

**30th Summer School – Conference on
“Dynamical Systems and Complexity”,
Chalkidi, Greece, August 28 – September 6, 2024**

&

**Celebration of Marko Robnik’s
Contributions to Physics
on the Occasion of his 70th Birthday**

I am honored and delighted to be able to participate in the celebration of Marko's 70th birthday and his numerous contributions theoretical physics, primarily in classical and quantum chaos!

I have great admiration and respect not only for his pioneering work and leadership in quantum and classical chaos but also for establishing highest international scientific standards in Slovenia and abroad, as the founder and the leader of the CAMTP.

I cherish him as a friend, a colleague and a collaborator.

Long-standing association with Marko:

personal (same elementary school, gymnasium & University)

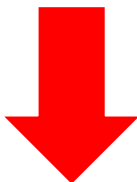
& professional (since 1994, the member of CAMTP since 2000)

We continue close scientific ties and discussions on progress in our respective fields

& joint work:

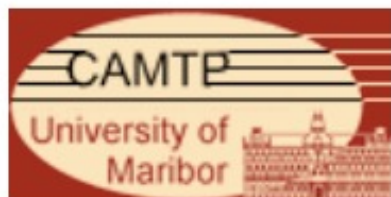
“Gravity Trapping on a Finite Thickness Domain Wall: An Analytic Study,”
Phys. Rev. D 77 ,124003 (2008) [arXiv:0801.0801 [hep-th]].

Non-linear aspects of gravitational physics and implications for
“The World on the Brane”, which fits into aspects of non-linear
physics for Fundamental Interactions.



Geometry and String Theory Confronting Particle Physics and Black Holes

Mirjam Cvetič

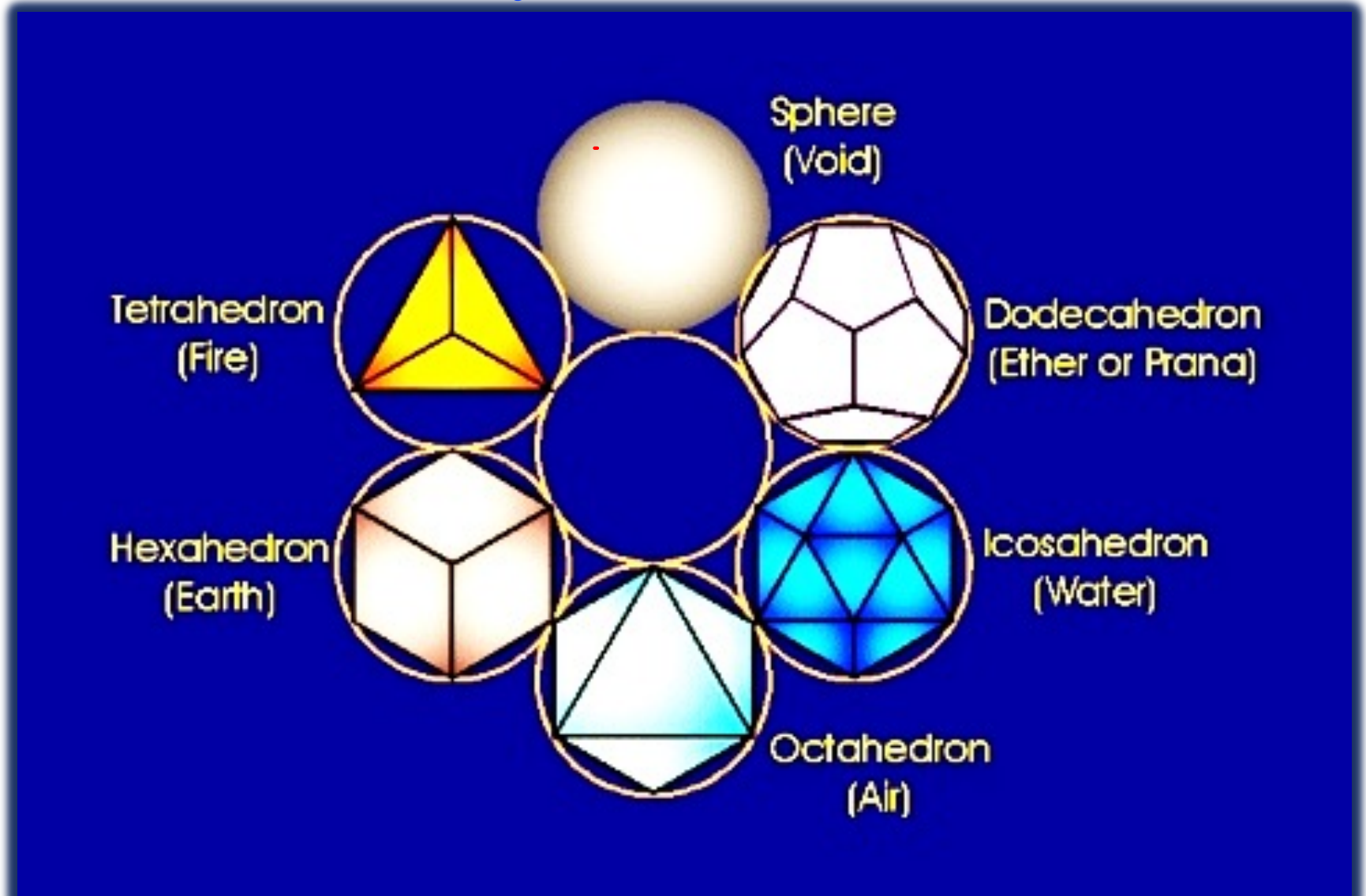


Univerza v Ljubljani
Fakulteta za *matematiko in fiziko*



Quests to uncover secrets of nature in a geometric way have a long history, dating back to Greek philosophers

Five Platonic Solids - Polyhedra as Five Elements of Nature



Modern examples of such links

Einstein's theory of gravity

&

String Theory:

prime candidate for unification of
electromagnetic, strong and weak forces
(Standard Model) with quantum gravity

Focus on implications for

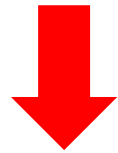
particle physics & black holes
&
quantum field theory

Apologies: UPenn-centric



Outline

I. Brief Summary of High Energy Physics



II. Modern String Theory
(Branes)



Implications for



III. Particle Physics & QFT

(Standard Model &
higher symmetries)

Time permitting

IV. Black Holes

(Microscopic Structure)

