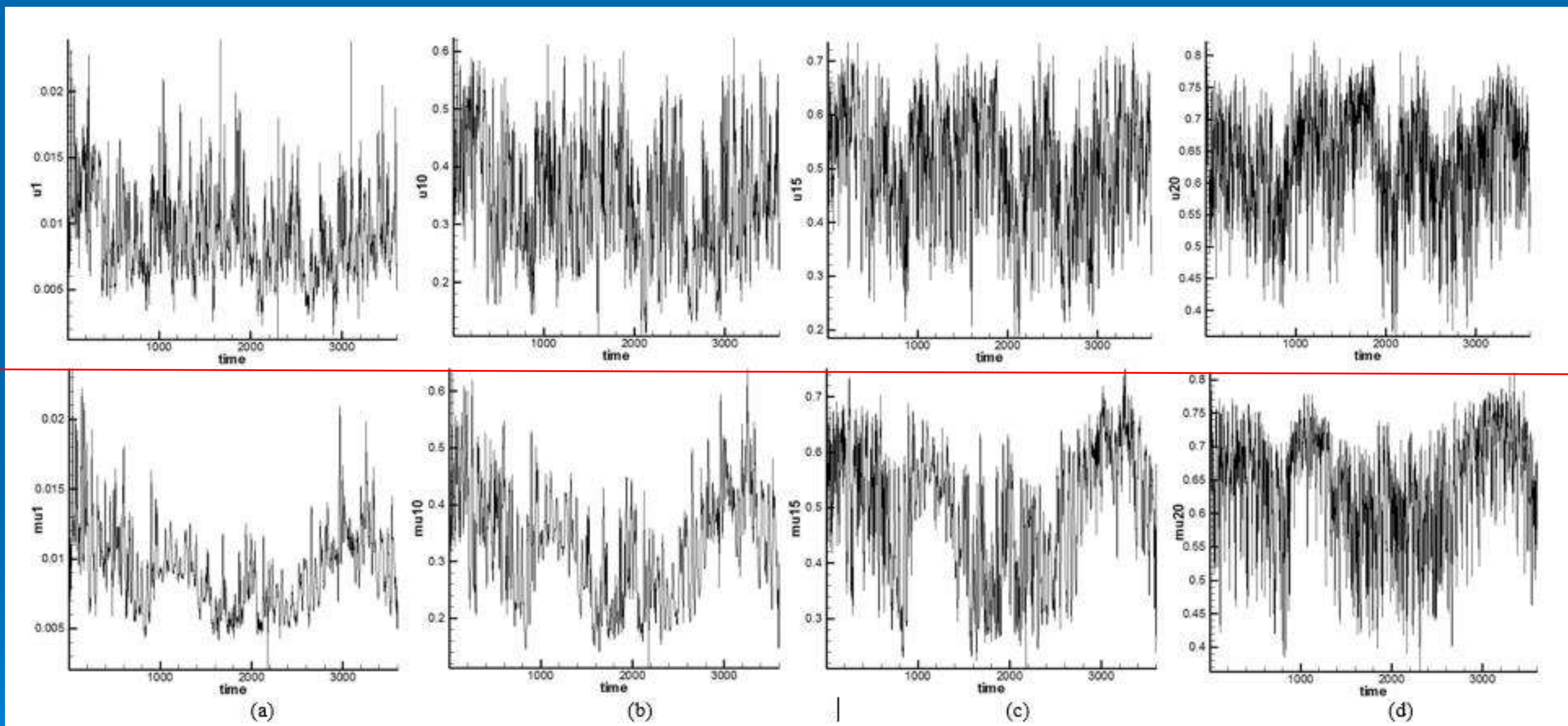
u velocity component

Close to the wall

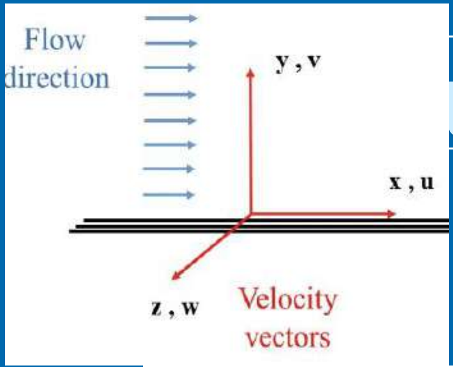
Intermediate distance

No B

with B



a) y=-0.99812, b) y= -0.94561 c) y= -0.89867 d) y= -0.76517,

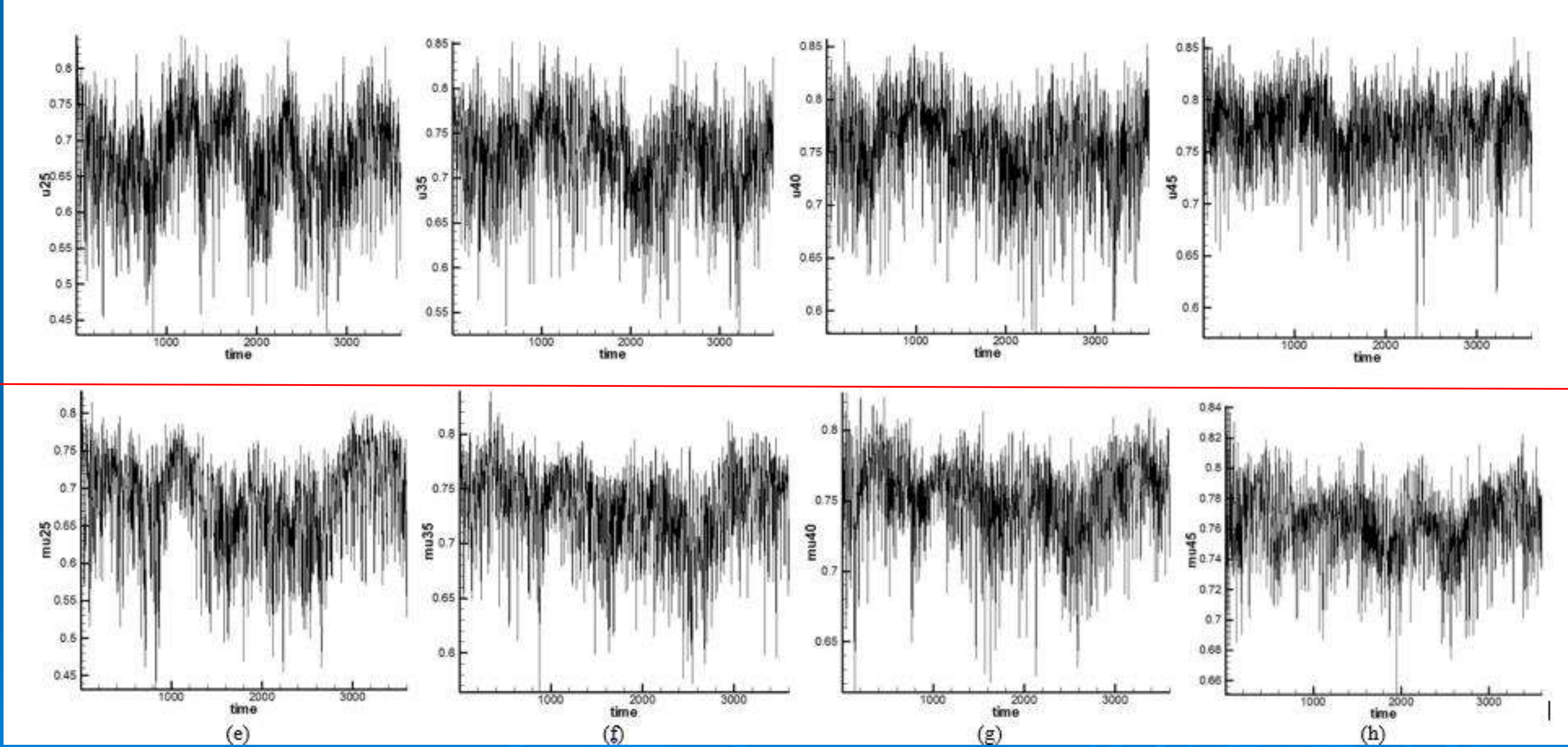


u velocity component

Towards the centerline

No B

with B



e) y= -0.64383, f) y= -0.37132, g) y= -0.20711 h) y= -0.012272.

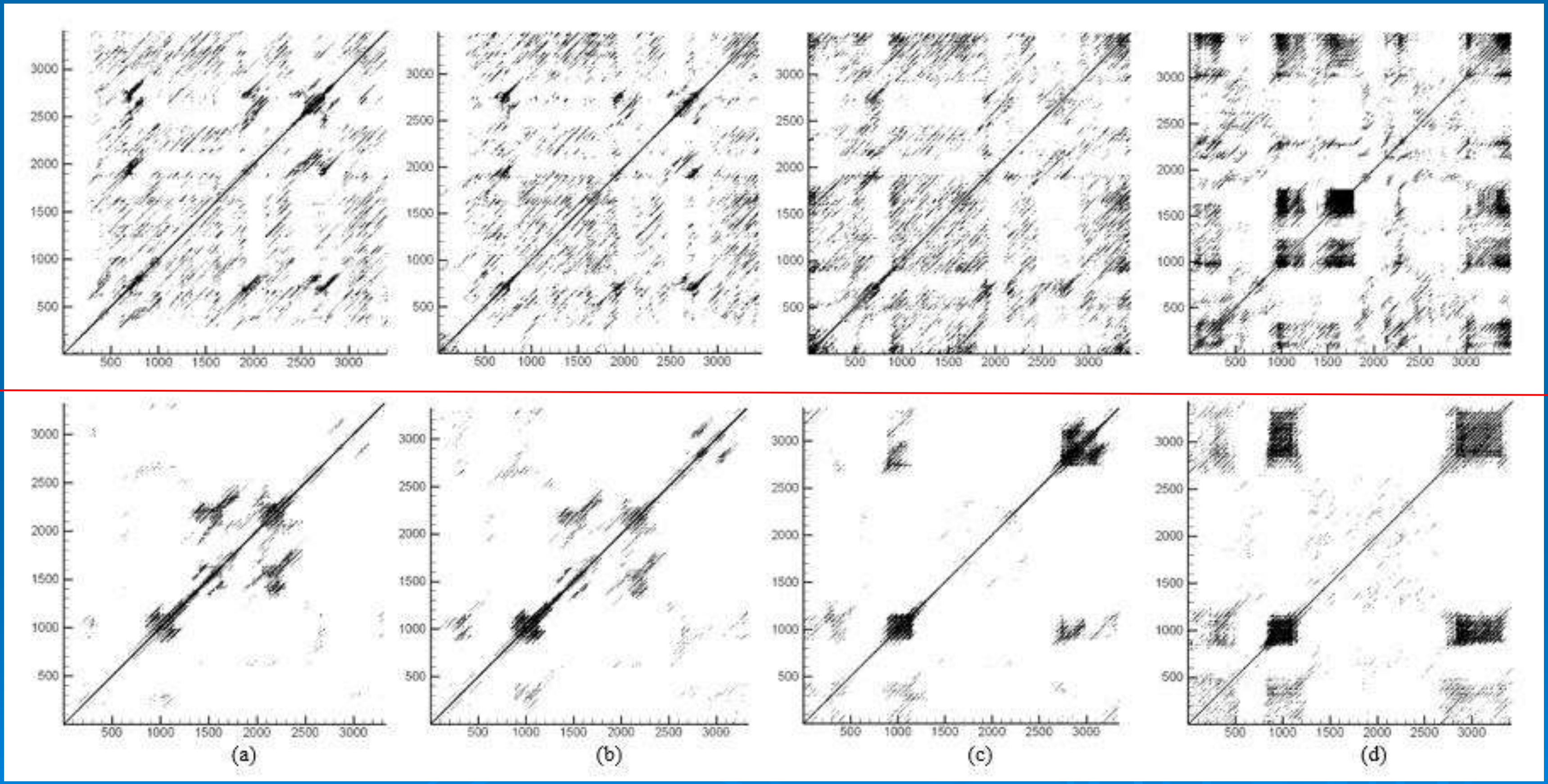
the u velocity component RPs

Close to the wall

Intermediate distance

No B

with B



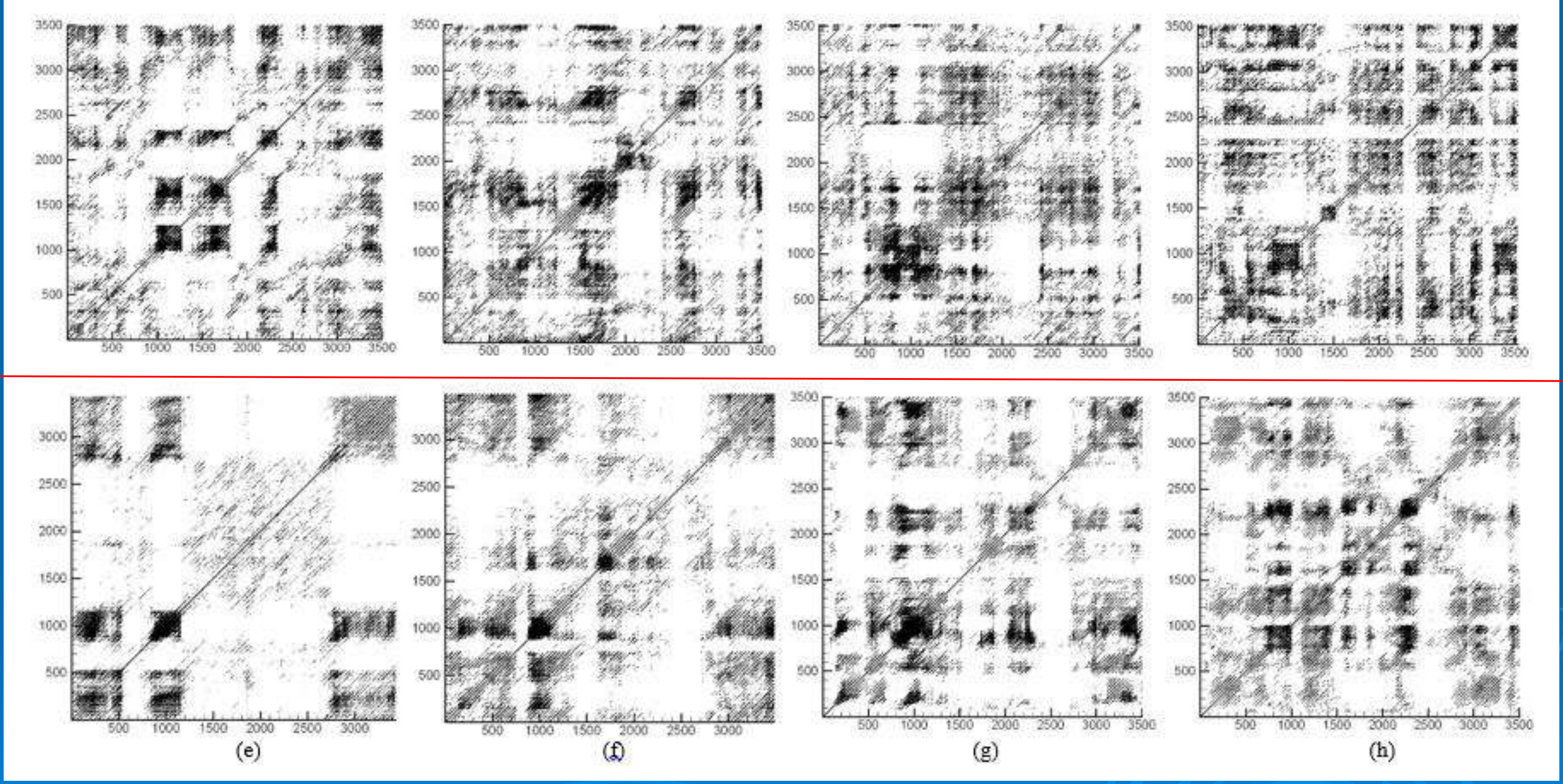
a) y=-0.99812, b) y= -0.94561 c) y= -0.89867 d) y= -0.76517,

the u velocity component RPs

Towards the centerline

No B

with B



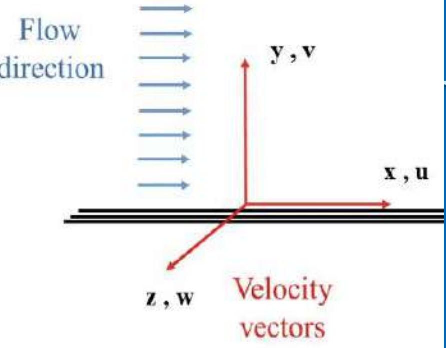
e) y= -0.64383, f) y= -0.37132, g) y= -0.20711 h) y= -0.012272.

Some reminders from fluid dynamics

Close to the walls
we have creation of streak structures which travel towards the center of the channel and they finish their trajectory in the fully developed turbulence region.

This affects the fluid region close to the walls for a long time while their effect in the central region is short in time (large values of Trapping Time close to the walls and small values of Trapping time in the central region).

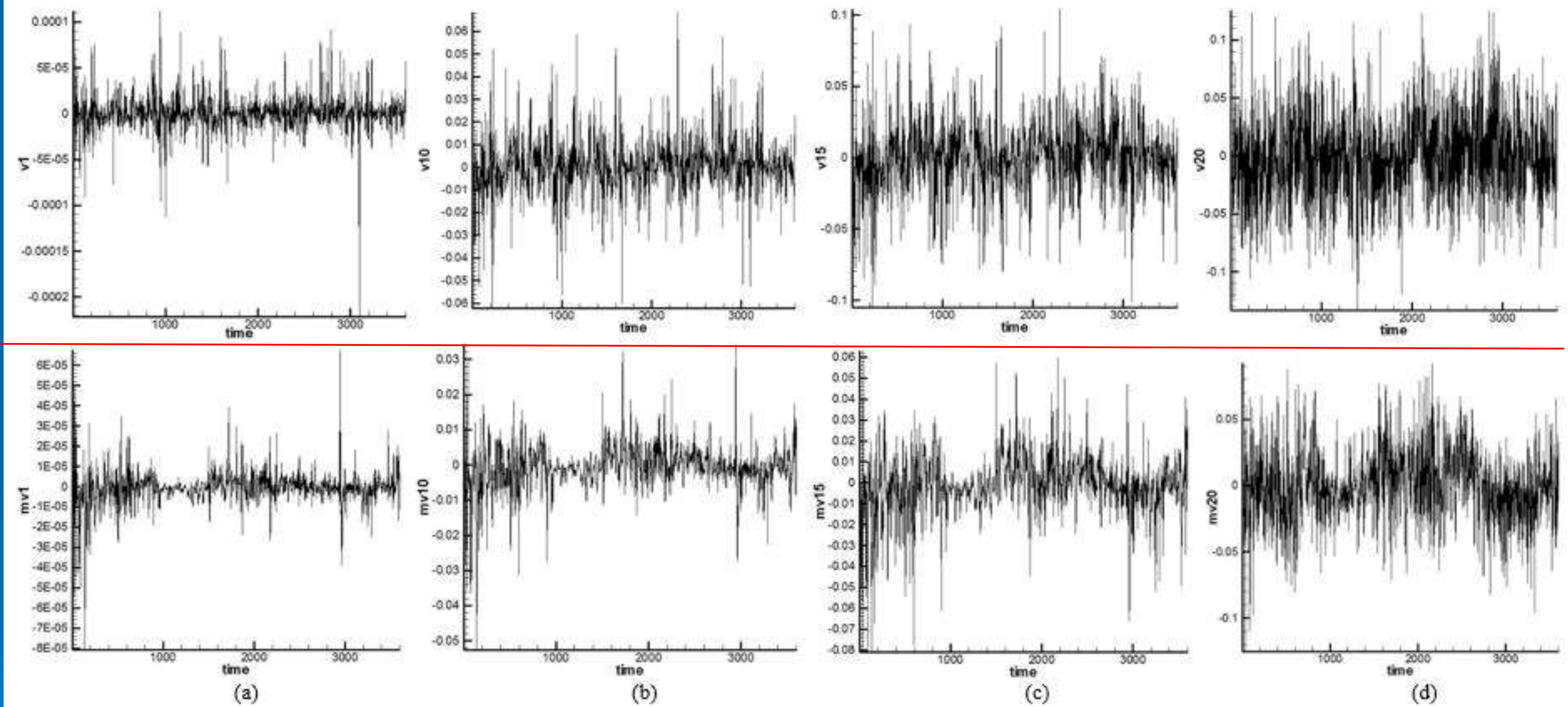
In the center of the channel: we expect to have fully developed turbulence coalescence of vortices very fast → the corresponding fluid layers will lose memory faster and their velocities become more uncorrelated (large white areas in the RPs) compared to the regions close to the walls, while we have small parallel lines in the RPs and of the parameters %DET, maximum diagonal line from RQA).



V velocity component

No B

with B



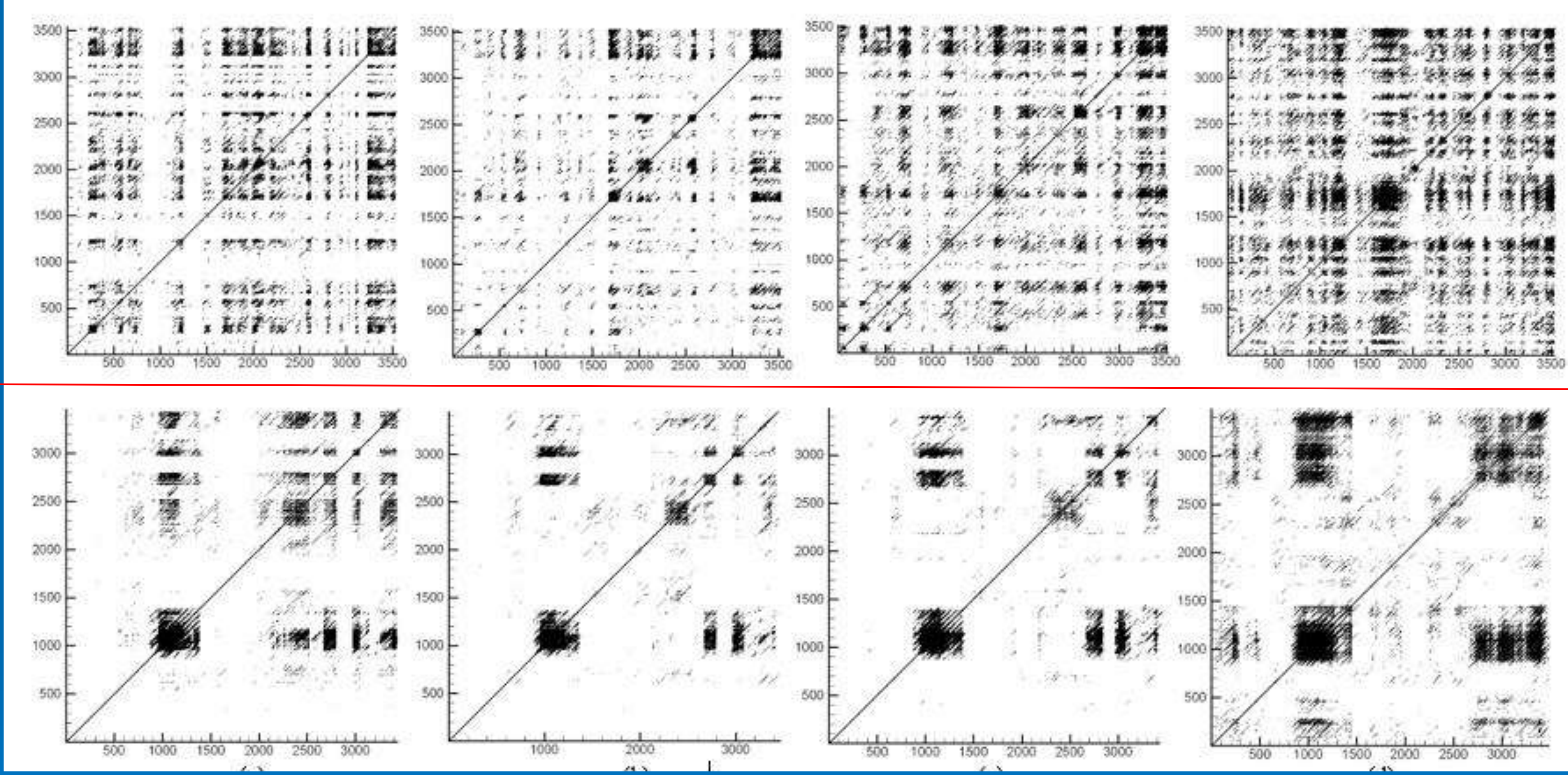
a) y=-0.99812, b) y= -0.94561 c) y= -0.89867 d) y= -0.76517,

Close to the wall

the v velocity component RPs

No B

with B



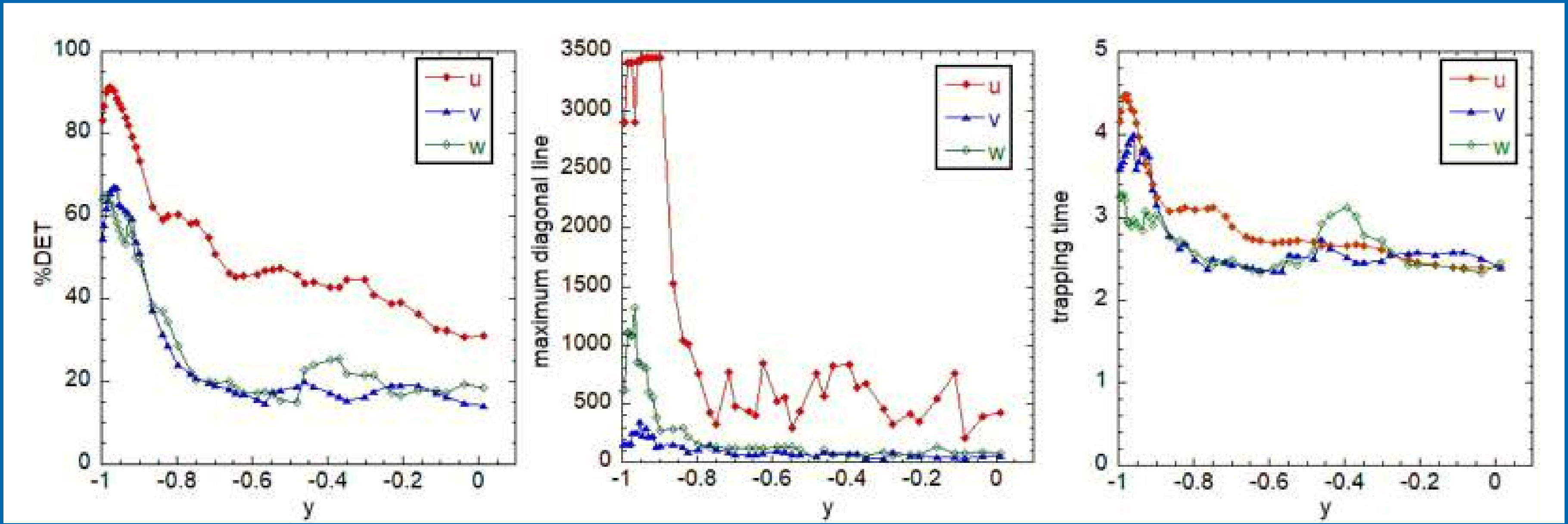
a) y=-0.99812, Close to wall

b) y= -0.94561 Close to wall

c) y= -0.89867

d) y= -0.76517,

RP quantitative results results with no magnetic field



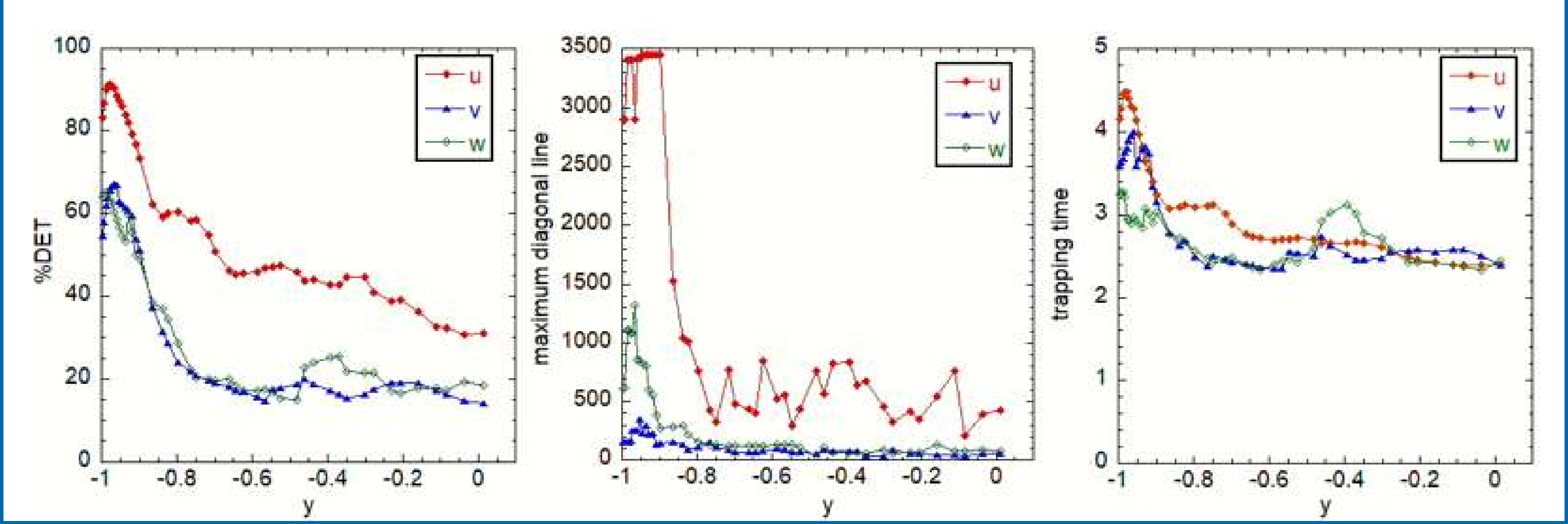
- A three-regime behavior in u component (along the flow)

(-1<y<-0.85, -0.85<y<-0.65, -0.65<y<0), values decreasing gradually from one region to the other but with different slops in general.

- A two-regime behavior in v, w (perpendicular to the flow)

(-1<y<-0.65 and -0.65<y<0)

RP quantitative results with no magnetic field



close to the wall $(y = -0.99812)$

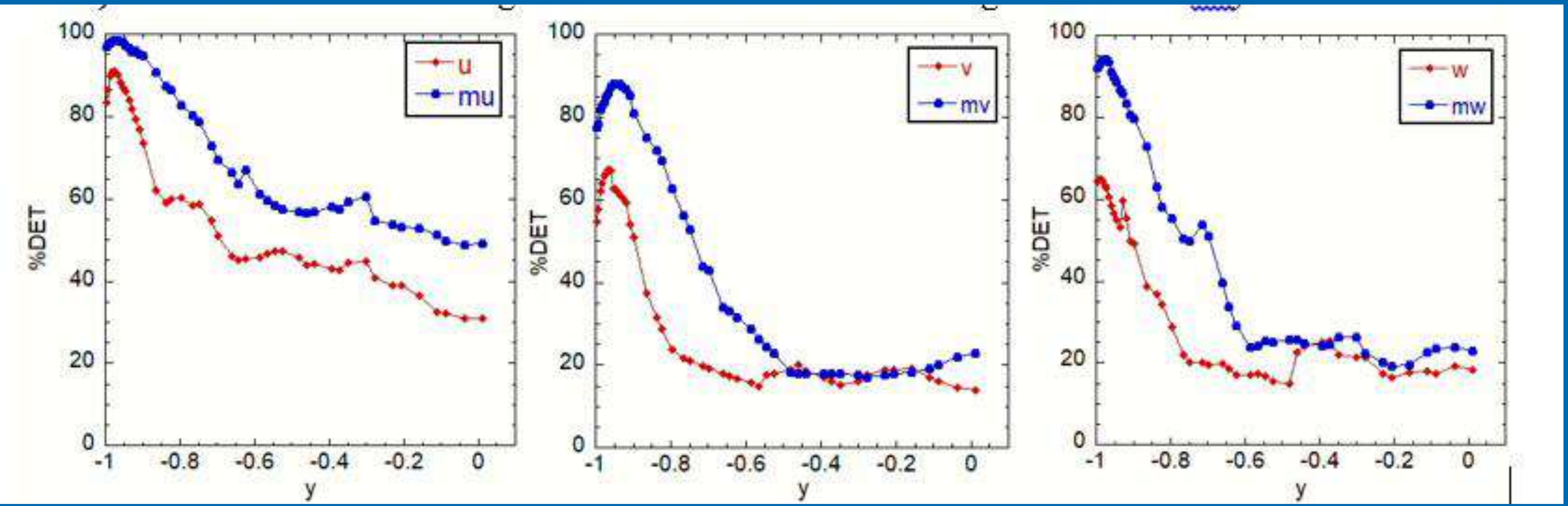
a) we have the viscous layer where diffusion phenomena are important with increased molecular viscosity due to molecular diffusion.

b) we have the creation of vortices which move towards the center of the channel.

➔ the correlation of states is more pronounced close to the wall

➔ in contrast towards the center of the channel we have less memory effects due to the multiplicity of events of vortices destruction as well the multiplicity of coexistence of vortices of various sizes and characteristic times

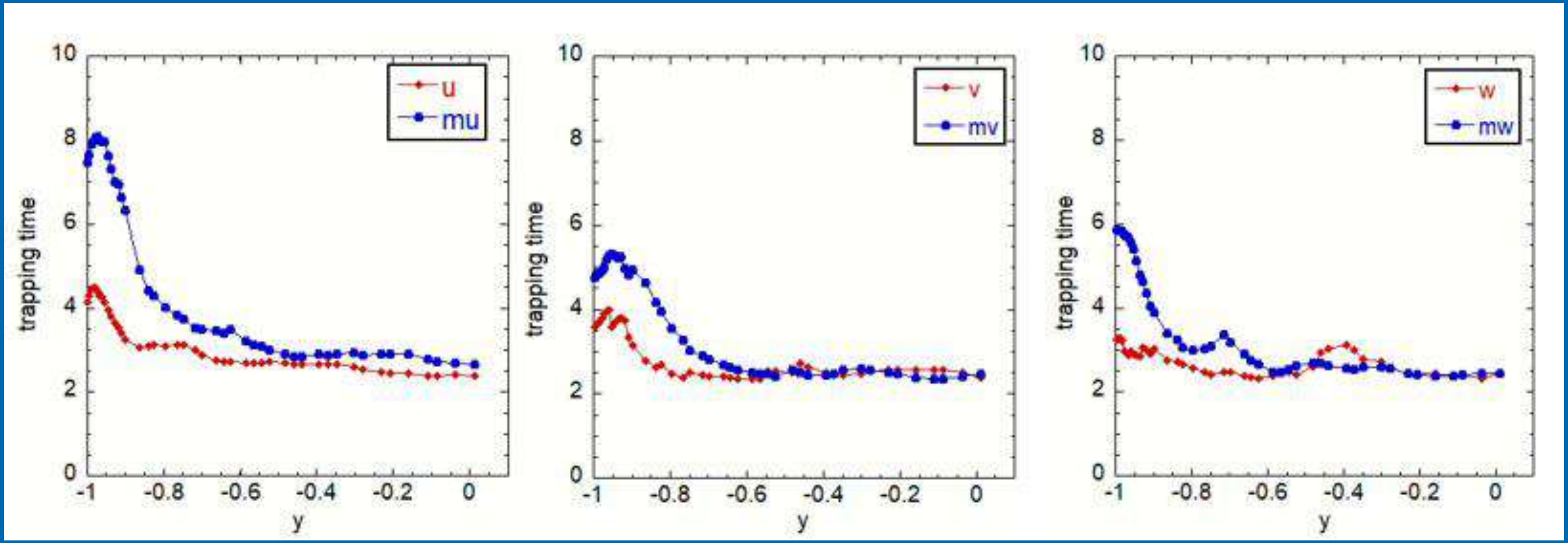
Comparison with and without B



The presence of the magnetic field extends this region towards the center of the channel and modifies the behavior of the transition region where turbulence and molecular diffusion effects are present.

The magnetic field enhances the effect of the molecular diffusion for the u and w component.
In fact these two directions are expected to be affected by the magnetic field since it will produce movement of particles in the direction normal to the plane formed by B and u. thus affecting significantly the u direction since they will tend to provoke a motion in the plane normal to the u direction affecting thus the w and v components.

Comparison with and without B



Close to the wall the system stays close to given state for longer times (streakcreatin and propagation)
While close to te center where we have fast coalescence the system looses fster memory

Summary MHD channel

We observe an important effect of the magnetic field along the flow and more specifically that the system becomes more “deterministic” (with larger values of parameters %DET, maxline) in comparison with the case without magnetic field.

The variations of the velocity present also smaller fluctuations compared to the case of absence of magnetic field.

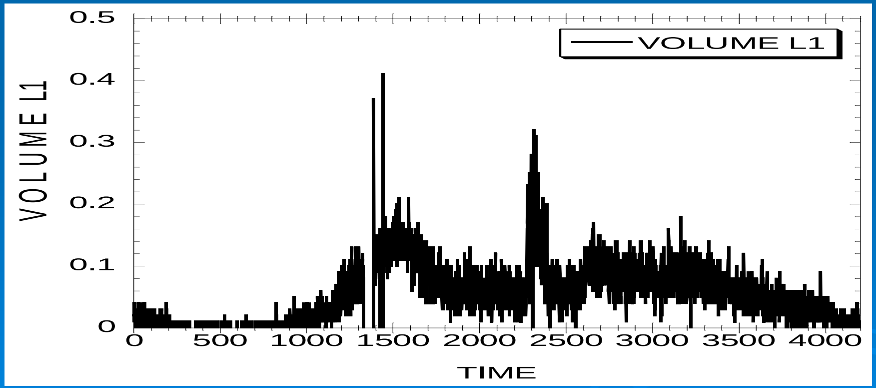
The effect of the field is pronounced up to < -0.5 . We believe thus that the magnetic field affects the streaks close to the channel walls since trapping time is increased compared to the absence of magnetic field.

Application on Traffic Data

Fragkou, A. D., Karakasidis, T. E., & Nathanail, E. (2018). Detection of traffic incidents using nonlinear time series analysis. Chaos, 28(6), 063108.

TRAFFIC DATA

Attica Tollway: metropolitan Highway 70 km long with 2 to 4 lanes each way

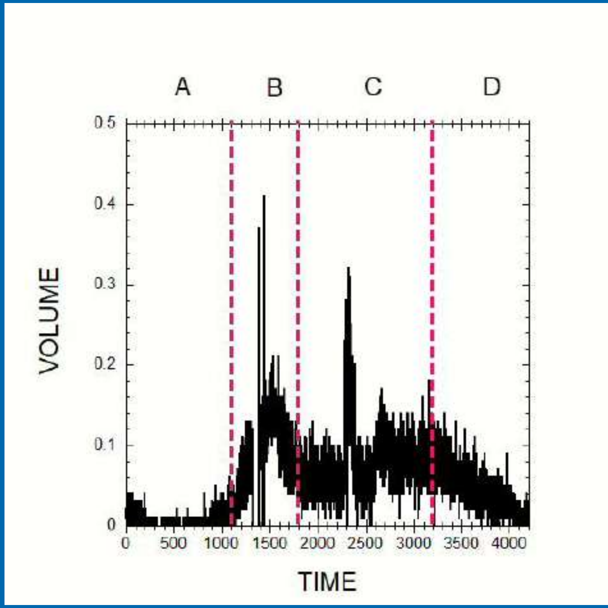


Sensors: count every 20 s

Timeseries: number of cars passing over sensor (volume) (4320 points)

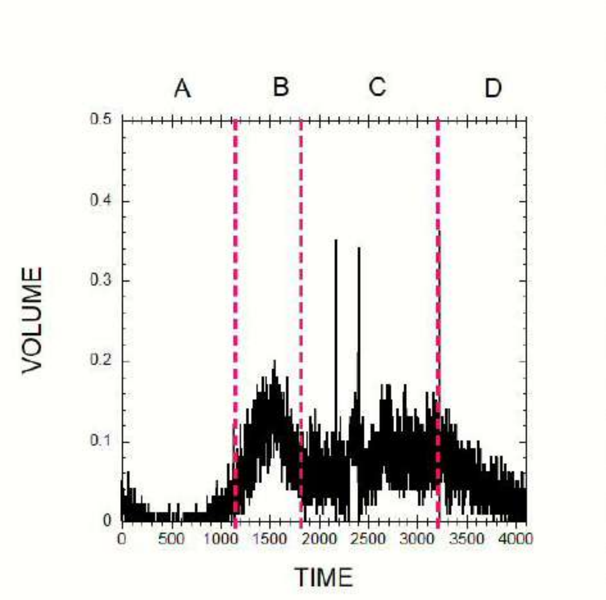
Aim of the study: Investigate the feasibility of incident detection by analyzing time series with RP and RQA methods