V. Concluding Remarks

Ia. Theory of Elementary Particles based on Quantum Field Theory

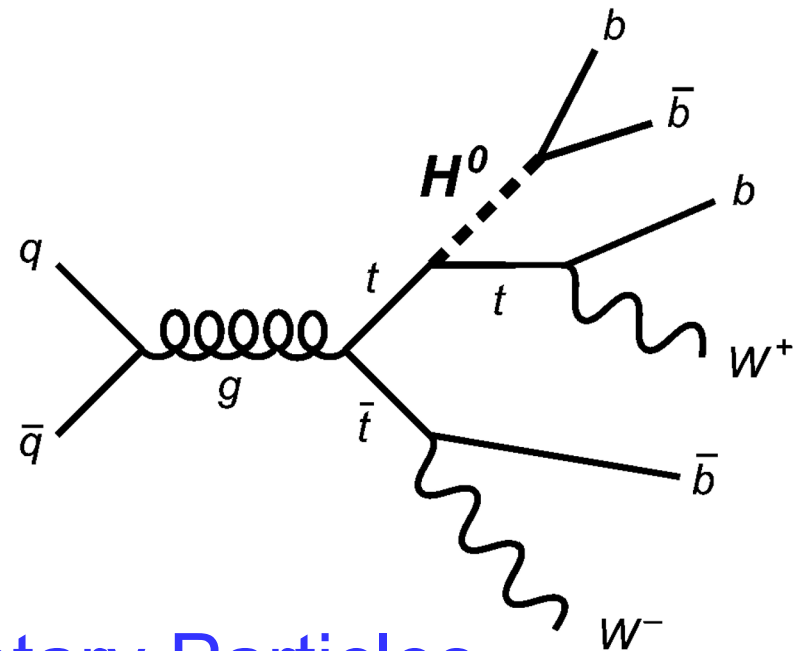
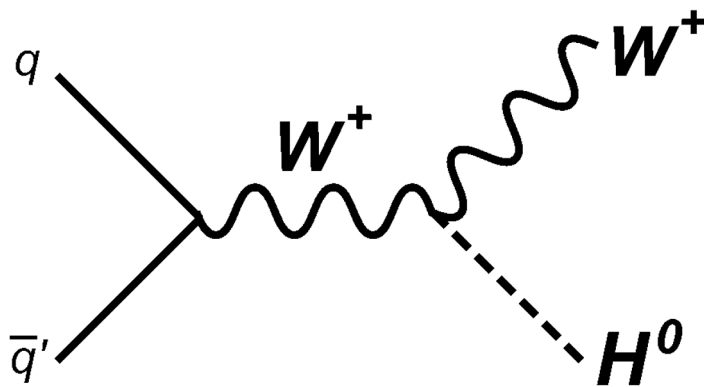
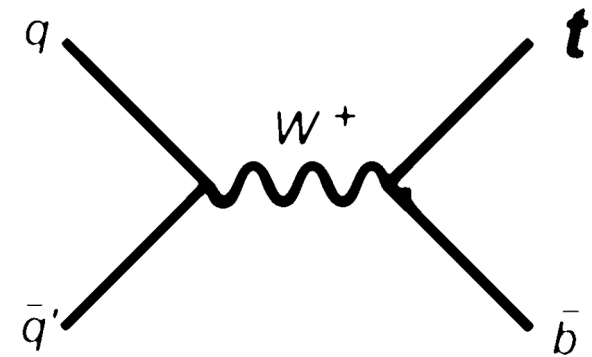
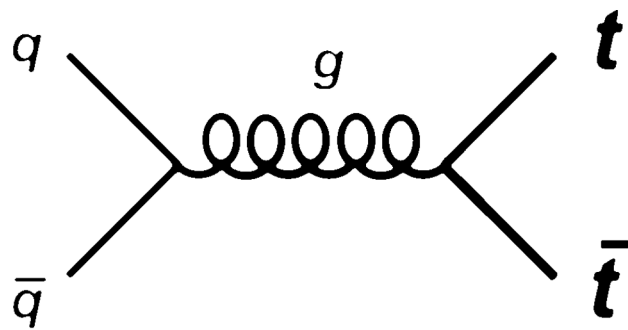
Quantum mechanics at high energies →
Interactions among elementary particles take place in space-time via exchanges of virtual particles

time



Quantified by
Feynman diagrams

'60-ies



Quantum Field Theory of Elementary Particles
 consistent (“renormalizable”) quantum theory
 of electromagnetic, strong and weak interactions →

Standard Model

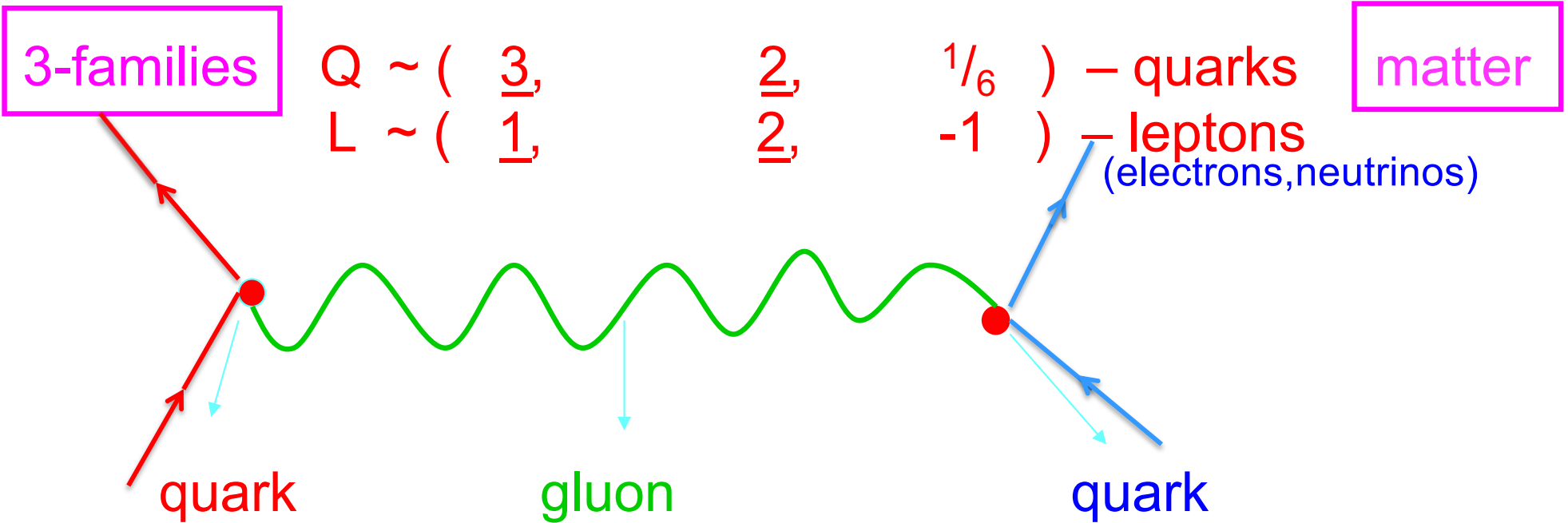
Glashow, Salam, Weinberg ‘70-ies

Standard Model

Quantum Field Theory based on **non-Abelian gauge symmetry:**

$$SU(3) \times SU(2) \times U(1)$$

Gauge bosons: gluons, W-bosons, photon
mediate quantum interactions among matter particles:



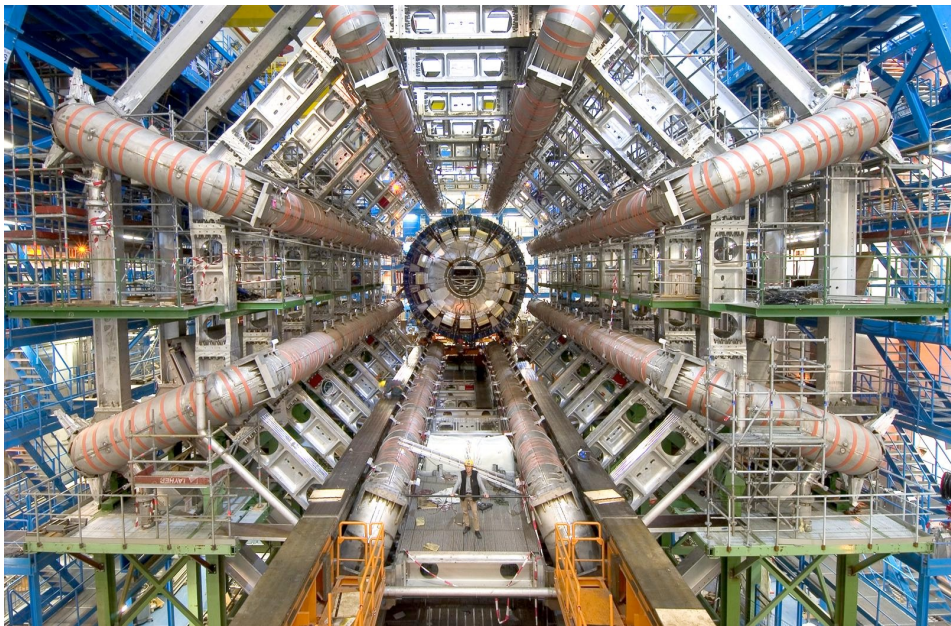
No time to explain

& Higgs particle (responsible for symmetry breaking & mass of matter)

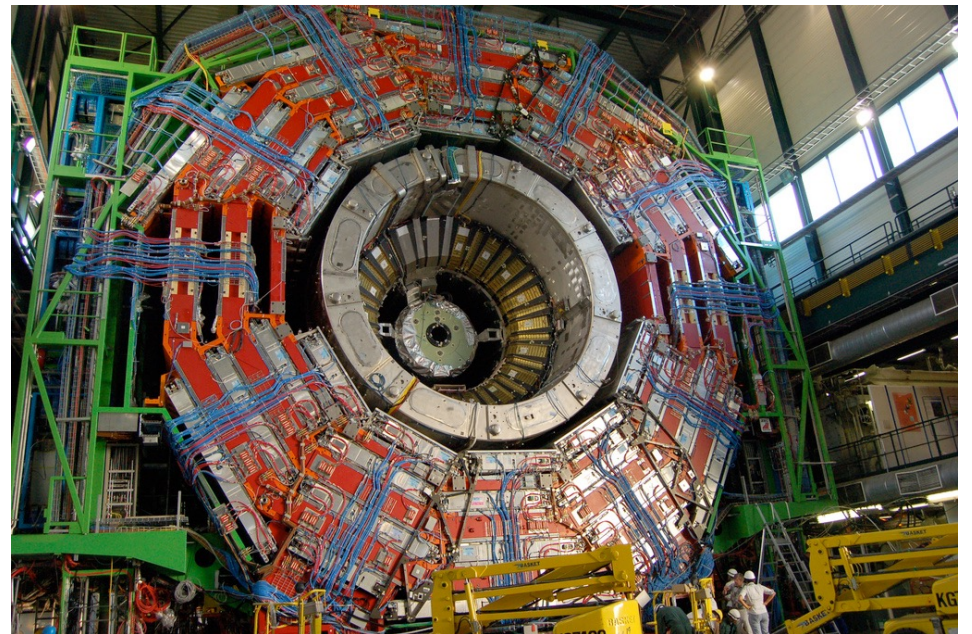
Building blocks tested at the Large Hadron Collider at CERN!



Large Hadron Collider
at CERN,
Geneva, Switzerland

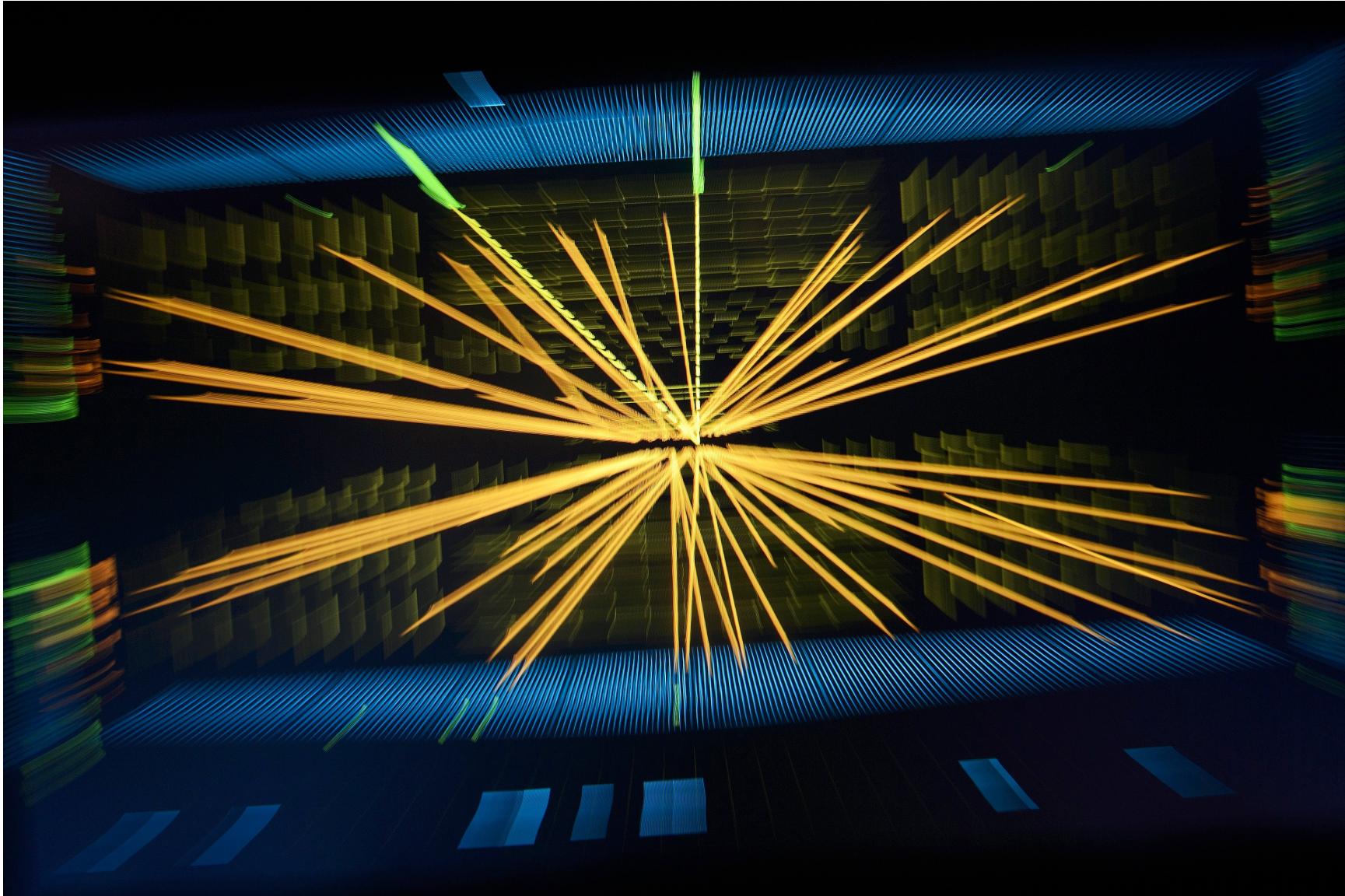


ATLAS detector



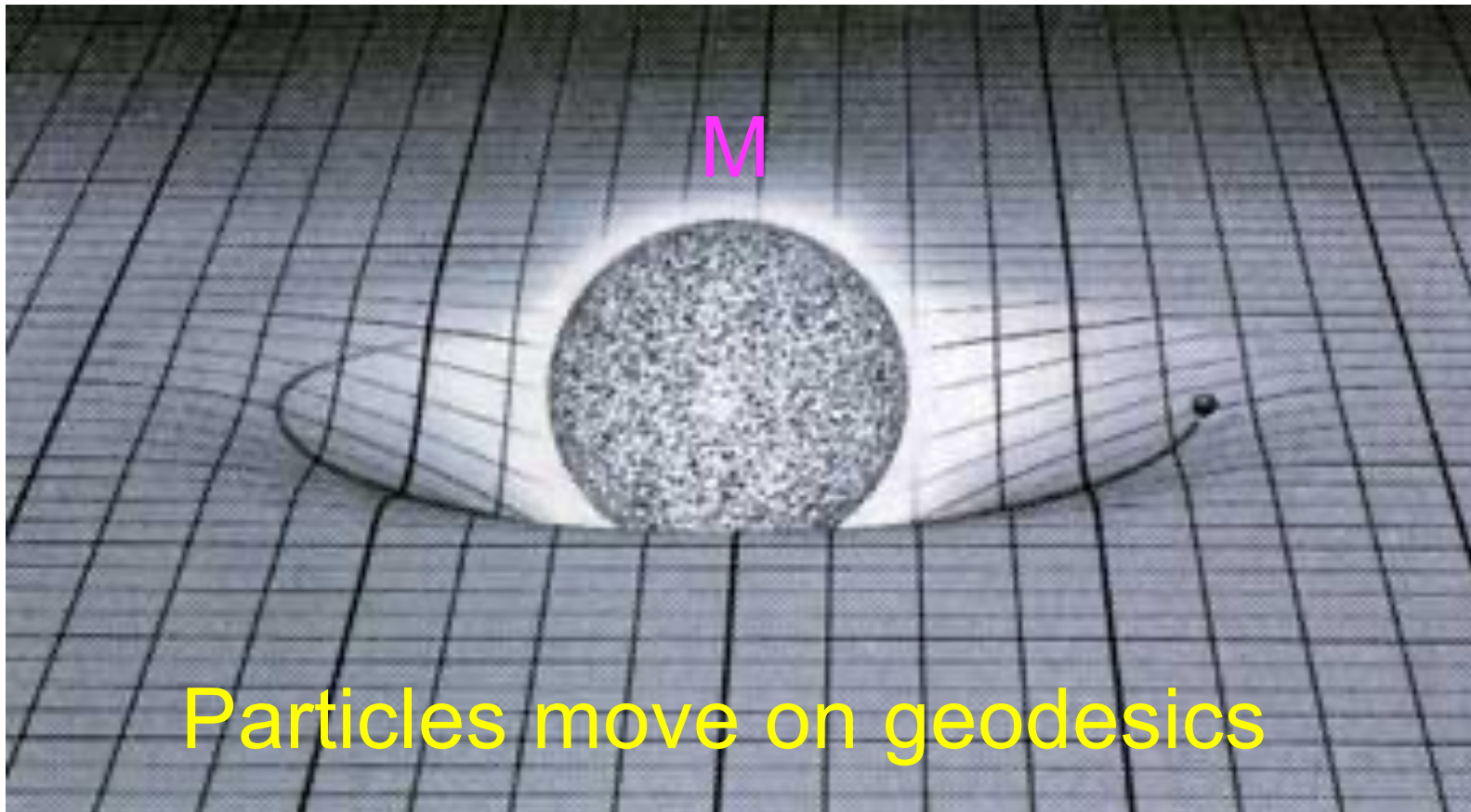
CMS detector

Culminated in the Higgs Particle Discovery, July 4, 2012



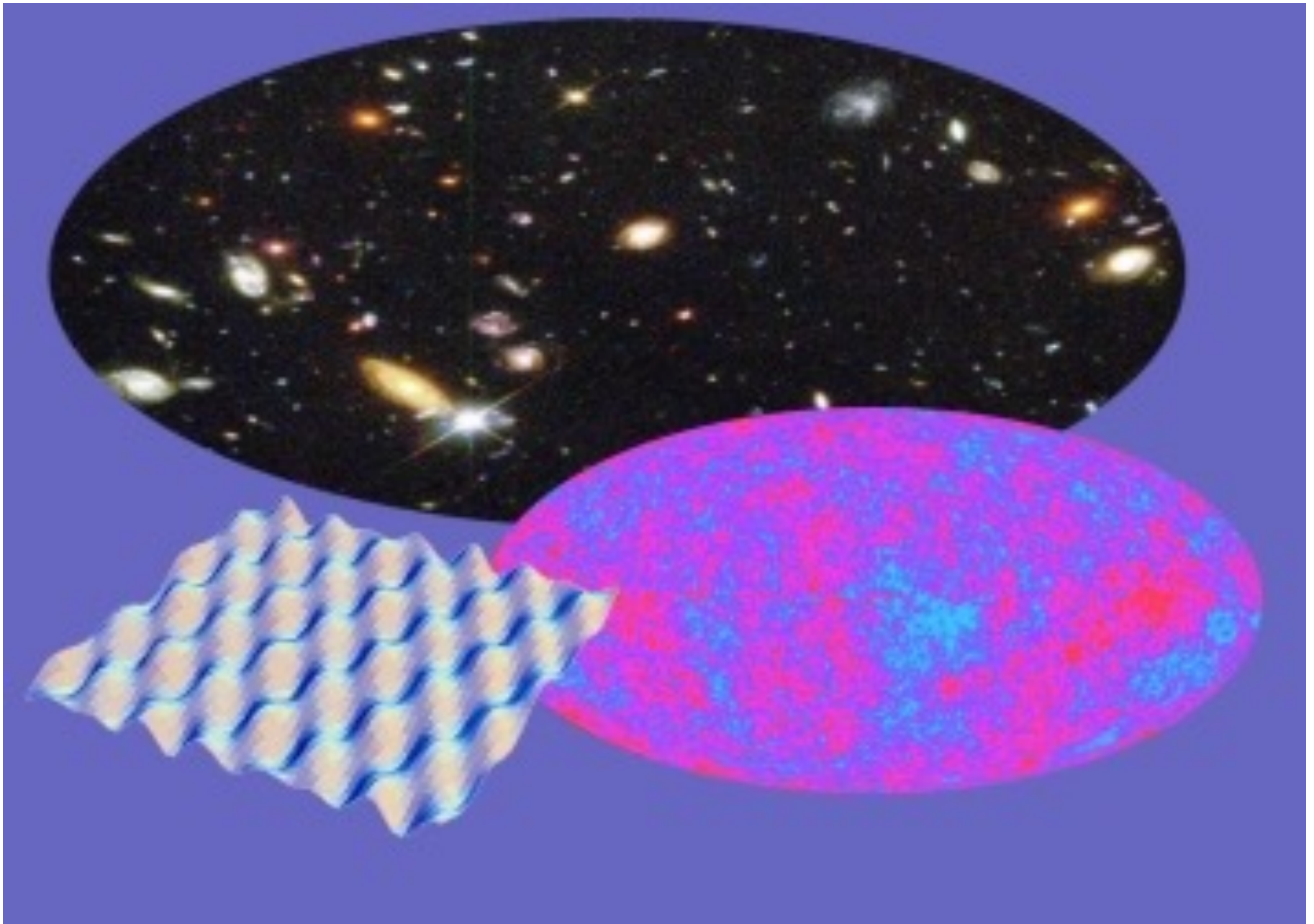