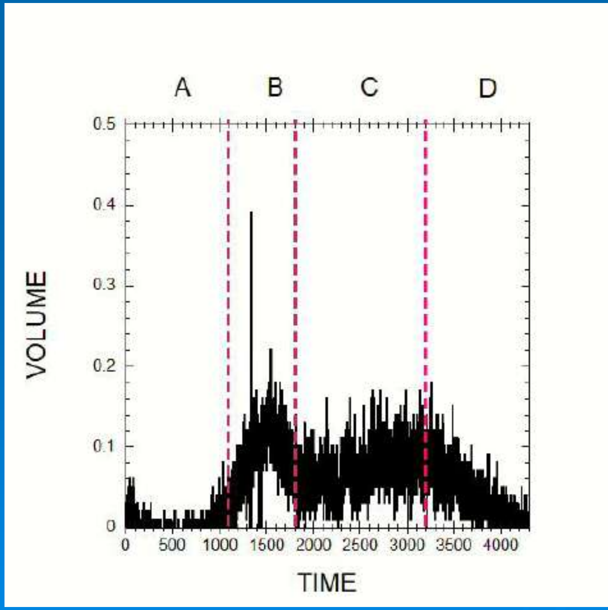
Timeseries from 4 sensors



3555

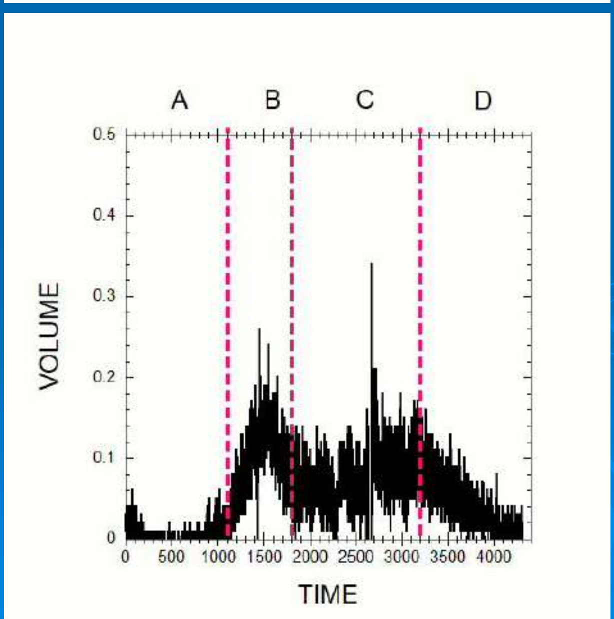
3557





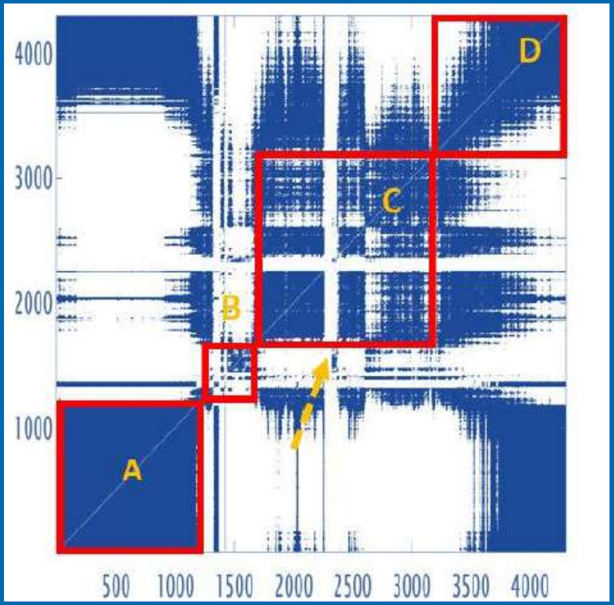
3604

3605



Separated in 4 Regions

Recurrence Plots S1



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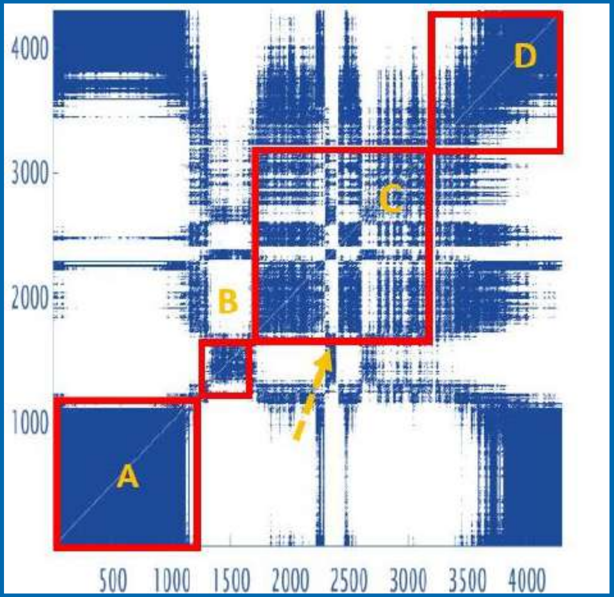

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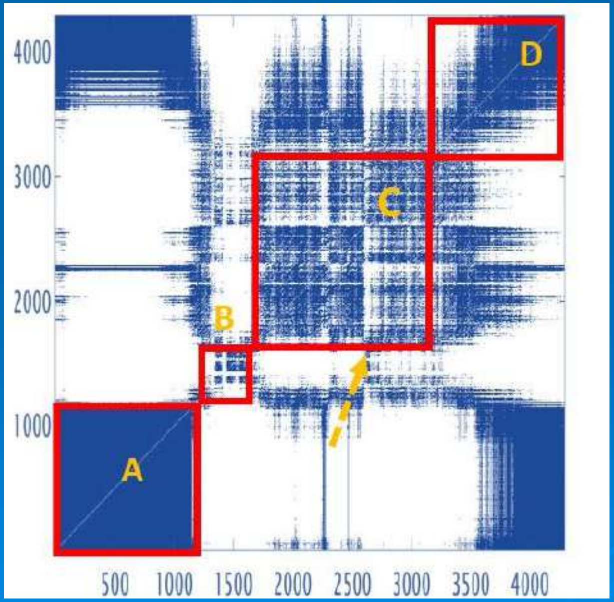
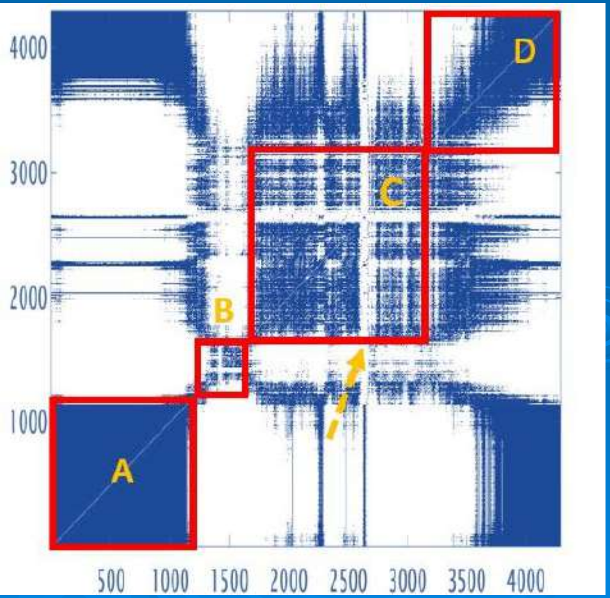


3604



3605

Separated in 4 Regions

Recurrence Plots S1 (explained)

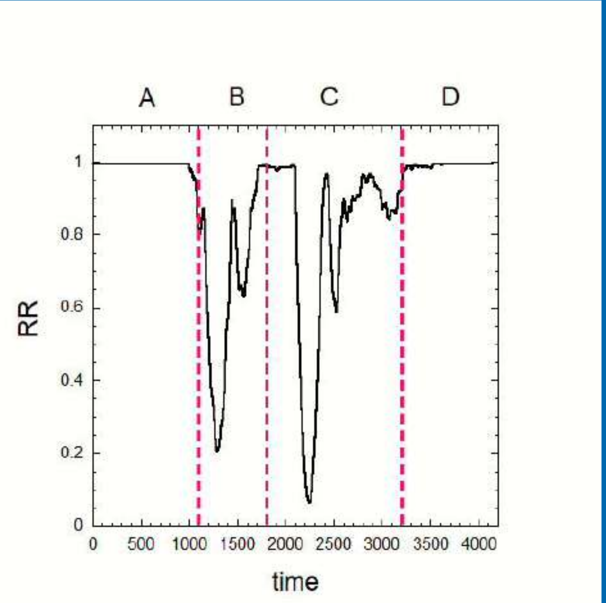
Regions A (1-1100) and D (3200 – 4350): large amount of recurrent points forming big deterministic lines, -> during those time periods the traffic flow over the sensors was relatively homogeneous and the corresponding volumes do not show any significant fluctuations

Regions B (1100-1800), C (1800 – 3200) : small diagonal lines and vertical white lines -> chaotic behavior with abrupt changes in the dynamics of the system.

Interesting characteristic: (yellow arrow) in sensors 3555 and 3557 a vertical white line (t=2250) An incident causes an abrupt change in the system behavior.

Same in sensors 3604 and 3605 at t=2550

Recurrence Quantification Analysis with epoqs S1



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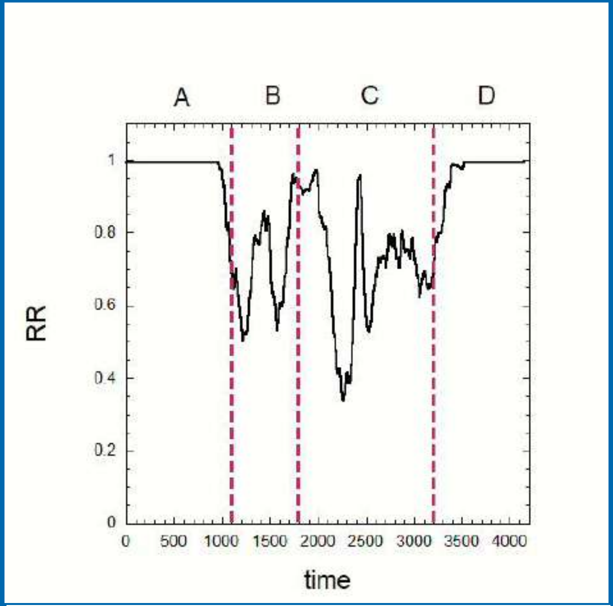

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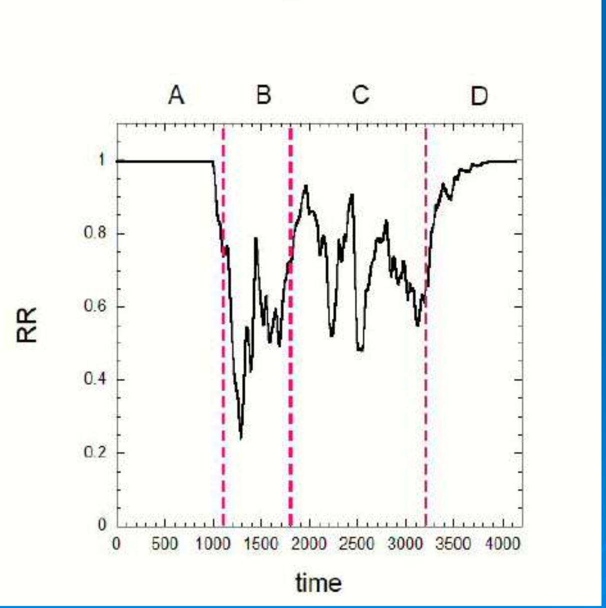

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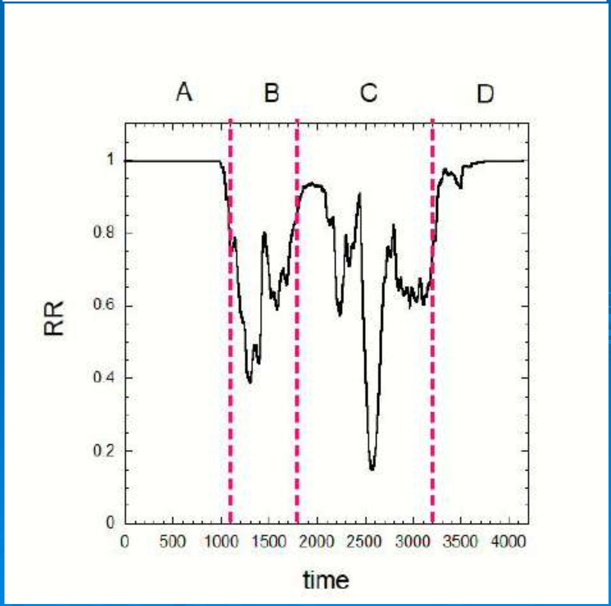


3605

epoqs of 180 successive records each with time shift of 1. (epoq = one hour. Results of Recurrence Rate

Recurrence Quantification Analysis S1 (explained)

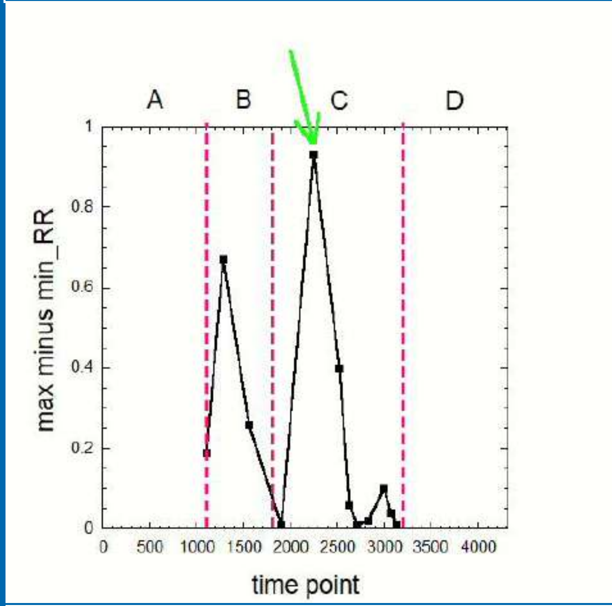
Regions A and D, high values, (almost 0.98): during morning and night hours volume fluctuation is smooth, without problems and possible incidents that may disturb traffic conditions and system dynamics

Regions B and C, main characteristic: the alternation of maximum and minimum values of RR parameter (max – min alternations): fast change affects the dynamics of the system

why this big change happened during the middle day hours having in mind the everyday normal traffic conditions during these hours.

abrupt change of the volume -> something happened on the traffic stream and changed abruptly the system dynamics

Recurrence Quantification Analysis S1 finding local max and min



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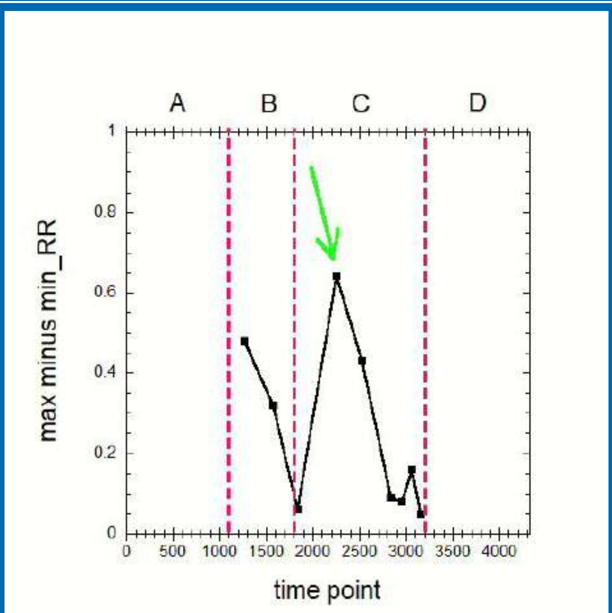
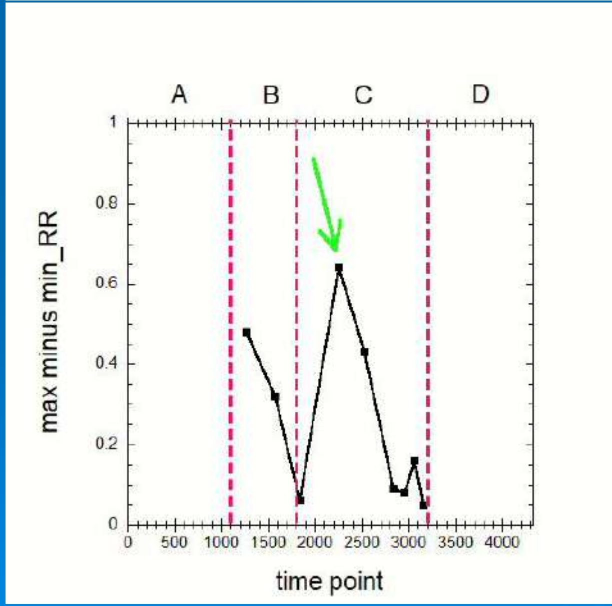
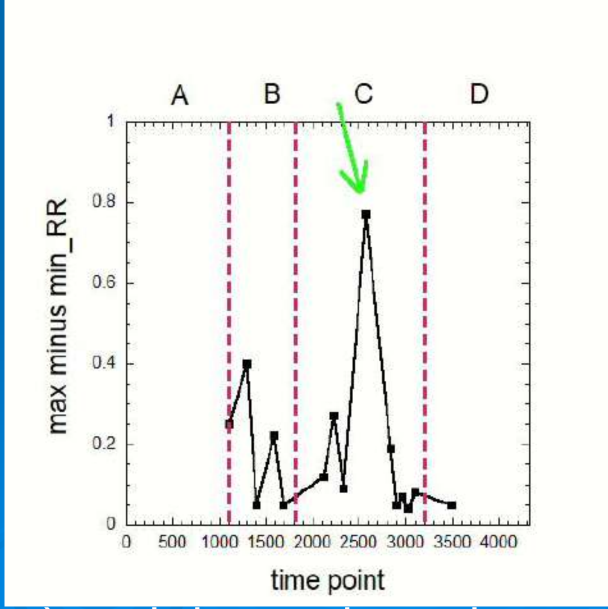




3604

3605



time points where RR has high values (local maximum) and time points where RR has small values (local minimum)

RQA S1 finding local max and min (explained)

Region's B and C biggest differences (green arrow):

sensor 3555 : t=2240

sensor 3557 : t=2251(nearest downstream sensor to the incident) more accurate location of the incident

sensor 3604 : t=2543 (further to the incident site) a time delay to locate any abrupt change.

sensor 3605 t=2566 (further to the incident site), a time delay to locate any abrupt change.

Incident S1	**Position A28.5, time point t=2268 (time 12:35:40)**			
Sensor	3555	3557	3604	3605
Sensor position	A28.00	A28.70	A29.30	A29.60
Upstream (B)/ downstream (A) the incident	B	A	A	A
t time point of incident detection	2240 (time 12:26:20)	2251 (time 12:30:00	2543 (time 14:07:20)	2566 (time 14:15:0

Conclusions

Recurrence Plots with epoqs → useful tool to detect dynamic state transitions during the evolution of a dynamical system.

Quantification of RPs (RR) → extra help on identifying those abrupt changes in dynamics of the system.
Empowered our belief that those changes are due to incidents that occurred during the time period of our data analysis.

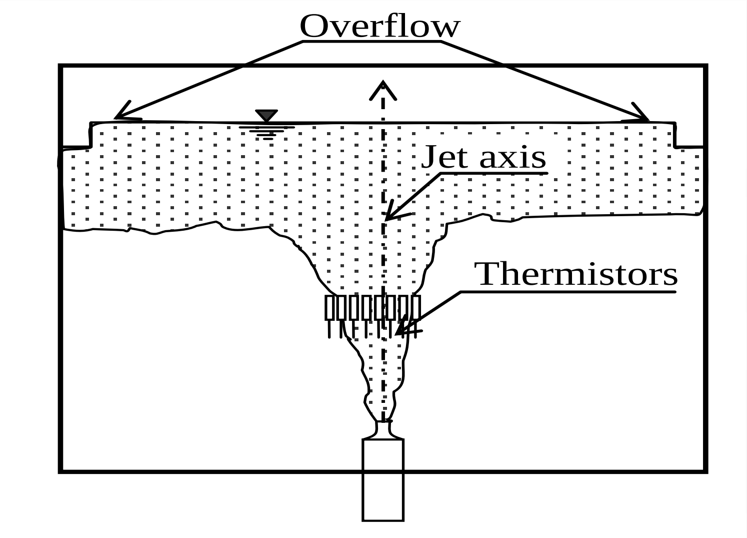
Bigger value of the max – min alternations between successive values of RR → allow us to find the time when the incident occurred and permits us to discriminate it from the recurrent traffic congestion

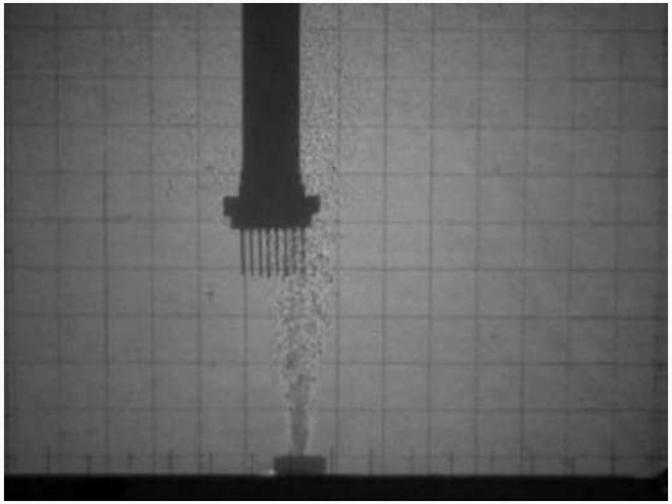
“Analysis of turbulent heated jets complex network time series mapping”

Charakopoulos, A. K., Karakasidis, T. E., Papanicolaou, P. N., & Liakopoulos, A. (2014). The application of complex network time series analysis in turbulent heated jets. *Chaos*, *24*(2), 024408.

Charakopoulos, A. K., Karakasidis, T. E., Papanicolaou, P. N., & Liakopoulos, A. (2014). Nonlinear time series analysis and clustering for jet axis identification in vertical turbulent heated jets. *Physical Review E*, *89*(3), 032913.

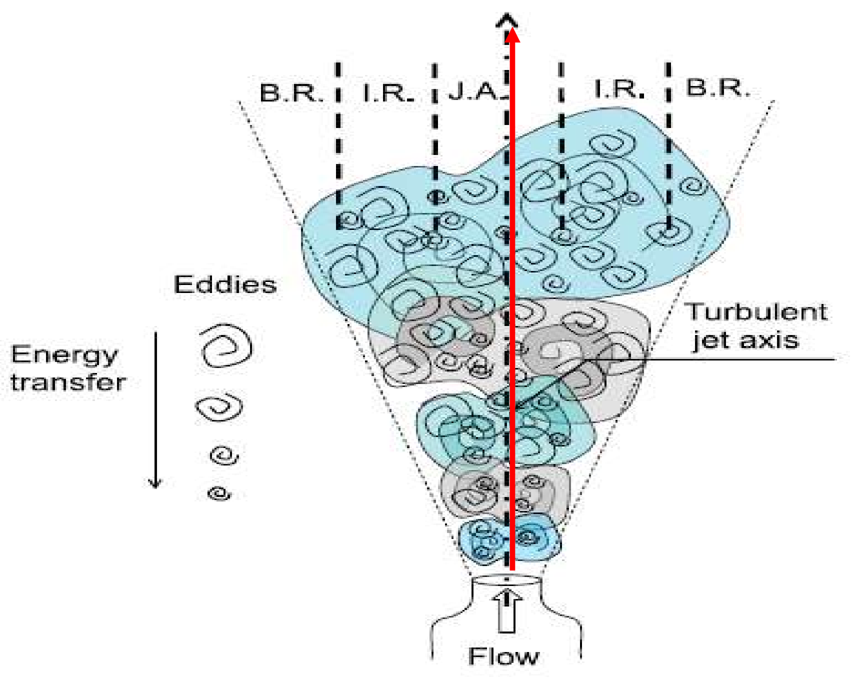
Experimental set up





- Data from different sets of experiments with various initial conditions and circular and elliptical shaped nozzles
- Ambient water temperature ranged between 18.4-24.6 °C while the jet water temperature ranged between 58.6 to 61.4 °C
- Temperature data sampling frequency at 80Hz and 100Hz

Vertical heated jet

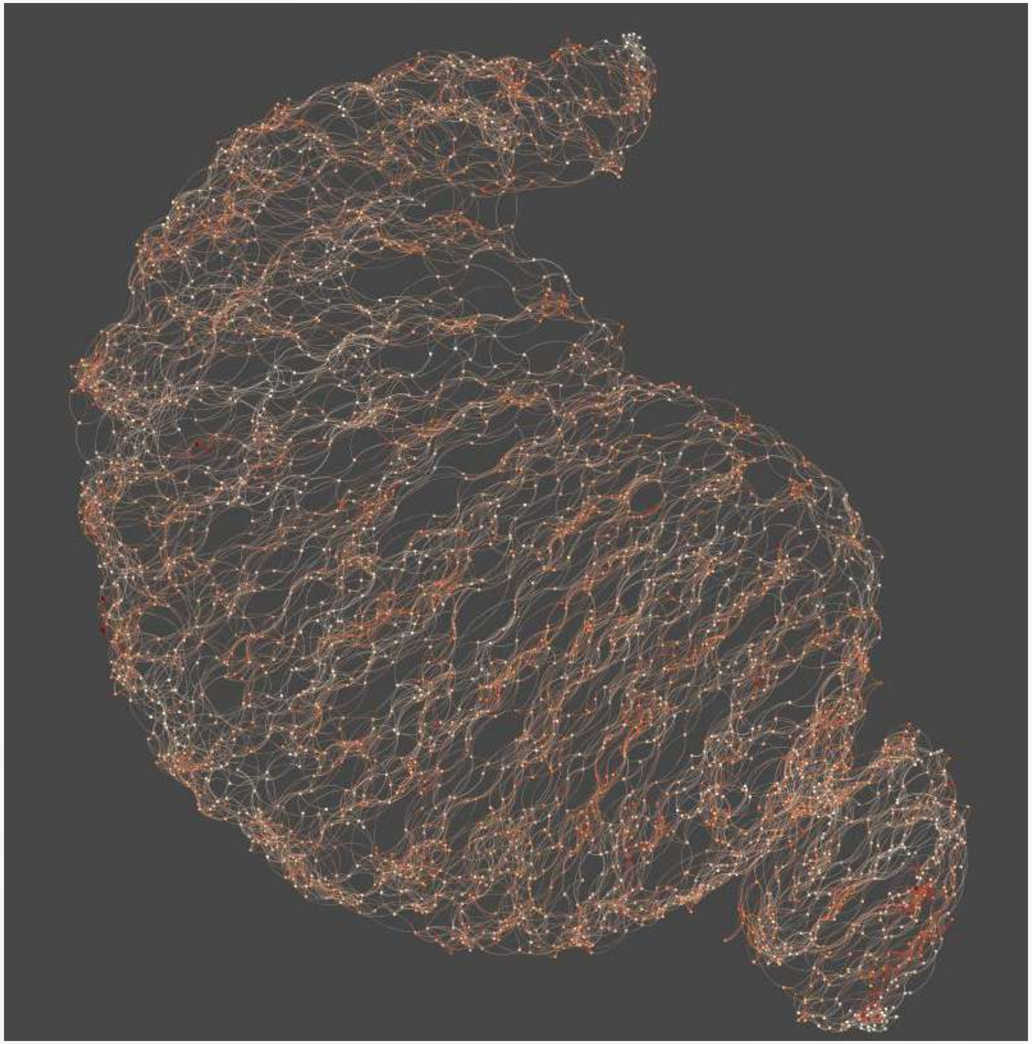


- ❖ **Three region behavior**
  - ➢ The first region corresponds to large distances from the jet axis, actually at the boundary with ambient water named boundary region (BR)
  - ➢ The second one, the inner region (IR), concerns the region between the boundary region and the core of the jet,
  - ➢ The third region, the jet axis region (JR), is the region near the core of the jet

The dynamics of these regions are characterized by the presence of small- and large-scale structures (vortices)

Complex Networks

Transforming time series into Complex Networks



Seems a promising technique

Can be further employed in the future for spatiotemporal analysis

Complex Networks

Some notions

- ✓ *A Network (graph) G=(V,E) consists of a set of nodes (V) that are interconnected with links or edges (E)*

- ✓ *A Network of N nodes can be described by the NxN adjacency matrix* $A=[a_{ij}]$

$a_{ij}=1$ *if the link i – j exists,*

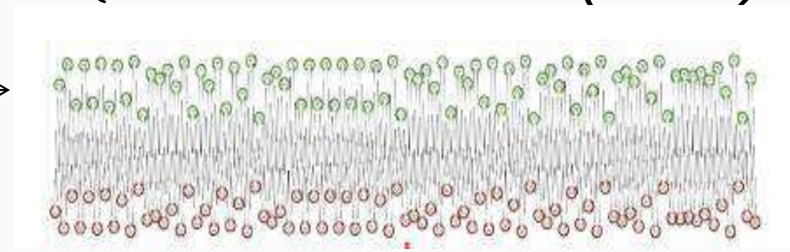
$a_{ij}=0$ *otherwise*

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Complex Networks and time series

Construction of the Complex Networks (Xiaoke Xu et. al (2008)

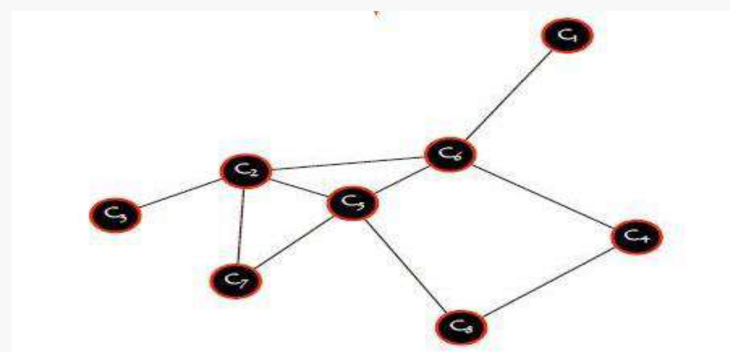
- ✓ *Time series*



- ✓ *Embedding the time series in an appropriate phase space and take each phase space point as a node in the network*
- ✓ *Select a fixed number of nearest neighbors for each point (node) and connect each point with its neighbors to form a complex network*

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	...
C_1	1	0	0	0	0	1	0	0	
C_2	0	1	1	0	1	1	1	0	
C_3	0	1	1	0	0	0	0	0	
C_4	0	0	0	1	0	1	0	1	
C_5	0	1	0	0	1	1	1	1	
C_6	1	1	0	1	1	1	0	0	
C_7	0	1	0	0	1	0	1	0	
C_8	0	0	0	1	1	0	0	1	
...									

- ✓ *Construct the adjacency matrix*
- ✓ *Construct the complex network*



Complex Networks Time Series

Construction of the Complex Networks (L. Lacasa et. al (2008))

Visibility graph method:

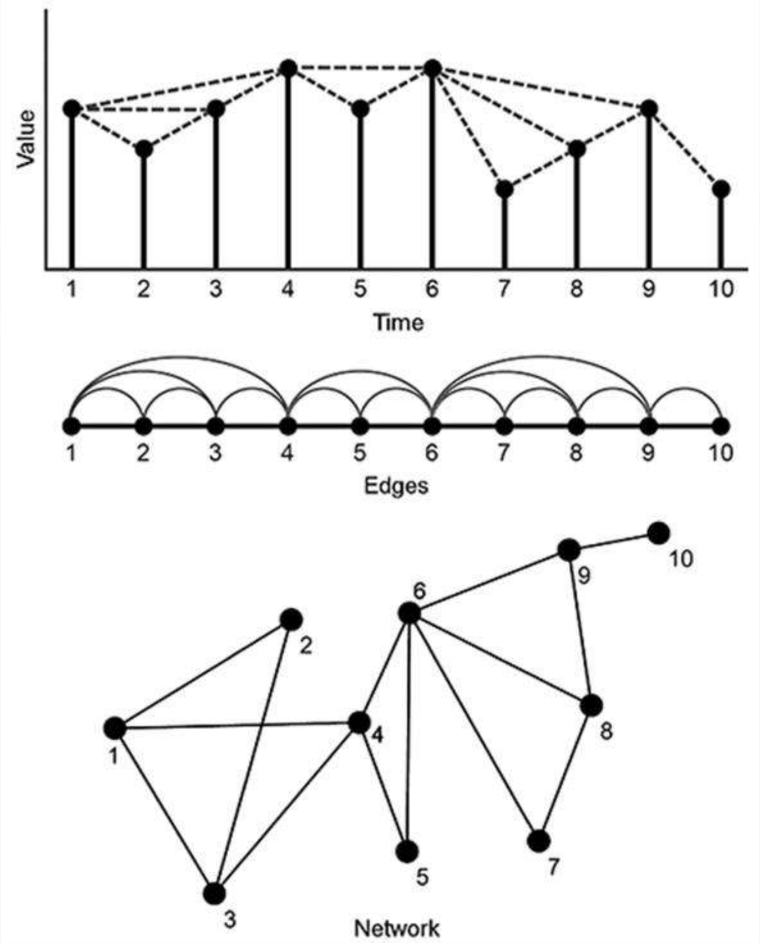
Let $x(t_i)_{i=1,\ldots N}$ is the time series

.

Two nodes $x(t_i)$ and $x(t_j)$ in the time series have visibility and become two connected nodes in the associated graph, if any other data $(t_k, x(t_k))$ placed between them $(t_i<t_k<t_j)$ fulfills

$$x(t_k) < x(t_i) + (x(t_j) - x(t_i))\frac{t_k - t_i}{t_j - t_i}$$

i and j are connected if one can draw a straight line in the time series joining the two points i and j, such that, at all intermediate points $(t_i<t_k<t_j)$, $x(t_k)$ falls below this line



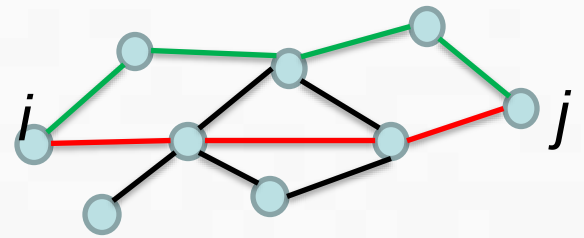
Complex Networks

Properties of Complex Networks

- ✓ *Shortest path (d_{ij}): Corresponds to the minimal distance between all paths that connect nodes i and j*

  d_{ij} is the red line (3) and

  not the green one (4)



- ✓ *Average path length (l): Is the average number of steps along the shortest paths for all possible pairs of network nodes. It measures the efficiency of information or mass transport on a network.*

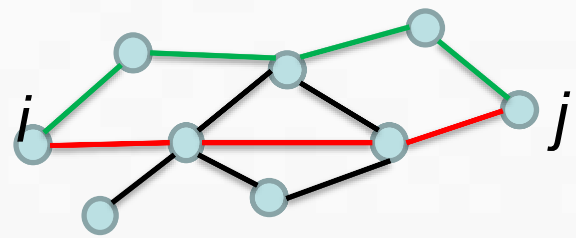
$$\langle d_{i,j} \rangle = \frac{1}{N(N-1)} \sum_{i,j} d_{i,j}$$

Complex Networks

Properties of Complex Networks

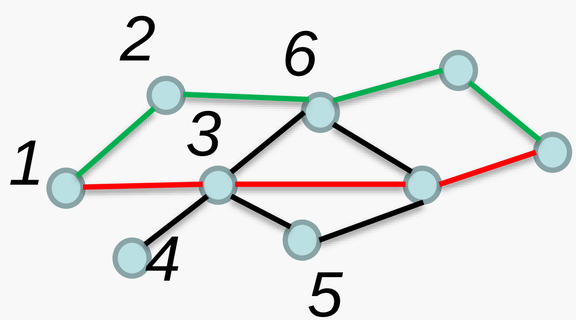
✓ Diameter (D): *The maximum length between all shortest paths*

D=max(dij)=3



✓ Degree (Ki): *The degree Ki of a node i is the number of connections of the node to other nodes*

K1=2, K2=2, K3=5, K4=1, K5=2, K6=4

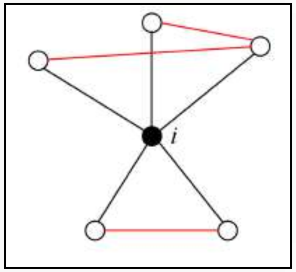


Complex Networks

Important properties of Complex Networks

- ✓ *Degree distribution: P(k) of a network specifies the fraction of nodes having exactly degree k*

- ✓ *Clustering coefficient: Ci is the ratio between the number of links E connecting the nearest neighbors of i and the total number of possible links between these neighbors.*



$$c_i = \frac{2e_i}{k_i(k_i - 1)} \qquad K_i = 5,\ e_i = 3$$

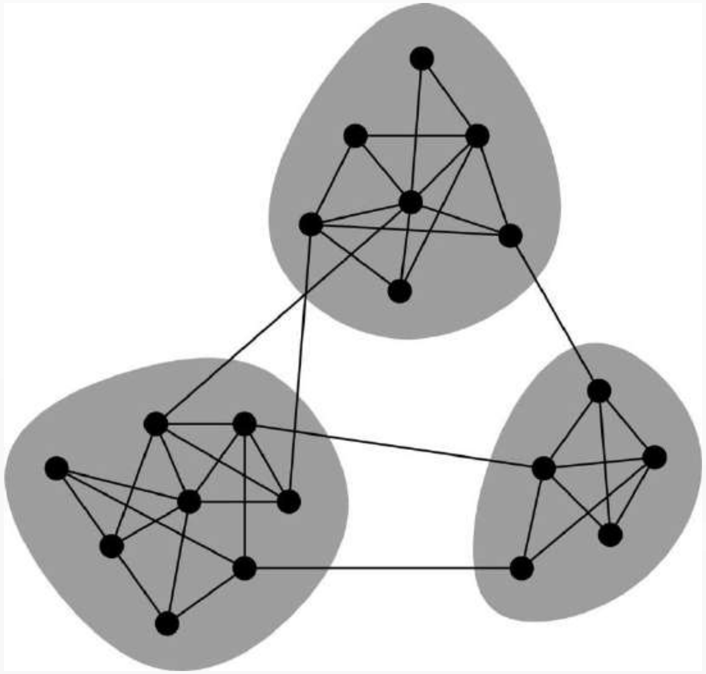
K_i is the degree of i, e_i is the number of links directly connecting neighbors of i

The clustering coefficient of a network C is the average of Ci over all nodes

$$C = \langle c_i \rangle = \frac{1}{N}\sum_i c_i$$

Complex Networks properties

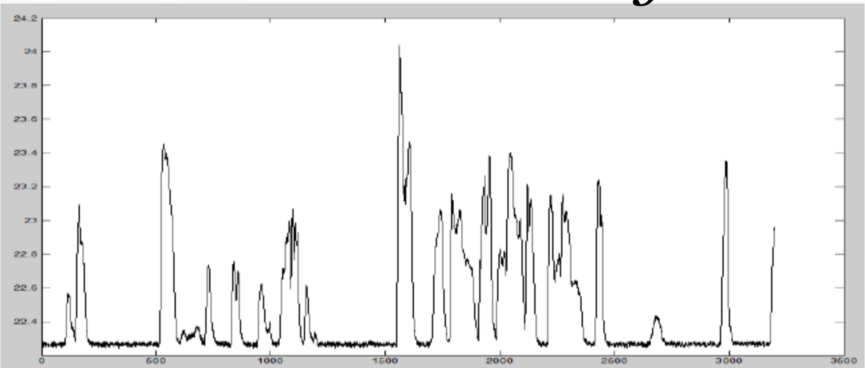
- ✓ *Modularity (M):* *a measure of the structure of a network. It measures the strength of division of a network into modules (also called groups, clusters or communities).*
- ✓ *Networks with high modularity have dense connections between the nodes within modules but sparse connections between nodes in different modules.*