Ib. Gravity as classical theory: Einstein's Theory of Gravity

Space-time is curved due to massive matter



M-mass

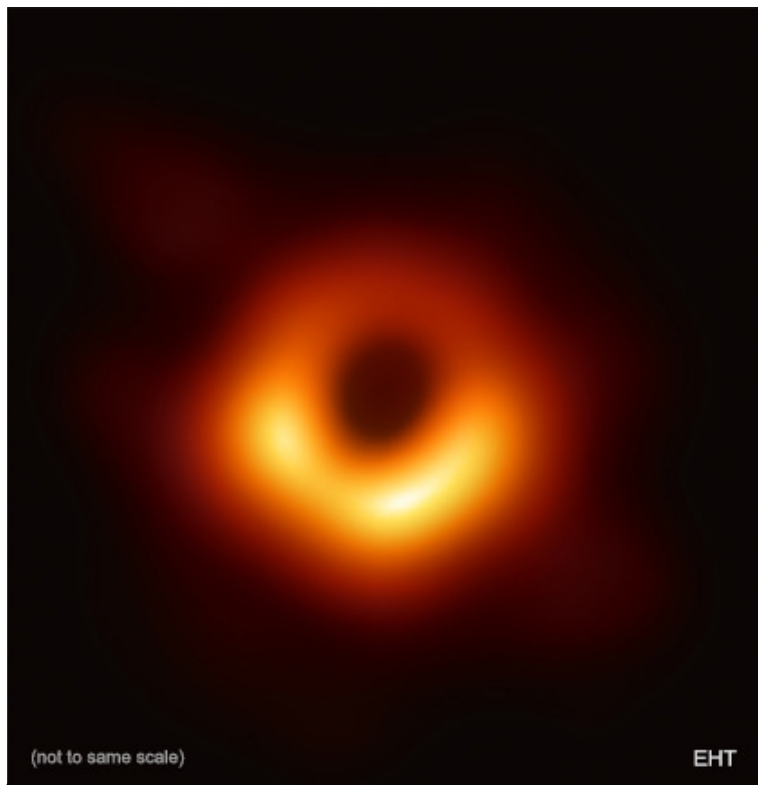
Planetary orbits can be explained geometrically



Einstein Theory of Gravity is relevant for cosmological questions of the **Early Universe** (not in this talk)

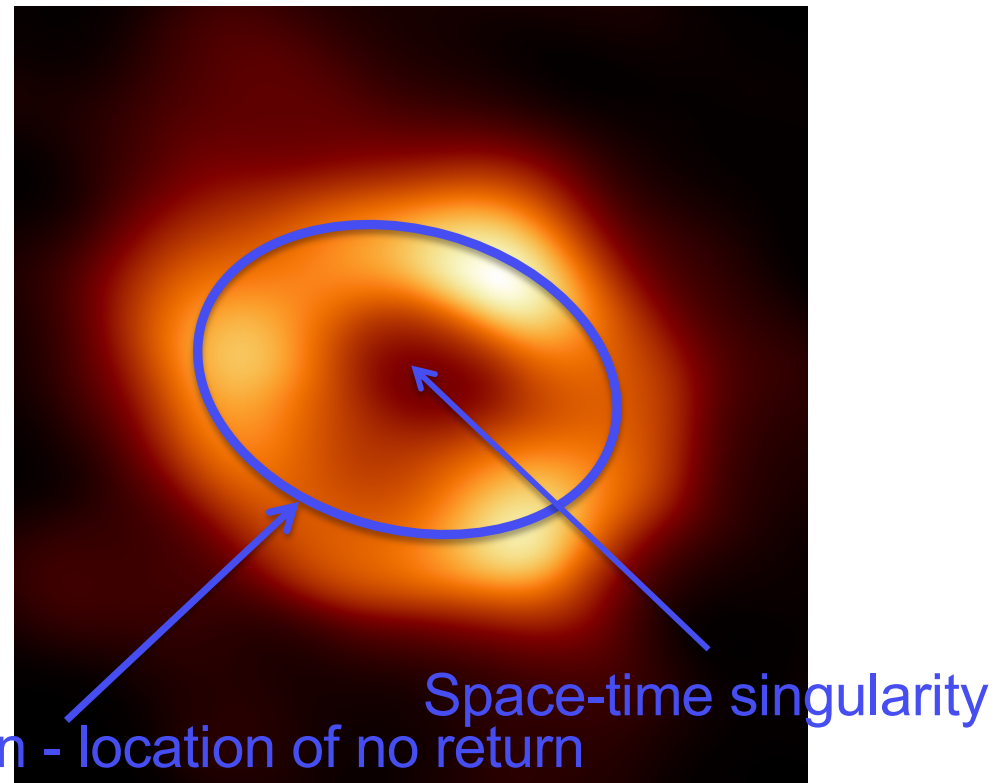
Einstein's Theory predicts Black Holes

Due to its high mass density the space-time curved so much that objects, traveling toward it, reach a location of no return - Horizon



2019

Event Horizon Telescope discovery (Sgt A*)