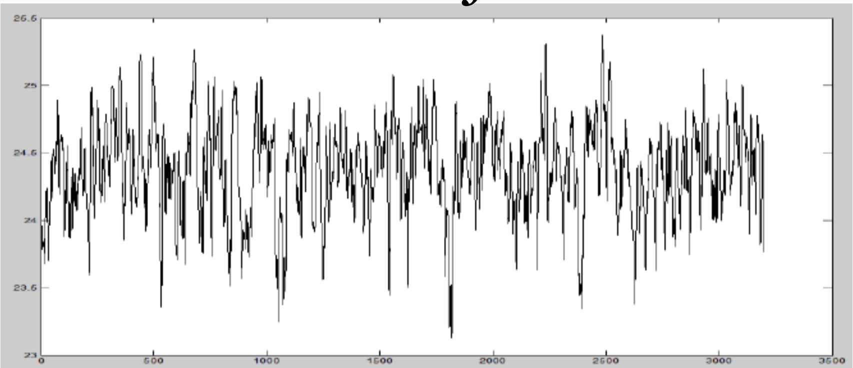
Newman M E J PNAS 2006;103:8577-8582

Complex Networks: Results

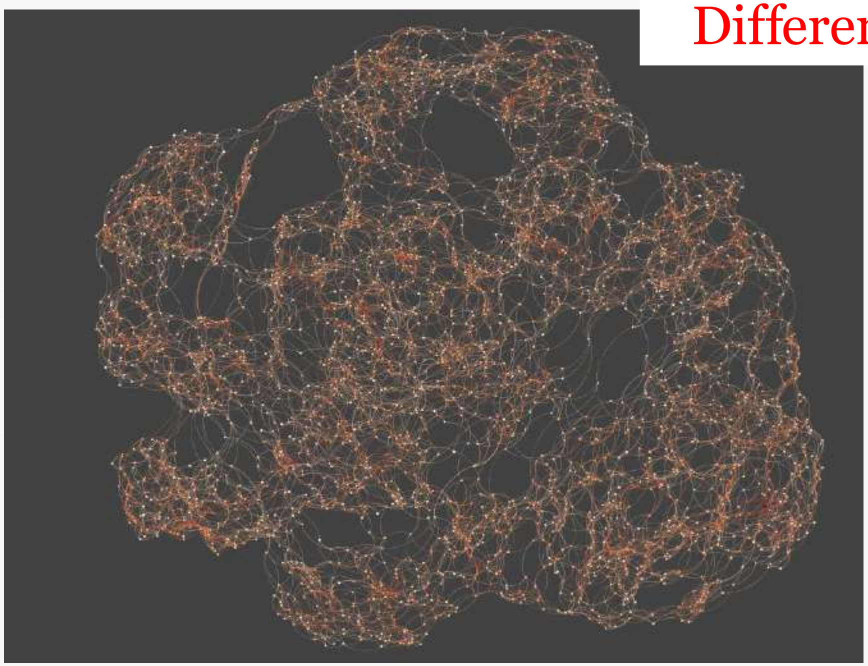
Near the boundary

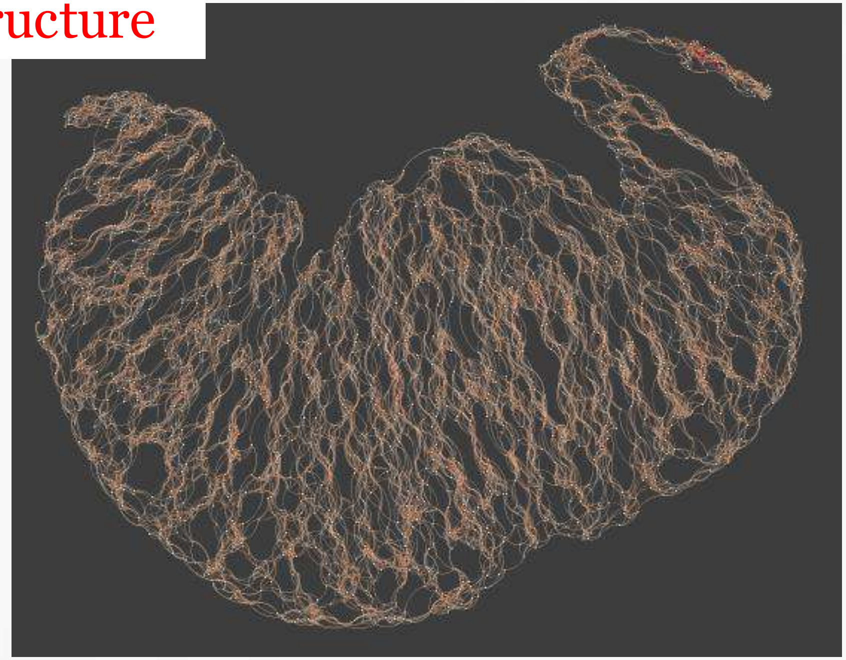


Near the jet line



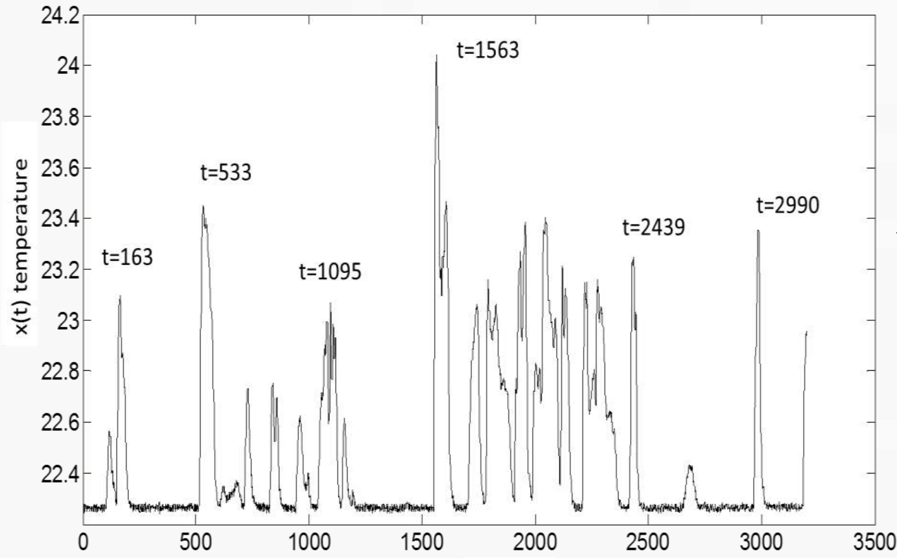
Different structure

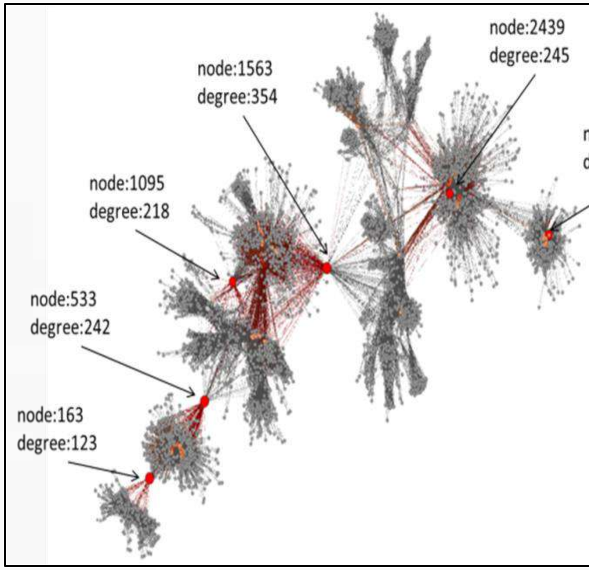




Complex Networks: Results

- *Visibility method*

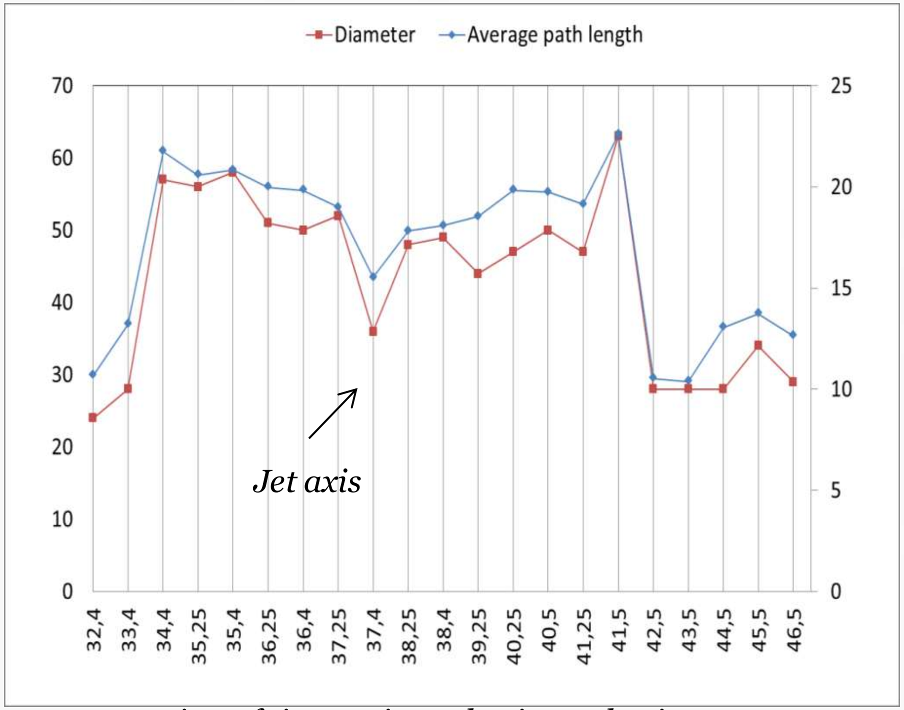




Node	Degree
163	123
533	242
1095	218
1563	354
2439	245
2990	165

- ✓ *Peak of the value are due to the interaction of small heated eddies to the bigger vortex*
- ✓ *Throughout the development of the time series if there exists a peak and the previous and next value would not be located very close this data tend to have higher degree than the other data.*
- ✓ *At the network each point represented as a hub with different degree*
- ✓ *Physically this means that at these points we have a strong influence of a heated vortex to less heated fluid*

Complex Networks: Results



Location of time series at horizontal axis

Case 1

Lowest values of

Diameter : The maximum length between all shortest paths

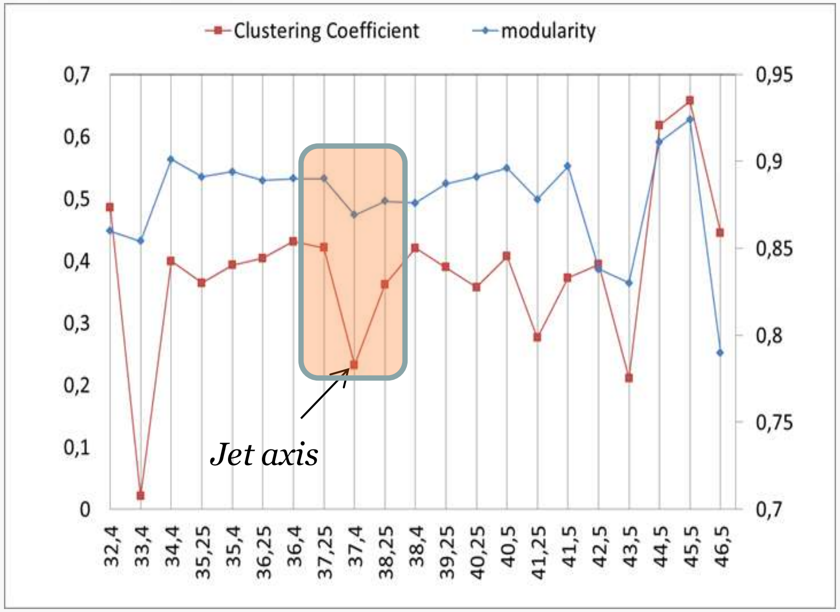
Average path length (l): average number of steps along shortest paths for all possible pairs of network nodes

In the jet axis region : fully developed turbulence → increased presence of short-lived small scales → change of states occur faster → successive states are less linked

As we move towards to boundary large scale longer living structures persist → change of states take more time to occur

Complex Networks: Results

Case1



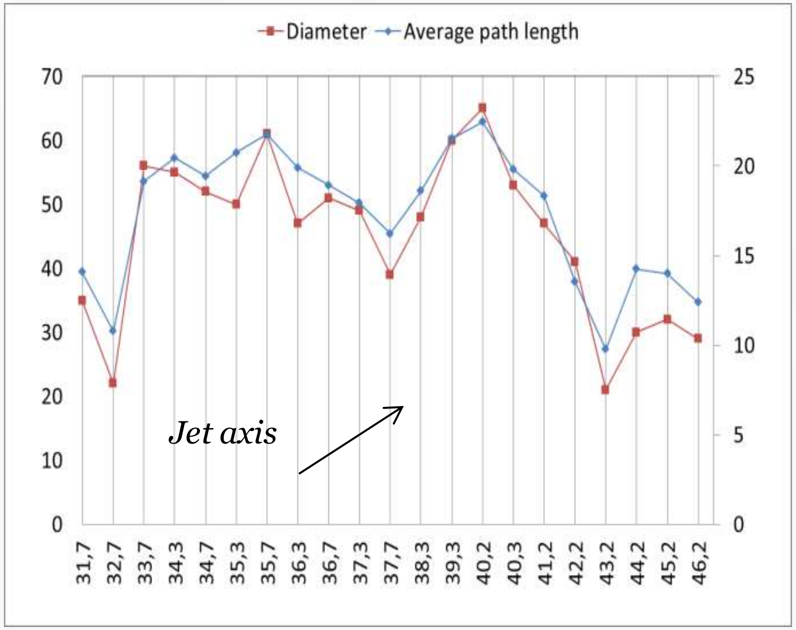
Location of time series at horizontal axis

Clustering coefficient: Ci is the ratio between the number of links E connecting the nearest neighbors of I and the total number of possible links between these neighbors.

Due to presence of short lived small structures connections between successive sates are reduced compared to regions where long-lived structure persist.

Modularity (M): a measure of the structure of a network. It measures the strength of division of a network into modules (also called groups, clusters or communities).
Short live structures result in fewer well defined separations. More interconnections are present.

Other Cases- *Diameter and Average path length*



- *Diameter (D): The maximum length between all shortest paths*
- *Average path length (l): Is the average number of steps along the shortest paths for all possible pairs of network nodes. It measures the efficiency of information or mass transport on a network.*

Results of jet axis location using our methodology and comparison with estimations from hydromechanics methods

Case Study	Shape of nozzle	Measurement station attributed to the jet axis using the clustering procedure (present study)	Estimated location of jet axis using a Gaussian fit
	Round	38.25	37.75

Conclusions

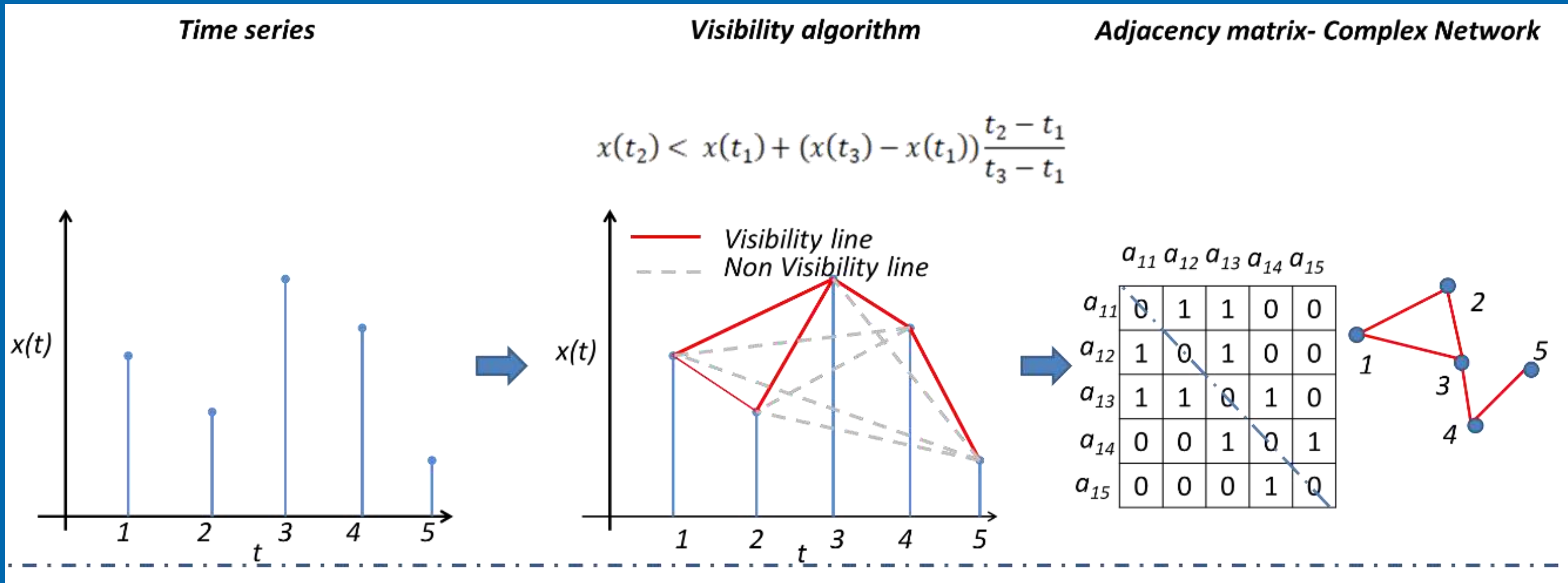
Aim: Distinguish the jet axis region from the regions near the boundary (ambient water) and the intermediate regions

- ✓ Various measures can provide information about various regions of the jet as well as about the location of the jet axis
- ✓ Using a combination of all the measures along with a clustering procedure can discriminate far better various regions of the flow based on a different behavior and lead to a methodology for obtaining the location of jet axis
- ✓ Analysis is capable of extracting information and can be useful for a more clear discrimination of the time series near jet axis from others that correspond to the region near the boundaries
- ✓ This methodology seems quite promising for application in complex flows, as well as in applications where several different state zones exist in a physical system where one can have access only to spatiotemporal data
- ✓ The time series derived from different regions along the horizontal line, exhibit different topological features of the network

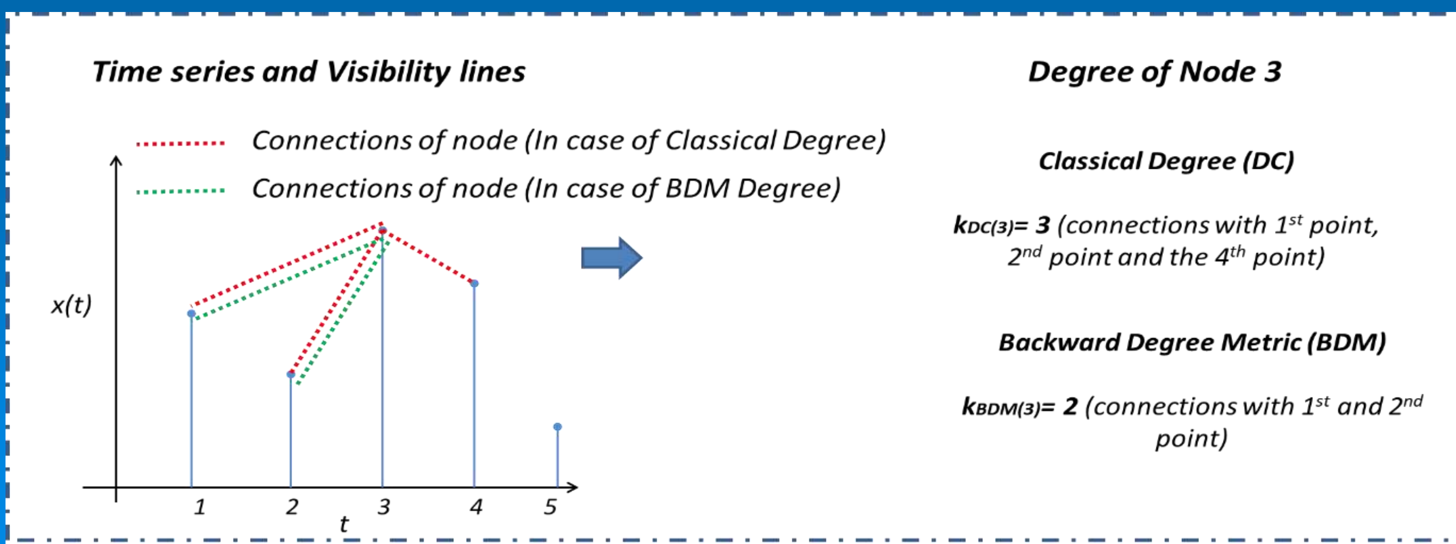
“Backward Degree: a new index for online and offline change point detection based on complex network analysis”

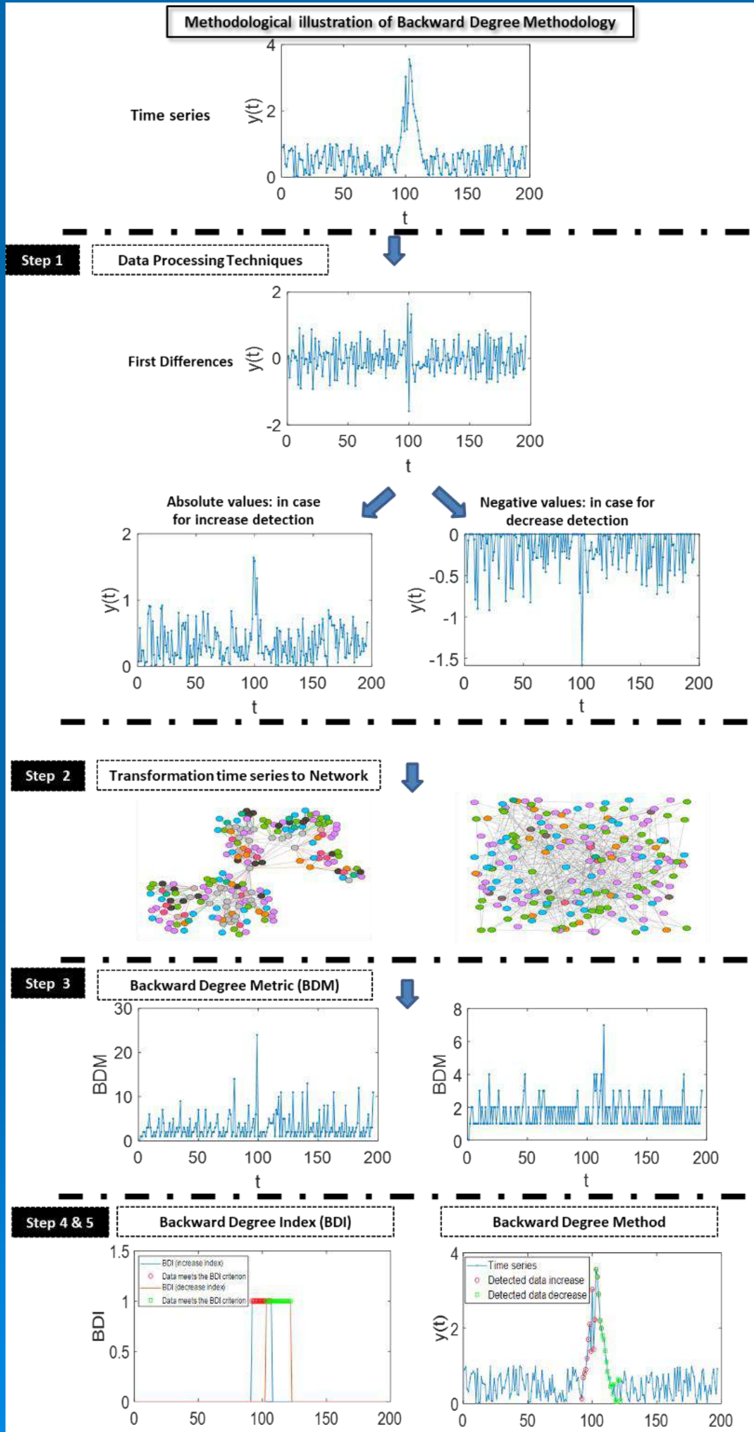
Charakopoulos, A., & Karakasidis, T. (2022) *Physica A*, 127929

Visibility algorithm for complex network construction (Lacasa et al. 2008)



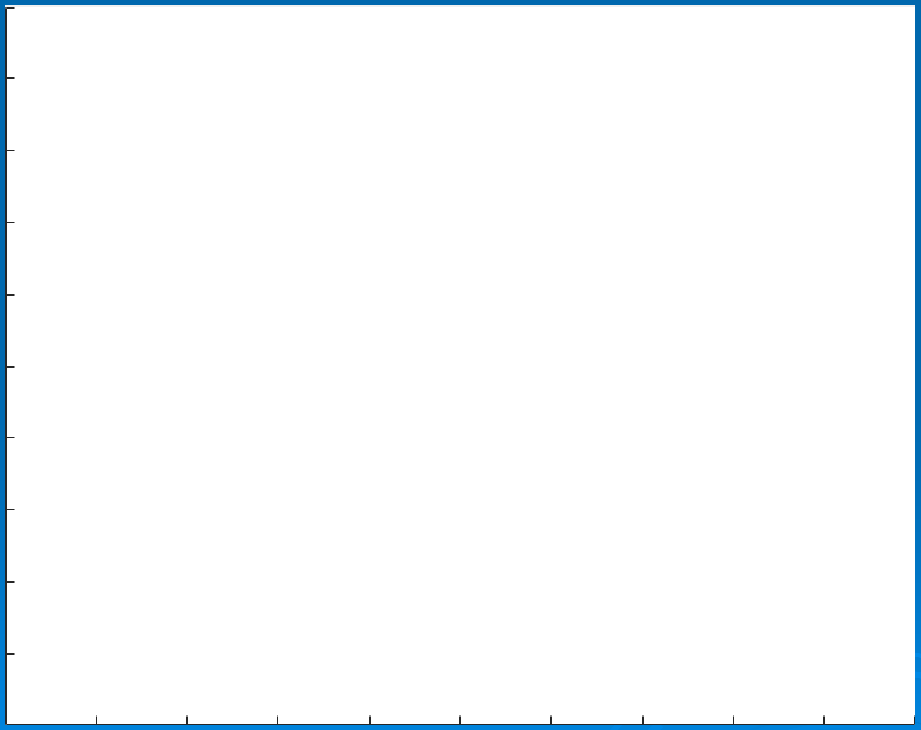
Degree of a node=number of connections with other nodes

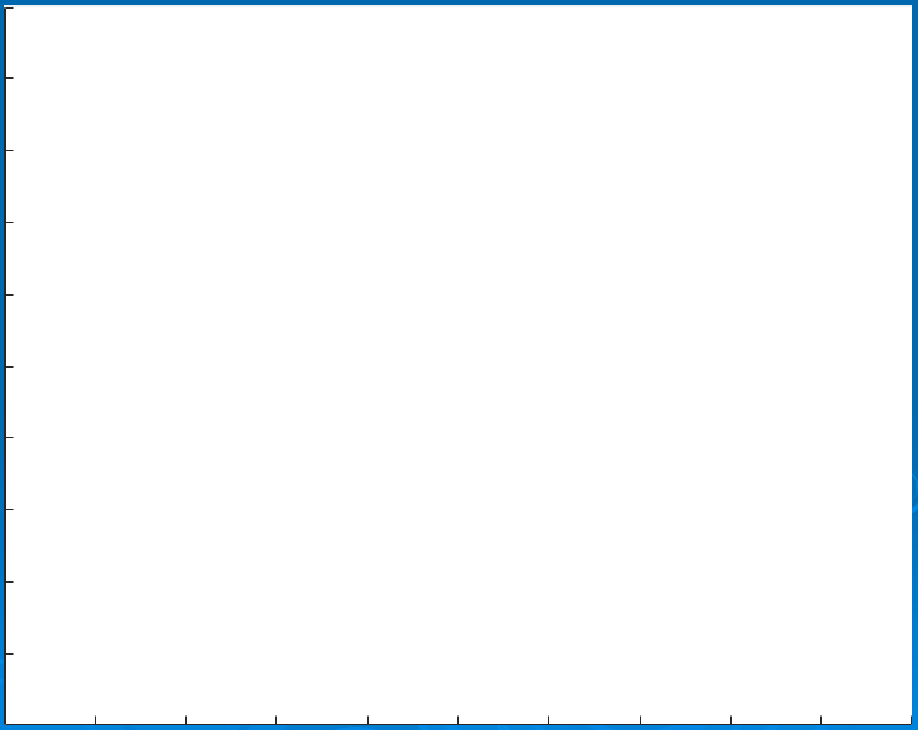


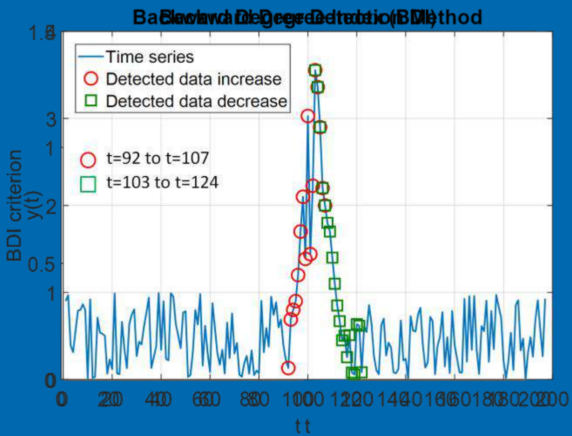


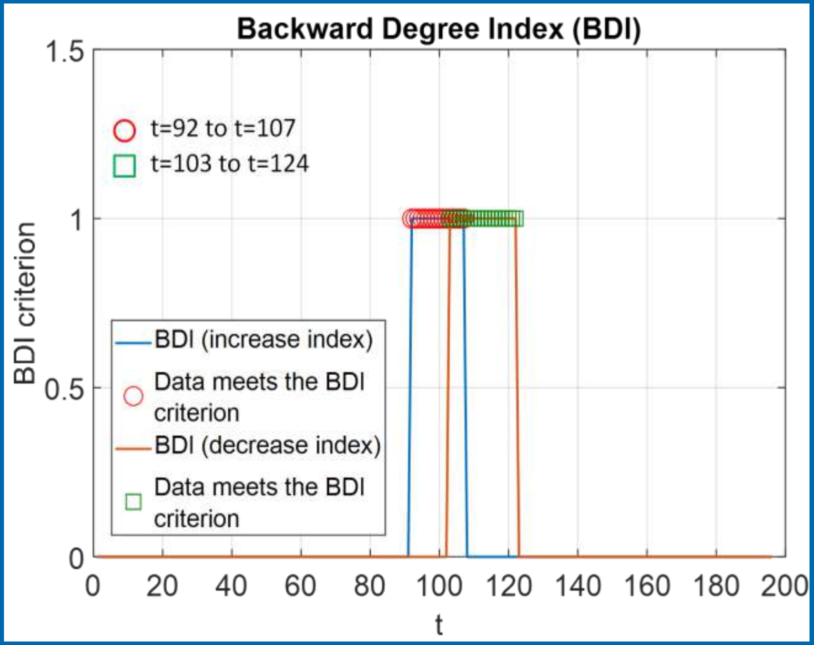
Input a time series	X(t);
Input the parameter	n;
Calculate First differences	DX(t)
Absolute value of DX(t) in case of increase detection	Abs{ DX(t)}
Zero negative values of DX(t) in the case of decrease detection	Z{ DX(t)}
Convert Abs{DX(t)} or Z{ DX(t)} to a graph	G(nxn);
Backward Degree Metric	BDM;
Moving average and moving standard deviation of the BDM	BDMma; &BDMmstd;
Define the Backward Degree Index	BDI=BDMma+BDMmstd;
Calculate sum of mean BDI plus twice standard deviation of BDI	BDIquant=mean (BDI)+2*std(BDI);
Test whatever the index satisfies the criterion.	If BDI> BDIquant then 1 If BDI< BDIquant then 0

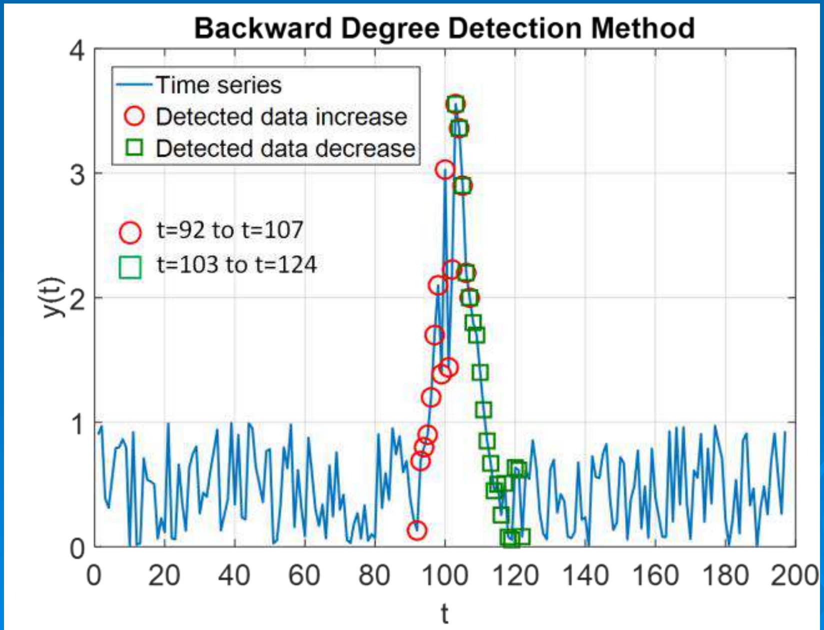
Online detection of sudden events



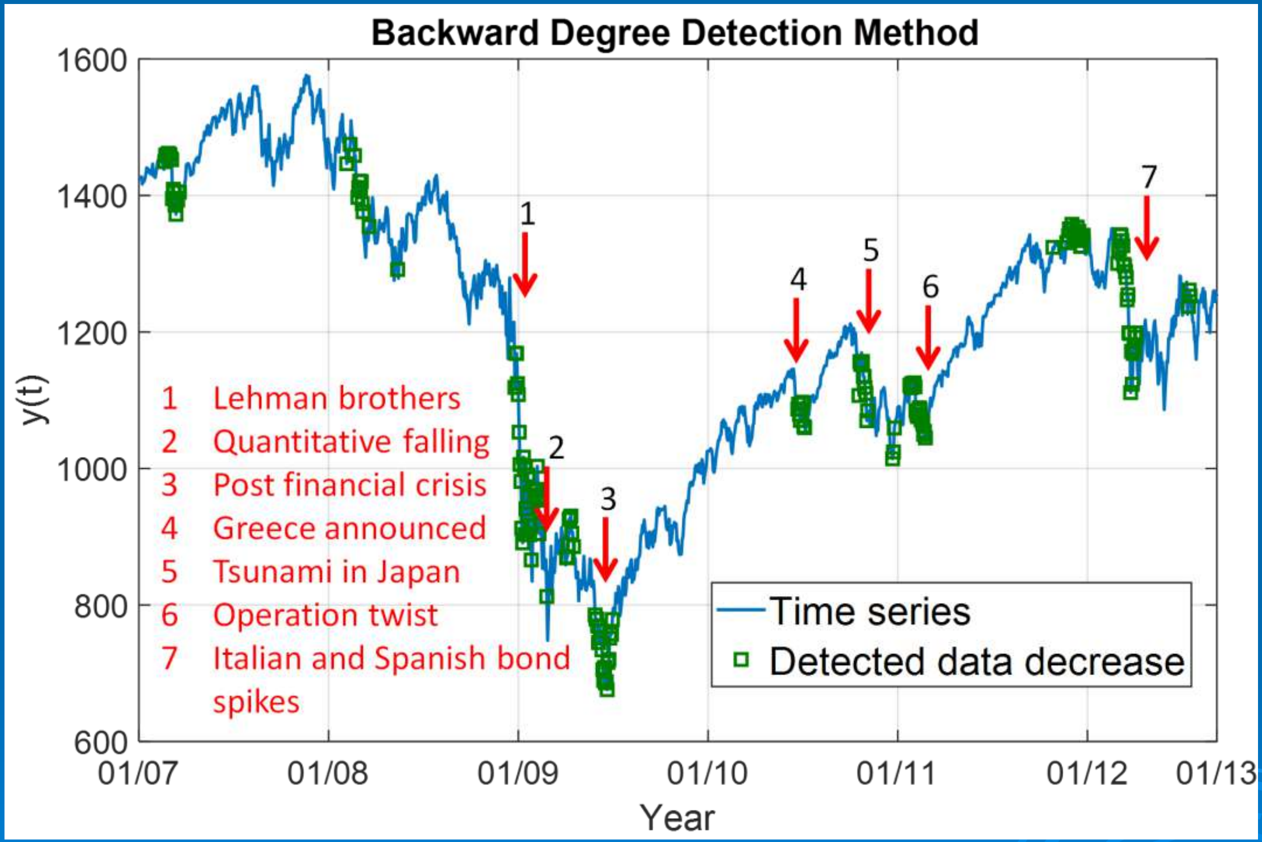



Time series
Detected data increase
Detected data decrease
t=92 to t=107
t=103 to t=124


Backward Degree Index (BDI)
t=92 to t=107
t=103 to t=124
BDI (increase index)
Data meets the BDI criterion
BDI (decrease index)
Data meets the BDI criterion
BDI criterion
t

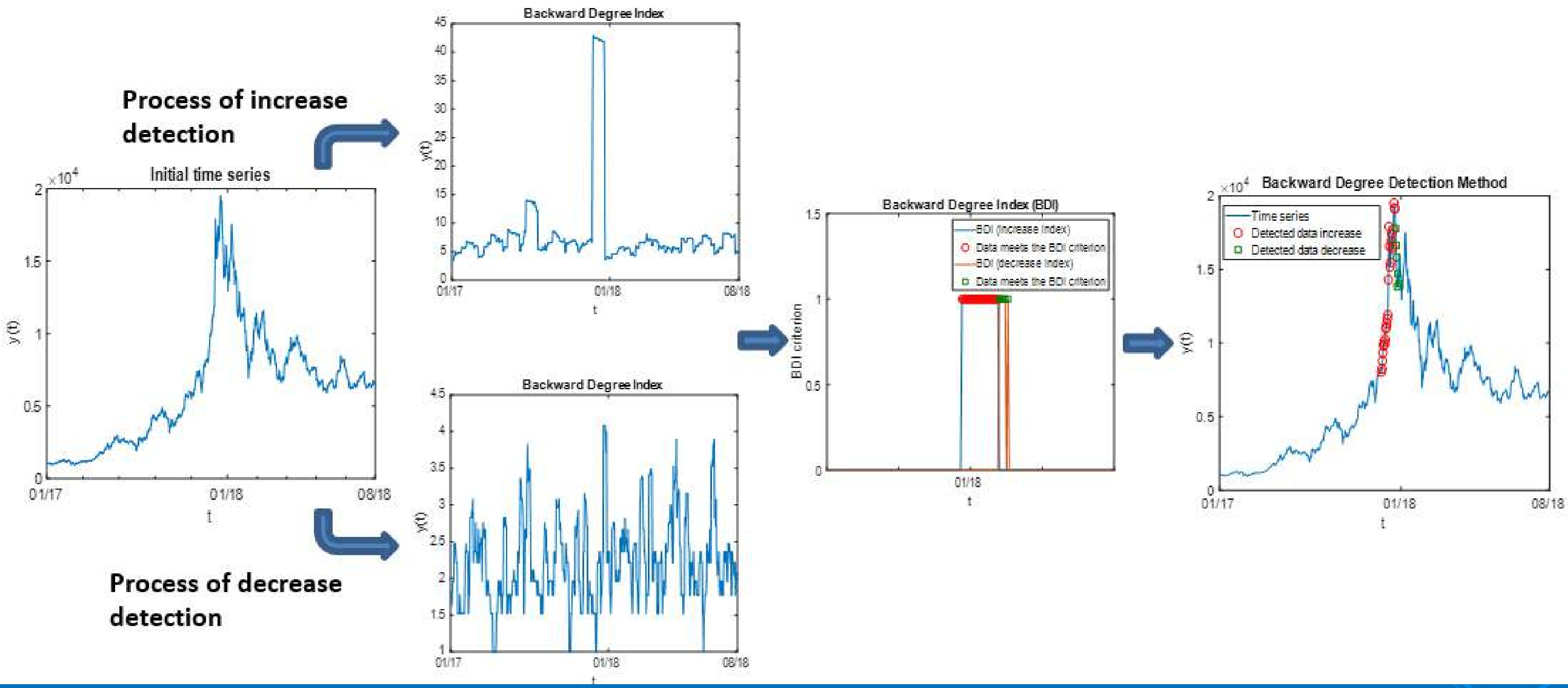

Backward Degree Detection Method
Time series
Detected data increase
Detected data decrease
t=92 to t=107
t=103 to t=124
y(t)
t

S&P500 close market index from January 2007 and December 2013 and the corresponding points, marked with green rectangular, where the method detects as abrupt decrease points



Charakopoulos, A., & Karakasidis, T. (2022) *Physica A*, 127929.

Bitcoin price index from January 17 to August 18.



Charakopoulos, A., & Karakasidis, T. (2022) *Physica A*, 127929.

Bubble detection in Greek Stock Market:

An approach with the DS-LPPLS Model

K Papastamatiou, T Karakasidis, Physica A: 587, 126533

Methodology

Positive bubble: A transient non-sustainable period in a stock market, where the present value of an asset exceeds by far its fundamental value due to the fact that there is excessive demand. (overpriced assets)

- A network of participants influenced by their "neighbors"
- Local mimetic behaviors of stockbrokers
- Universal cooperative behavior causes collapse
- Values are determined by the properties of the system
- Super exponential distribution

$$p(t) = A + B(t_c - t)^{\beta}\{1 + C\cos[\omega\, ln(t_c - t) + \varphi]\} \Rightarrow$$

$$p(t) = A + B(t_c - t)^{\beta} + C_1(t_c - t)^{\beta} cos[\omega\, ln(t_c - t)] + C_2(t_c - t)^{\beta} sin[\omega\, ln(t_c - t)]$$

(Anders Johansen et al., 1999)
(Johansen & Sornette, 1999)

Indicators

- **DS LPPLS Confidence** is the fraction of fitting windows,

$$Conf\ .Ind. = \frac{N_f}{N}$$

  - N_f number of the fittings which satisfy condition 1 of table (1),
  - N total number of fittings in the selected window

Confidence Indicator gives a measure of the sensitivity of the observed pattern at a given time scale dt.

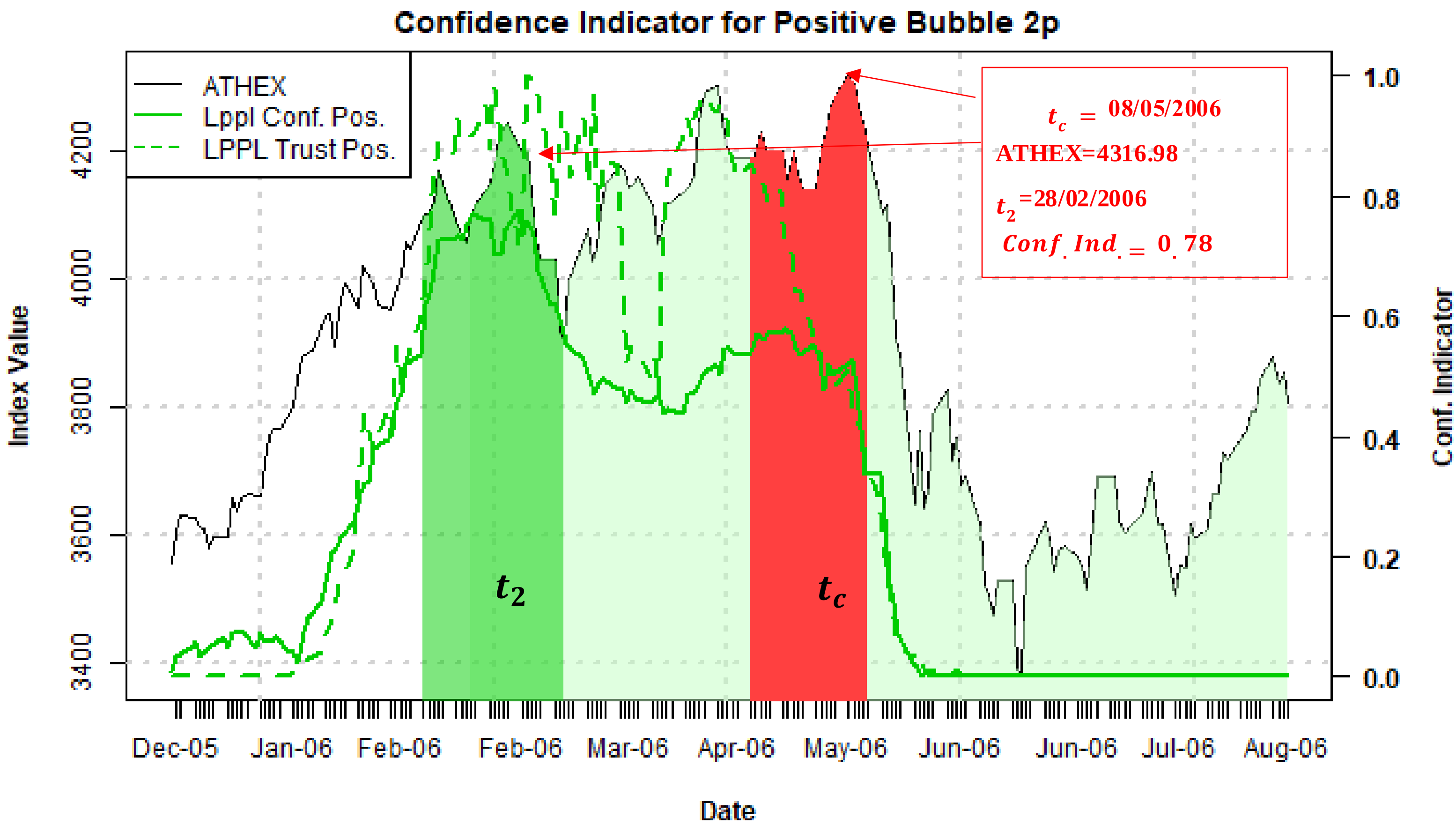
- **DS LPPLS Trust**, is the median level of the fraction among the synthetic time series generated that satisfy the filtering conditions 2 of table (1). It measures how closely the theoretical LPPLS model matches the empirical time series.