2022

Black Holes have M-mass, J-angular momentum,
can have Q-charge & Horizon

“Behave” as thermodynamic objects with

Bekenstein, Hawking '70-ies

Entropy: $S = \frac{1}{4} A$ (degree of disorder) &

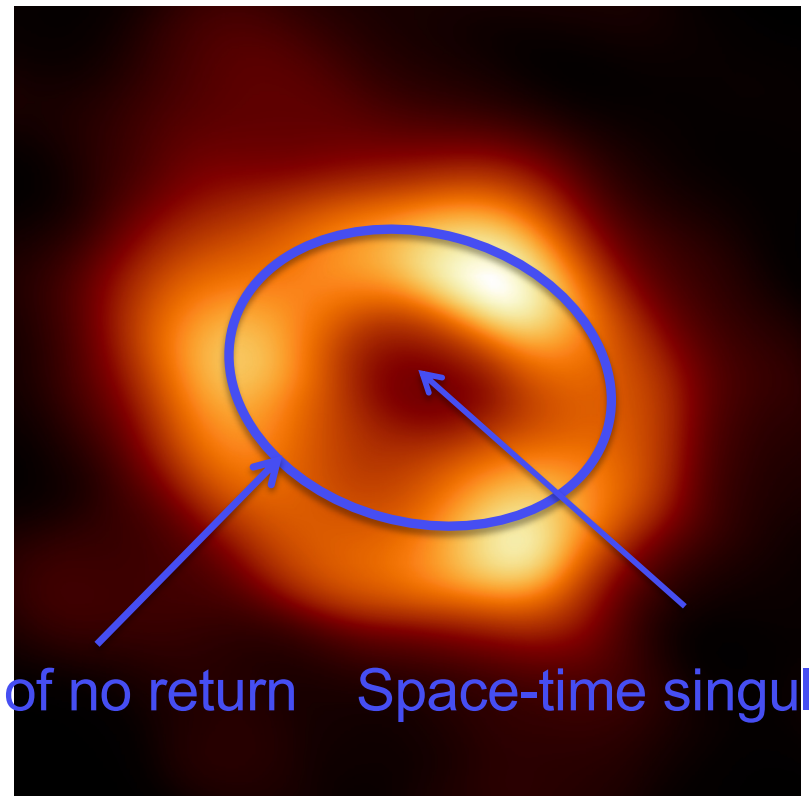
Temperature: $T = \kappa / 2\pi$

A = area of horizon

κ = surface gravity at the horizon

Statistical/microscopic origin
of black hole entropy? → later,
time permitting

Horizon-surface of no return Space-time singularity

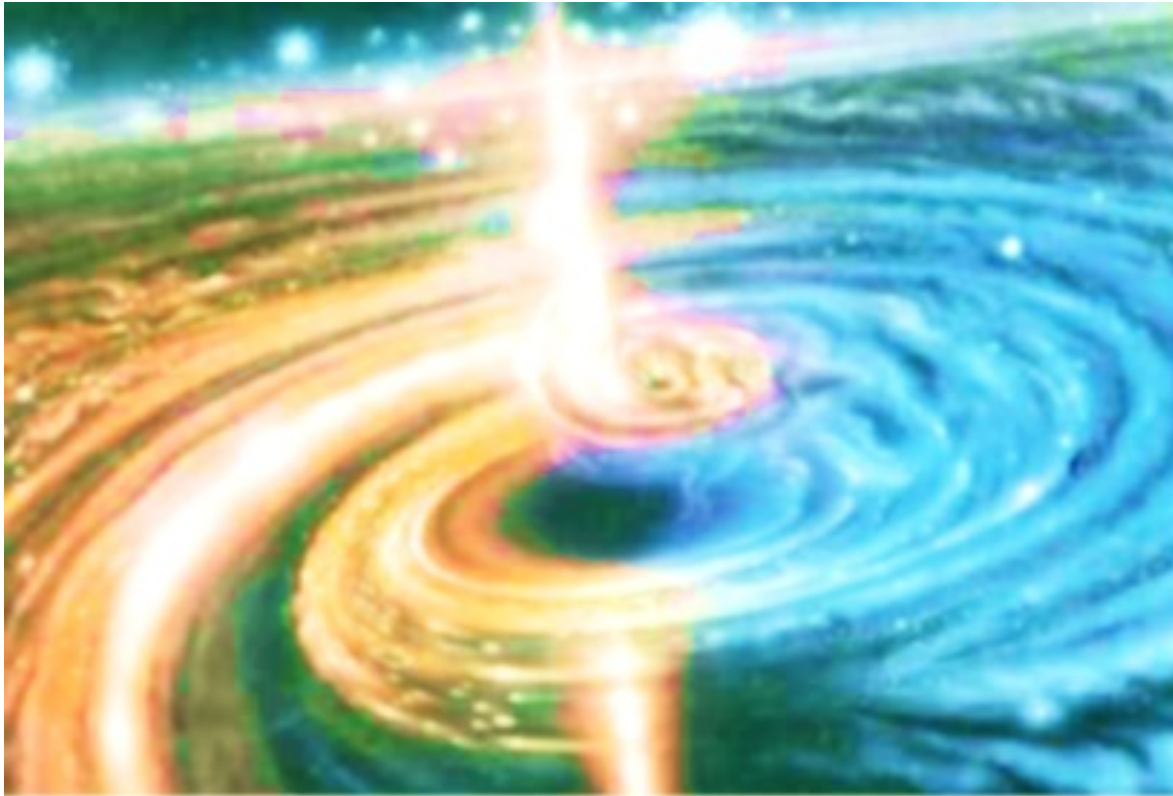


k_B (Boltzman constant) = G_N (Newton constant) = \hbar (Planck constant) = c (speed of light) = 1

Gravity as Quantum Field Theory

with gravitons mediating quantum interactions with matter

→ infinities (“non-renormalizable”) - inconsistent

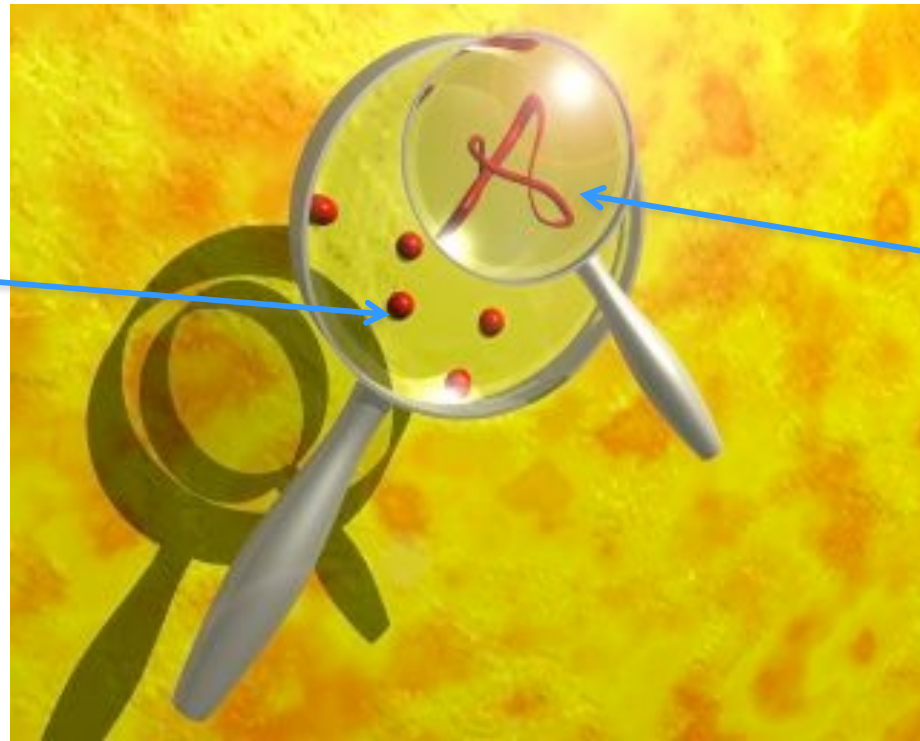


Search for consistent Quantum Theory of Gravity

The Holy Grail of Theoretical Physics!

II. String Theory

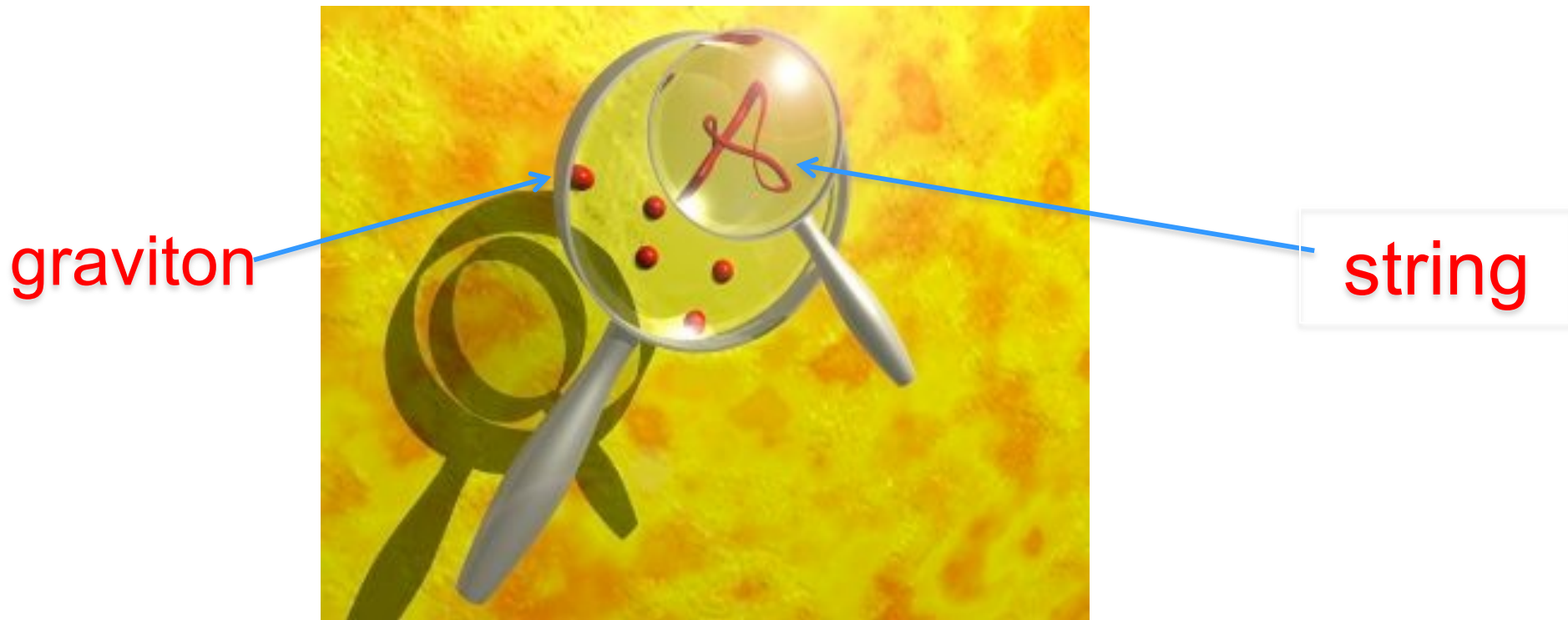
quark



string

distances
 $\sim 10^{-33}$ cm

Elementary particles as quantum excitations of strings



Graviton (mediating quantum gravitational interactions)
 always appears a massless quantum excitation of strings