(Fantazzini, 2016)
(Sornette & Cauwels, 2015)

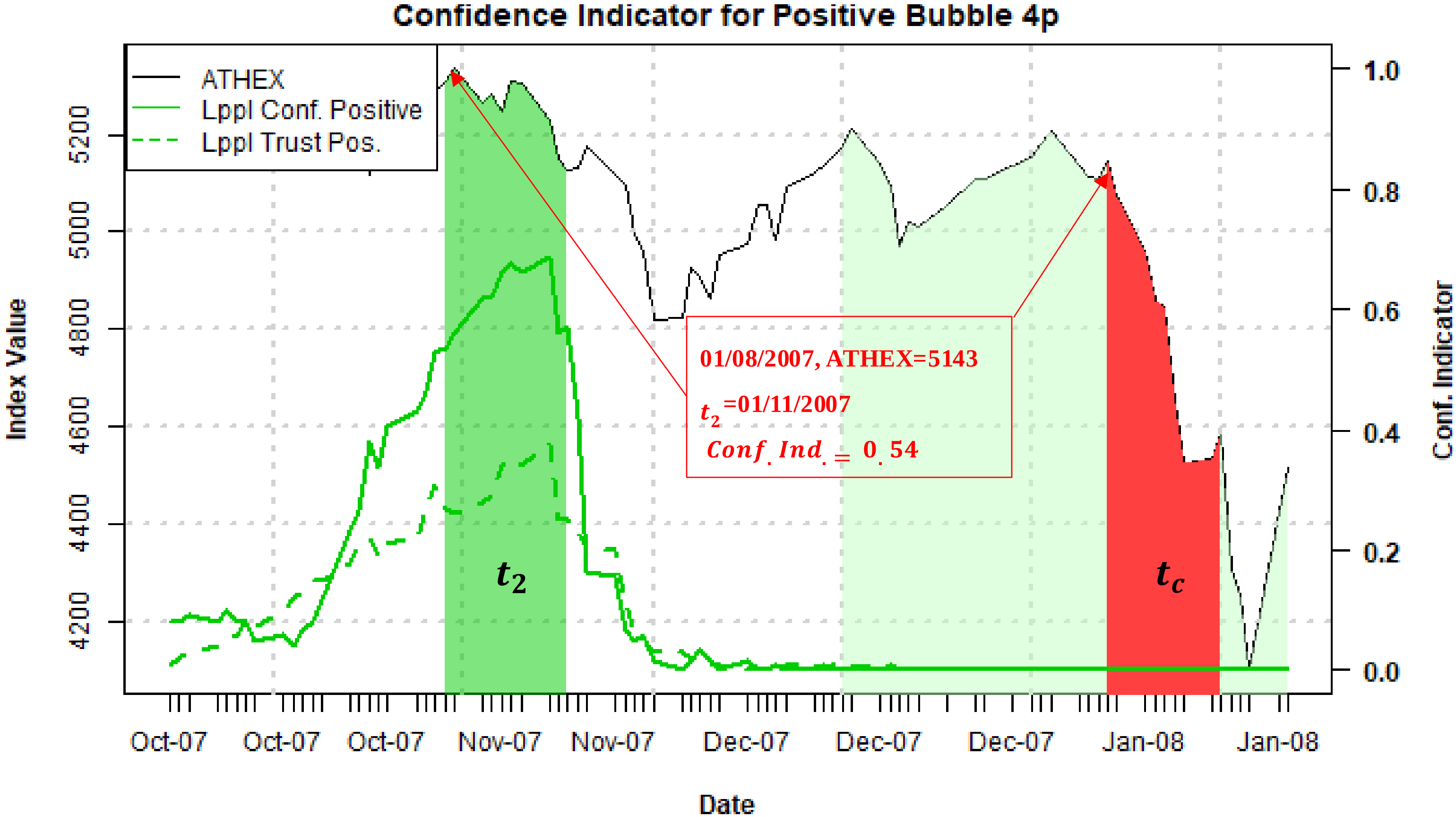
Methodology

- Rolling window of 400 days
- $p(t) =$ logarithmic value of Index (or stock)
- $t_1 =$ starting date of
- $t_2 =$ ending time of the rolling window
- $\Delta t =$ time step of the rolling window
- $t_c =$ estimation of the critical time (burst for positive, rebound for negative)

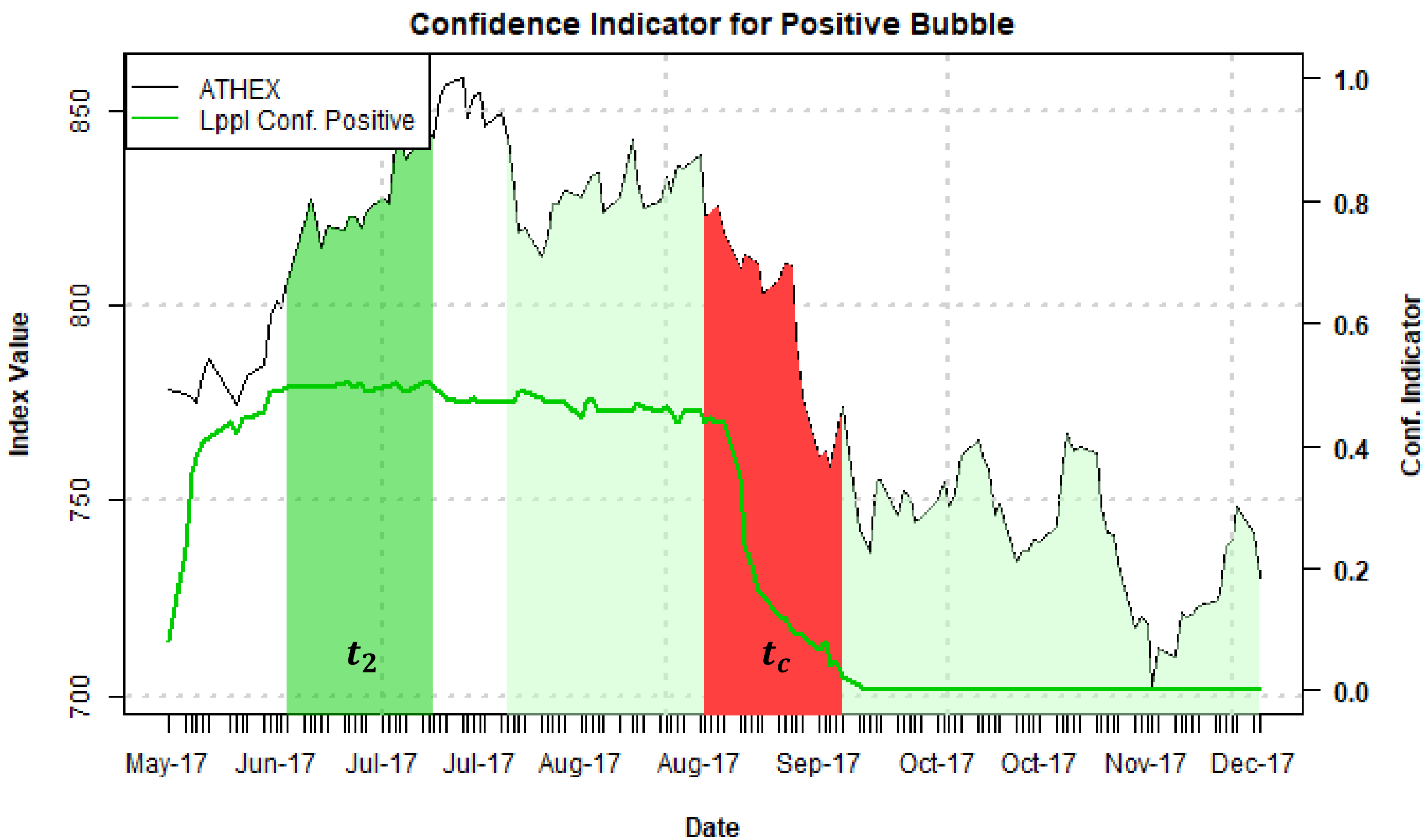
Positive Bubble 2p



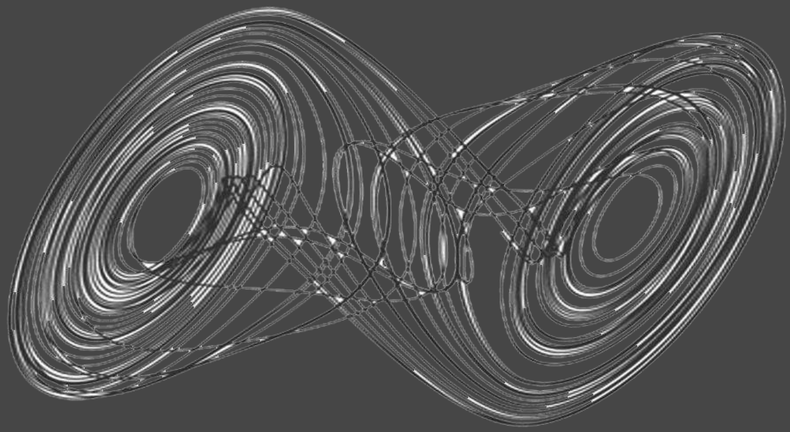
Positive Bubble 4p



Positive Bubbles



APPLICATION OF DEEP LEARNING AND CHAOS THEORY FOR LOAD FORECASTING IN GREECE



- Stergiou, K., Karakasidis, T.E. Application of deep learning and chaos theory for load forecasting in Greece. *Neural Comput & Applic* (2021). https://doi.org/10.1007/s00521-021-06266-2

Electricity and load consumption

- Predicting electricity / load consumption is of great importance
- Modern societies increasingly dependent on electricity
- Task → Minimize the energy waste → lead to the optimization of smart grids
- Forecasting load consumption → manage the production
- Short term forecasts → information to those involved with the energy markets
  Medium / Long term forecasts → information for the system operators

Neural Network Models

- **FNN (Forward Neural networks)**

  <u>total input calculation:</u>

  $z = w_1 \cdot x_1 + w_2 \cdot x_2 + \ldots\ldots + w_n \cdot x_n + b$

  <u>total output calculation:</u>

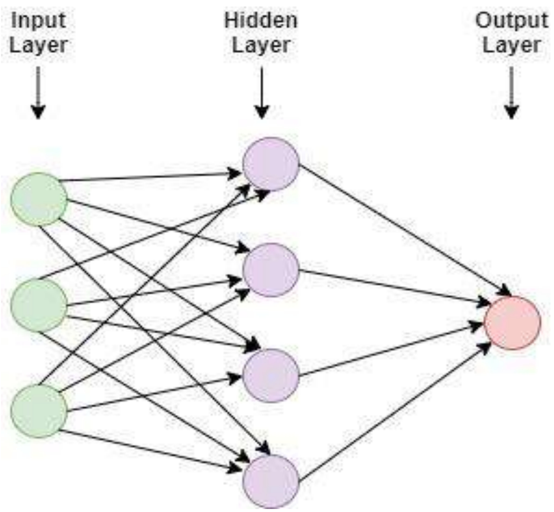
  $y = f(z) = f(w_1 \cdot x_1 + w_2 \cdot x_2 + \ldots\ldots + w_n \cdot x_n + b)$

x = input

w = weights

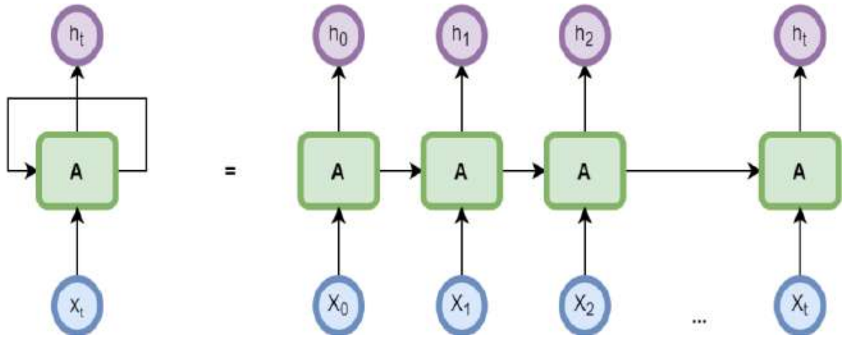
b = bias

y = ouput



!!! NO FEEDBACK COMMUNICATIONS BETWEEN THE LAYERS !!!

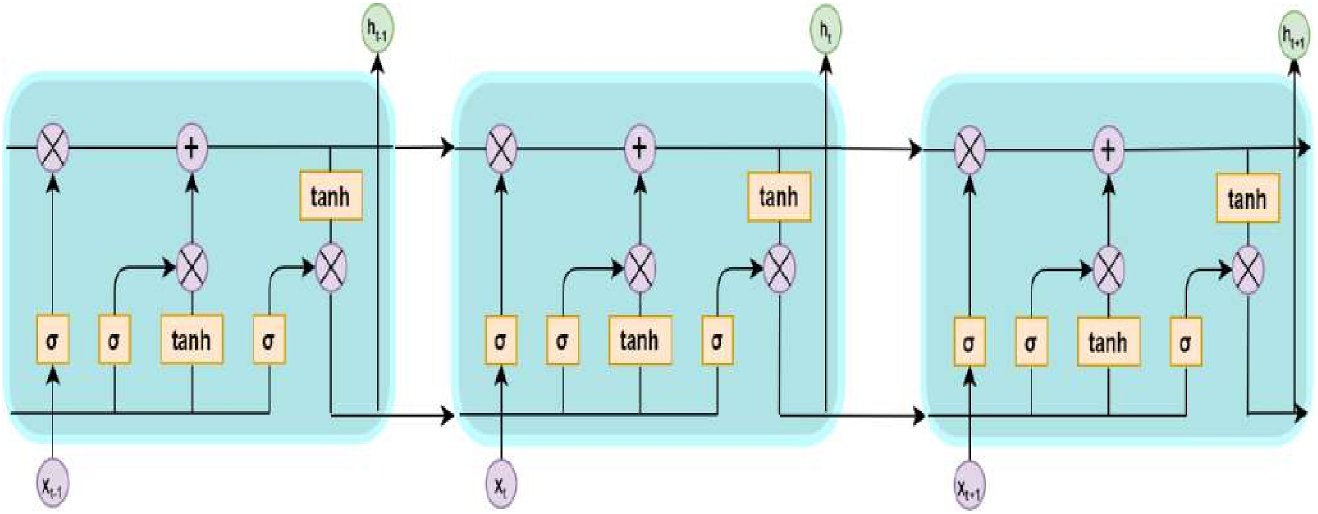
Neural Network Models (2)



- **LSTM - Long Short-Term Memory Network**
- → Recurrent Neural Networks (RNN's) using temporal information of the input data
  - ➢ Memory cell is a neuron structure of the LSTM that has the ability to store information.
  - ➢ Input, output, and forget gate are responsible for the flow of information.
  - ➢ Every gate possesses an activation function
  - ➢ To predict a value p_t at time t, past samples of data must be propagated through the neural network $\{p_{t-n}, \ldots, p_{t-1}\}$.
  - ➢ Advantage of LSTM network : overcomes the vanishing gradient by using structures such as forget gates, for the optimization of the information transferred between memory cells.

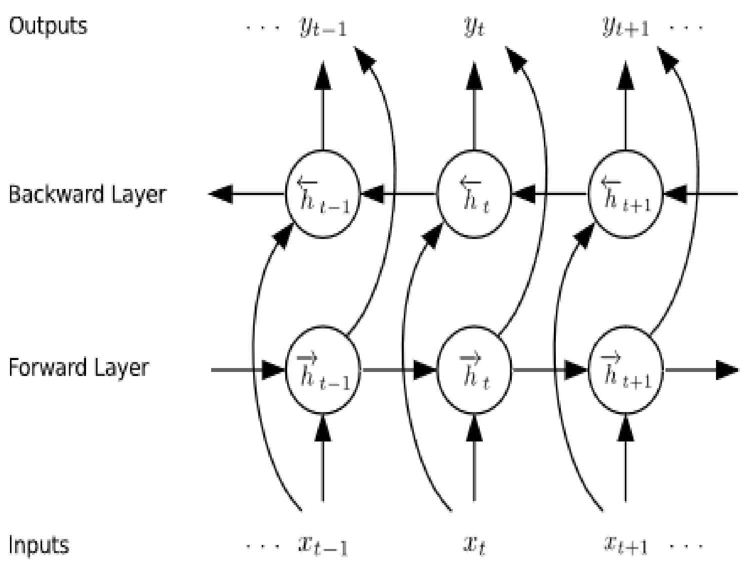
Neural Network Models (3)

- **LSTM**
  - Through the forgetting gate retains or rejects the information
  - the gate generates a value which is between 0 and 1 for time t and t -1
  - The value produced → multiplied by the hidden state at time t-1
  - The generated information is stored, while the outputs from the processes have been multiplied



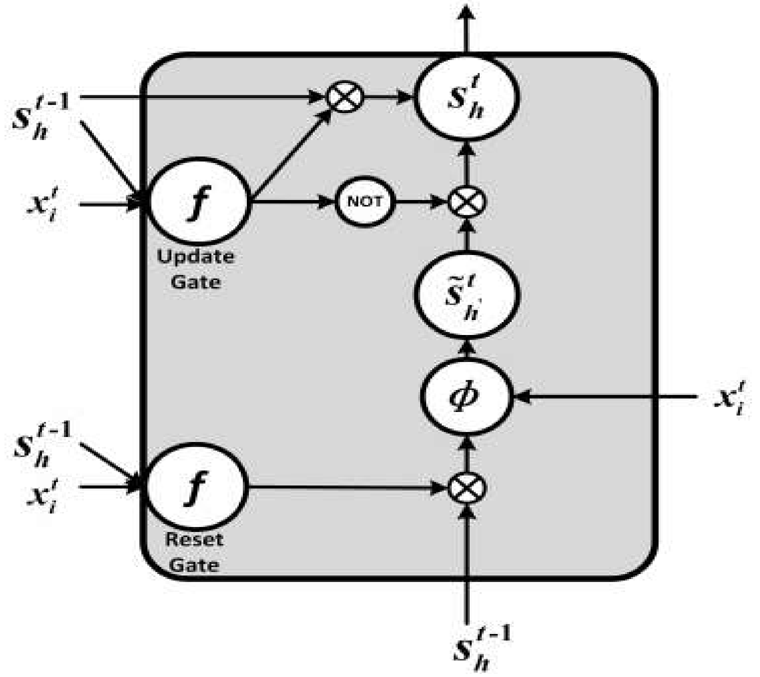
Neural Network Models (4)

- **Bi - LSTM**
  - ➢ The flow of information is two-way
  - ➢ Processing the data in both directions, with two separate hidden layers that are fed to the same output layer
  - ➢ Computes the forward/ backward hidden sequences and the output sequence by iterating
    - the backward layer from t = T to 1
    - the forward layer from t = 1 to T
    - updates the output layer



Neural Network Models (5)

- **GRU - Grating recurrent unit**
  - ➢ Improved framework based on RNNs
  - ➢ Similar with LSTM model but consists of two gates (LSTM has three gates)
  - ➢ Simpler to compute and implement
  - ➢ Better results in small datasets



Evaluation Indexes

- **RMSE (Root Mean Square Error)**
  - Standard deviation of the prediction errors
  - How far data points are located from the resulting regression curve

$$RMSE = \sqrt{\frac{1}{n}\Sigma_{i=1}^{n}(y_i^{obs} - y_i^{for})^2}$$

- **MAPE (Mean Absolute Percentage Error)**
  - Measure of prediction accuracy of a forecasting method
  - Statistical measure of how accurate a forecast system is

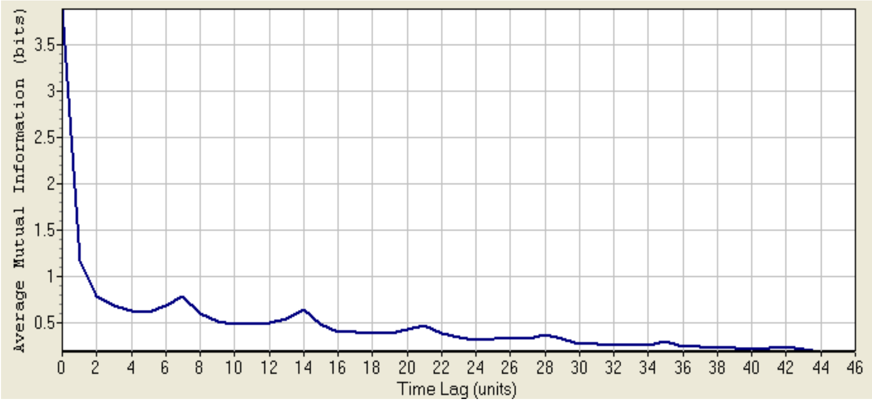
$$MAPE = \frac{1}{n}\sum_{i=1}^{n}|\frac{y_i^{obs} - y_i^{for}}{y_i^{obs}}|$$

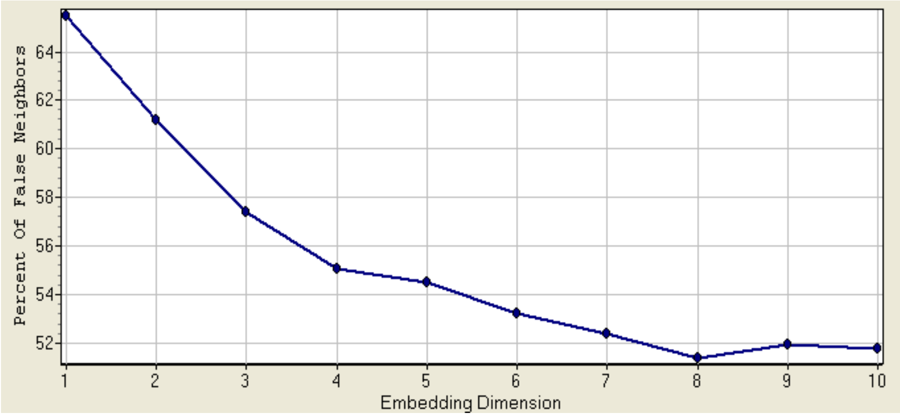
Lyapunov exponent

- Chaos → alternative cause of randomness → establishes a limit on long term – prediction / offers possibilities for short-term predictions
- Non – Linear Analysis → determination of the forecasting horizon
- Spectrum of Lyapunov exponents → indication of a system's chaotic behavior
- Lyapunov exponents → Indicators of the divergence or convergence of two system trajectories in the phase space, initially close to each other
- At least one positive Lyapunov exponent → CHAOS
- Maximun Lyapunov exponent → Lyapunov Time → $h = 1/\lambda max$

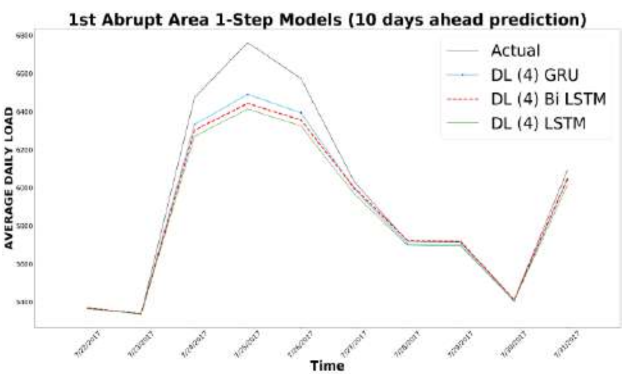
Safe Prediction Horizon

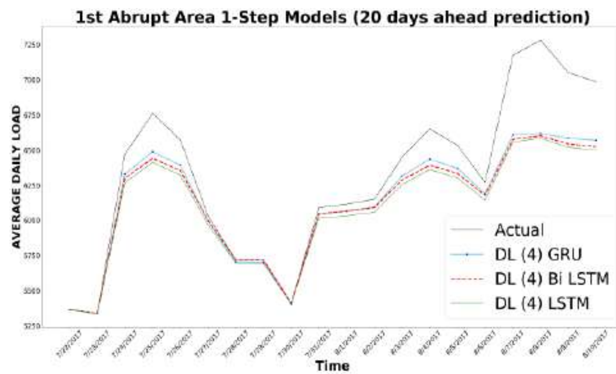
- Extract the maximum Lyapunov exponent (VRA software)
- Calculated the time delay τ = 4
- From False Nearest Neighbors diagram → embedding dimension extraction m = 8
- $\lambda_{max} = 0.1 > 0$ (chaotic)
- $h = 1/\lambda max = 10$
- Feed the Lyapunov Time into the Neural Network
  models as sequence

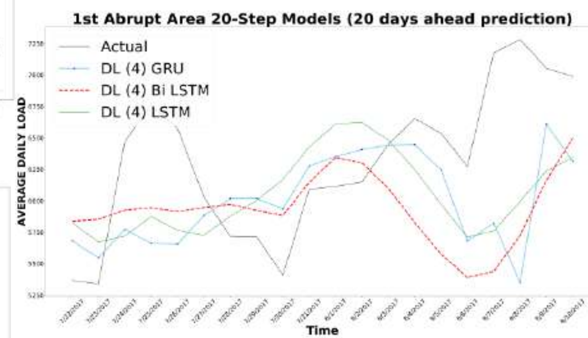


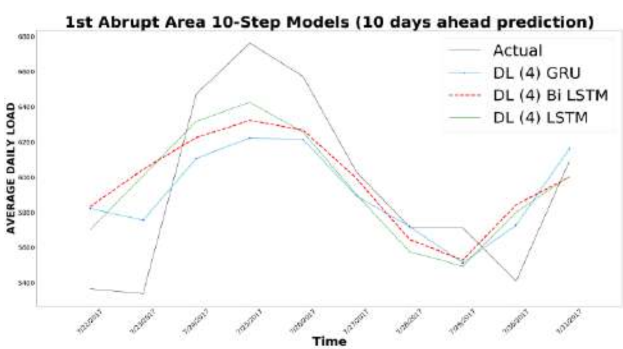


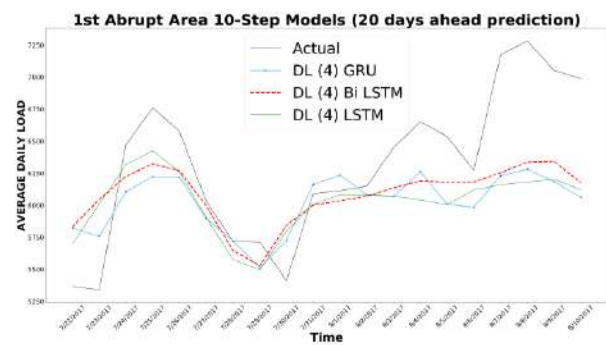
Abrupt Areas



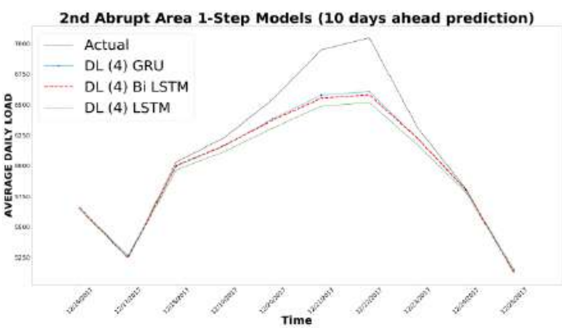


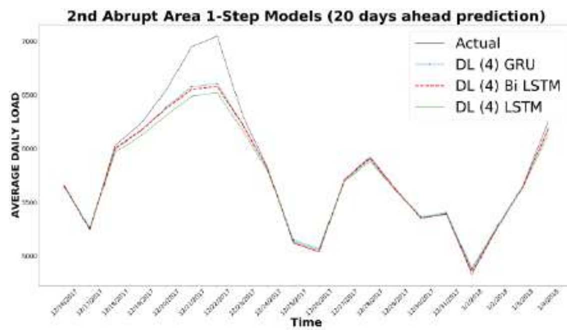


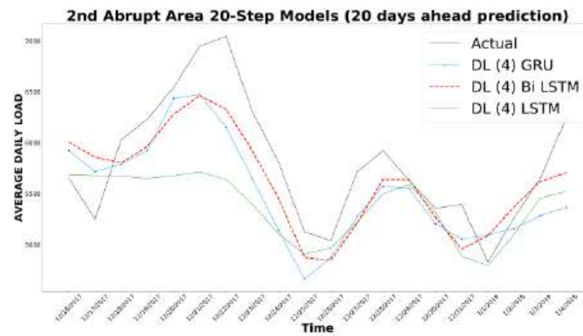


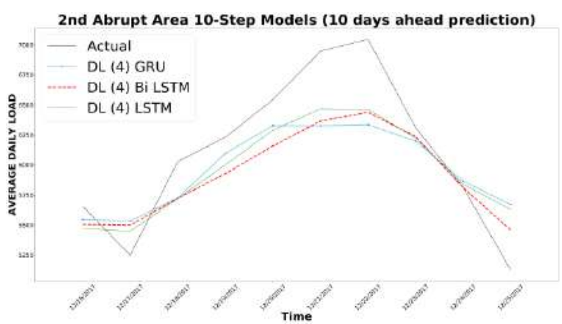


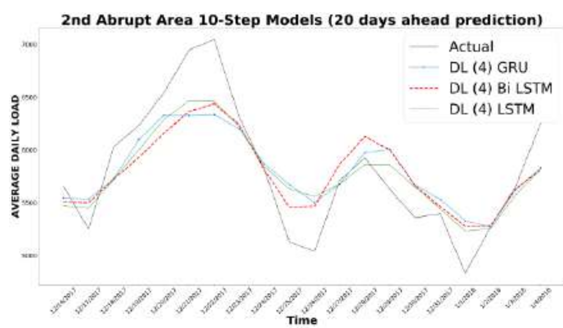
Abrupt Areas



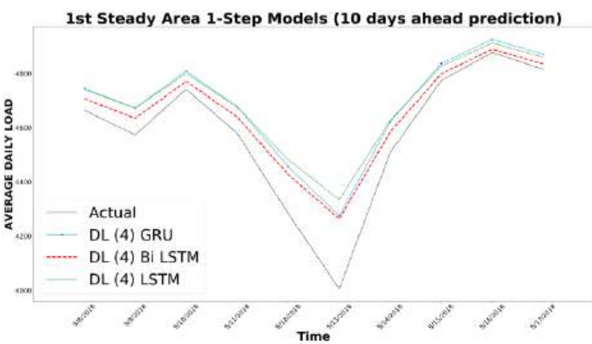


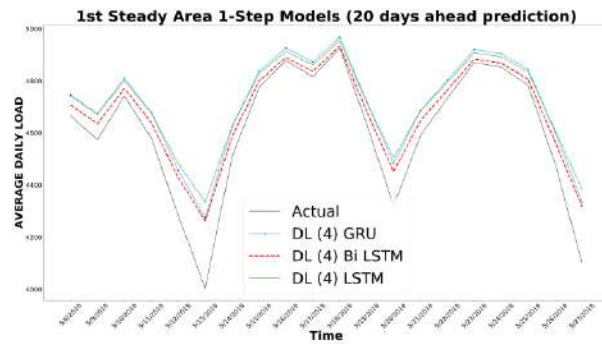


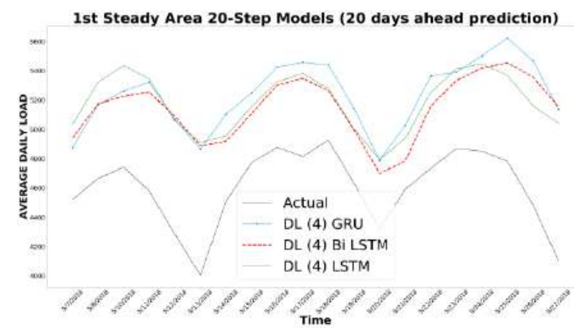


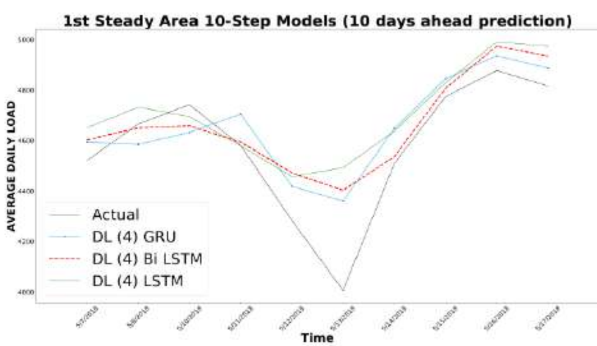


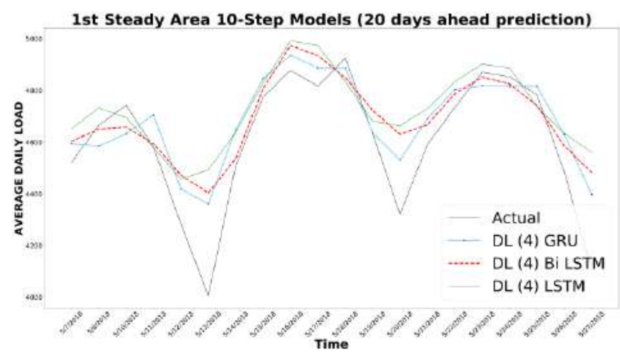
Normal Areas



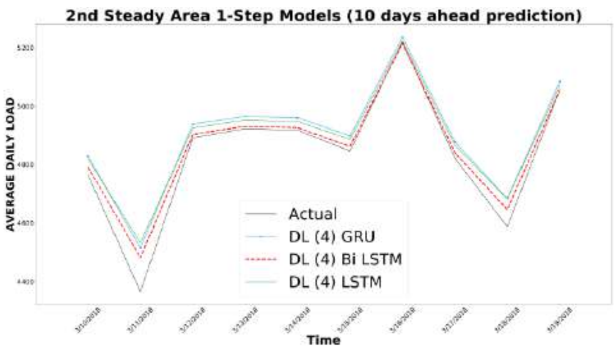


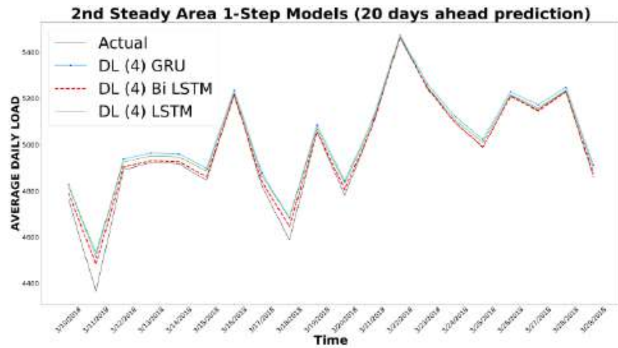


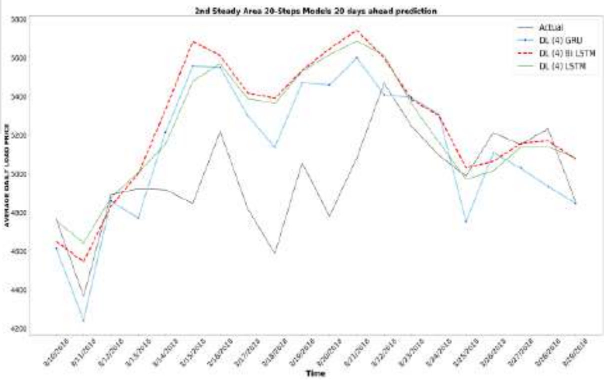


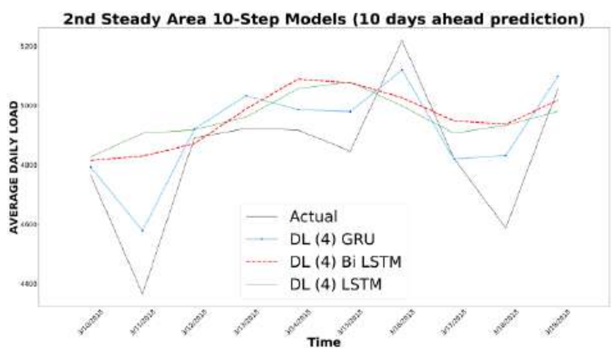


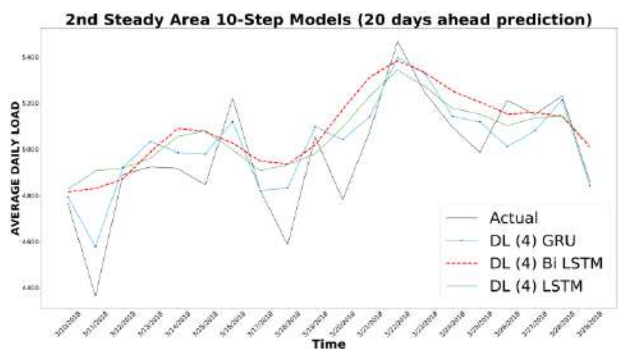
Normal Areas











Evaluation Indexes – DL (4) GRU

Evaluation Indexes for the Deep Learning – 4layers – GRU model

DL (4) GRU										
	1 - step				10 - step				20 - step	
	RMSE		MAPE		RMSE		MAPE		RMSE	MAPE
	+10 days	+20 days	+10 days	+20 days	+10 days	+20 days	+10 days	+20 days	+20 days	+20 days
1st Abrupt region	113.82	262.5	1.08	2.55	331.15	513.5	4.80	6.40	707.42	8.15
2nd Abrupt region	192.45	137.91	1.80	1.10	380.71	343.20	5.01	4.68	450.12	6.50
1st Steady region	125.09	119.26	2.43	2.43	144.95	138.69	2.61	2.42	657.80	13.81
2nd Steady region	69.98	54.49	1.27	0.91	160.17	142.50	2.46	2.18	349.46	5.72

Evaluation Indexes – LSTM (4) GRU

Evaluation Indexes for the Deep Learning – 4layers – LSTM model

DL (4) LSTM										
	1 - step				10 - step				20 - step	
	RMSE		MAPE		RMSE		MAPE		RMSE	MAPE
	+10 days	+20 days	+10 days	+20 days	+10 days	+20 days	+10 days	+20 days	+20 days	+20 days
1st Abrupt	153.40	301.71	1.60	3.17	322.71	528.40	4.79	6.44	678.57	9.11
2nd Abrupt	241.32	173.47	2.38	1.43	334.59	307.28	4.32	4.28	613.71	7.61
1st Steady	140.78	133	2.55	2.39	258.91	288.82	4.2	4.82	602.69	12.68
2nd Steady	68.60	51.05	1.11	0.72	235.56	196.62	3.77	3.11	388.32	5.87

Evaluation Indexes – Bi LSTM (4) GRU

Evaluation Indexes for the Deep Learning – 4layers – Bi LSTM model

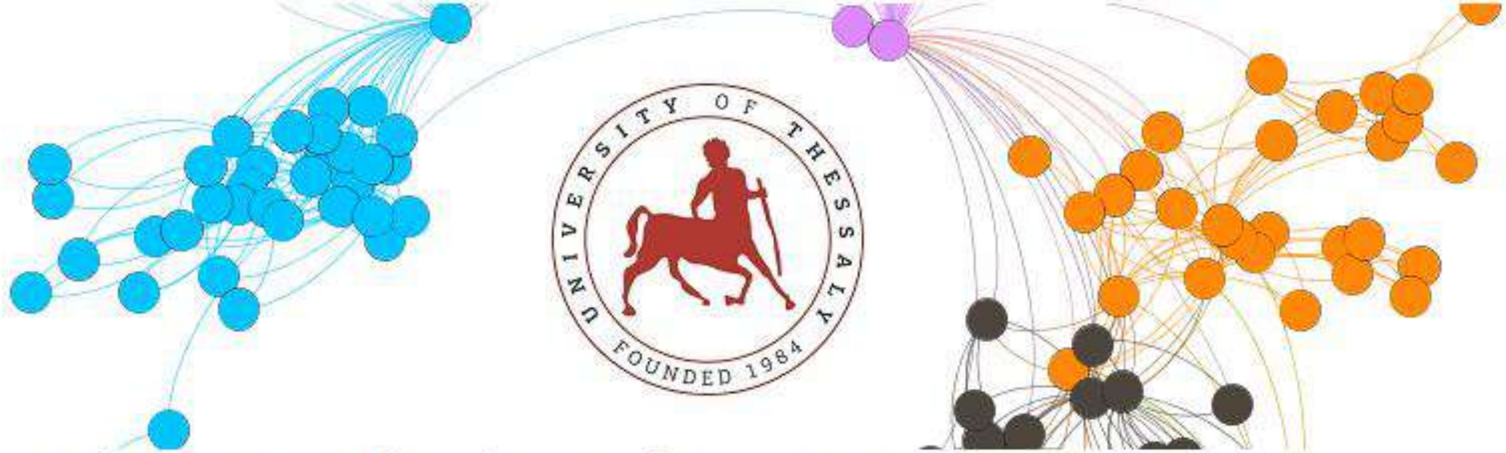
DL (4) Bi LSTM											
	1 - step				10 - step				20 - step		
	RMSE		MAPE		RMSE		MAPE		RMSE	MAPE	
	+10 days	+20 days	+10 days	+20 days	+10 days	+20 days	+10 days	+20 days	+20 days	+20 days	
1st Abrupt	134.65	284.16	1.21	2.81	360.34	482.79	6.44	7.00	750.69	9.30	
2nd Abrupt	203.66	144.95	1.79	1.00	352.35	325.86	4.79	4.66	368.54	5.40	
1st Steady	102.65	95.95	1.69	1.57	153.55	160.63	2.50	2.59	581.92	11.89	
2nd Steady	43.51	31.49	0.89	0.80	220.04	201.54	3.64	3.29	427.46	6.57	

General Conclusions

- Nonlinear methods seem to identify system transitions in time or in time and space
- Indications of chaos in the systems studied
- System dynamics are reflected on the R.P.
- Using RQA we can extract useful information about the dynamics of a system
- The methodology seems promising for the study of spatiotemporal and multiscale phenomena in engineering and environmental sciences

Future Applications

- Analysis and predictions
  - Prices of energy assets, Prices of building materials
- Predictions of energy production from intermittent renewable energies
  - Correlations-causalitites with other parameters (eg. Temperature, wind…)
  - Spatiotemporal variation for energy stock market use
- Combine with machine learning methods
  - Neural networks
  - Symbolic regression: find equations from data
  - Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations



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- D. Angelis (Ph.D. candidate)
- K. Stergiou (Ph.D. candidate)
- K. Papastamatiou (Ph.D. candidate)

Department of Physics Graduate courses

ΠΜΣ – Εφαρμοσμένη Φυσική

Τμήμα Φυσικής, Σχολή Θετικών Επιστημών, Πανεπιστήμιο Θεσσαλίας

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THANK YOU

FOR YOUR ATTENTION