Built as a quantum state: $g_{\mu\nu} = |\tau+\sigma\rangle_{\mu} \otimes |\tau-\sigma\rangle_{\nu}$

μ, ν - space-time indices

τ, σ - string world-sheet coordinates

Left-moving Right-moving sectors

→ string theory contains quantum gravity!

Quantum interactions: joining & splitting of strings

Interactions ``softer''

No infinities!

Green & Schwarz '84

string coupling

g_s



String Theory, a finite theory of quantum gravity!

Things seems to be in place:

- String Theory, a consistent quantum theory where particles, including graviton, appear as string excitations at $R_{\text{planck}}=10^{-33}$ cm
- However, string theory connection to fundamental forces of nature “more complex”:
as a consistent quantum theory
(without infinities, due to “anomalies”)
→ String Theory in 10 dimensions (9-space, 1-time)
- But our world is 4 dimensional (3-space, 1-time)!

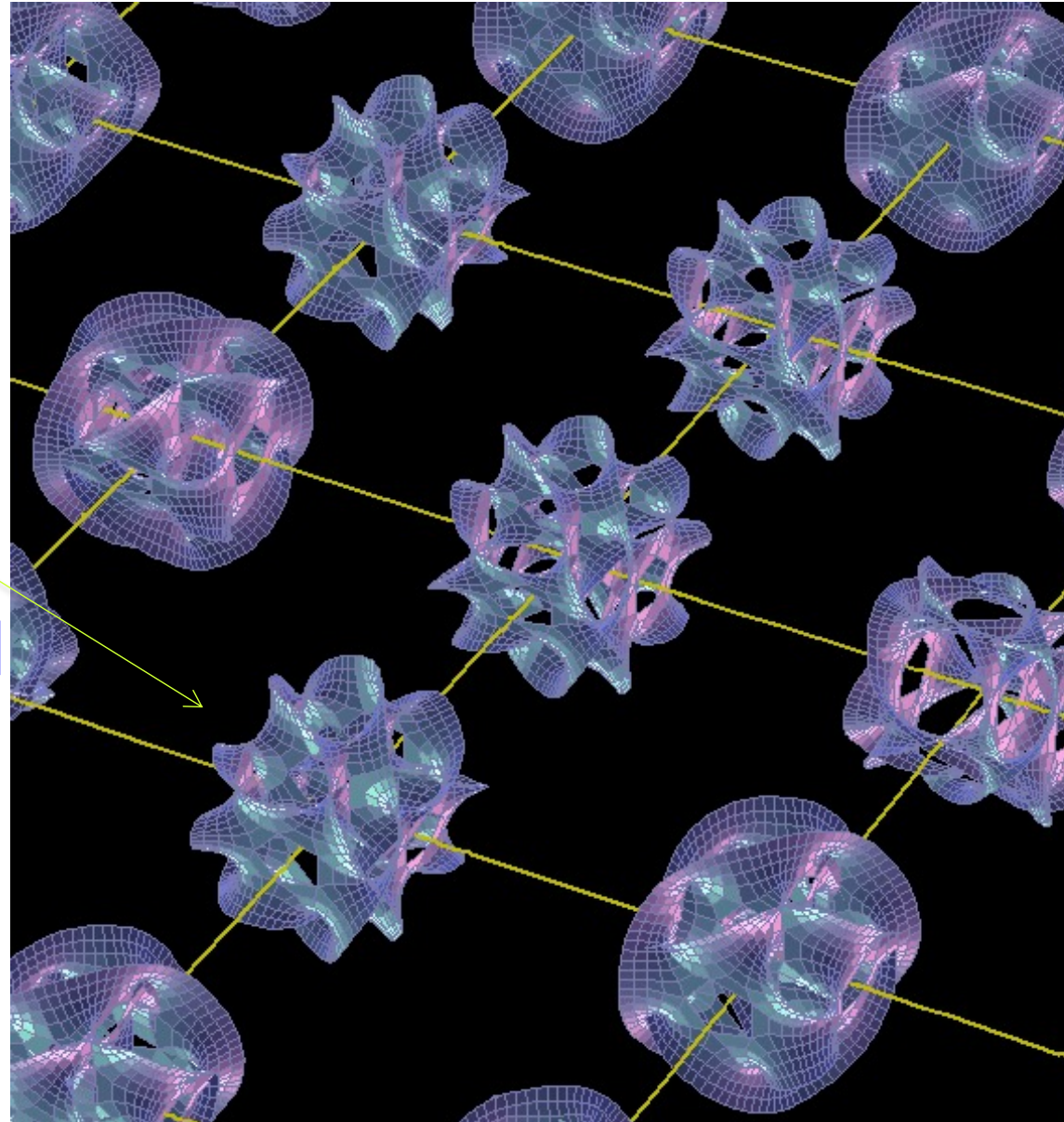
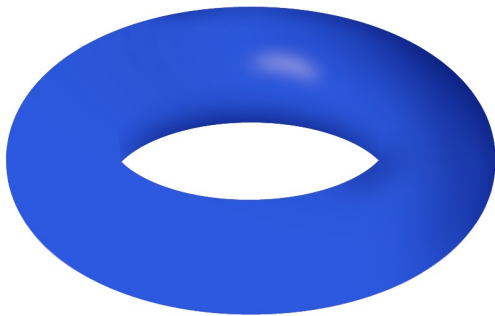
What to do with extra 6 dimensions?

Six extra dimensions are compactified

[curled up into a tiny compact space, thus rendered unobservable]

Special curvature -
Calabi-Yau space

[torii - "doughnuts" special cases]



The role of extra dimensions

for physics in 4 dimensions (3-space, 1-time)?

New perspective brought with introduction of extended objects → Branes

[generalization of membranes in dimensions >4]

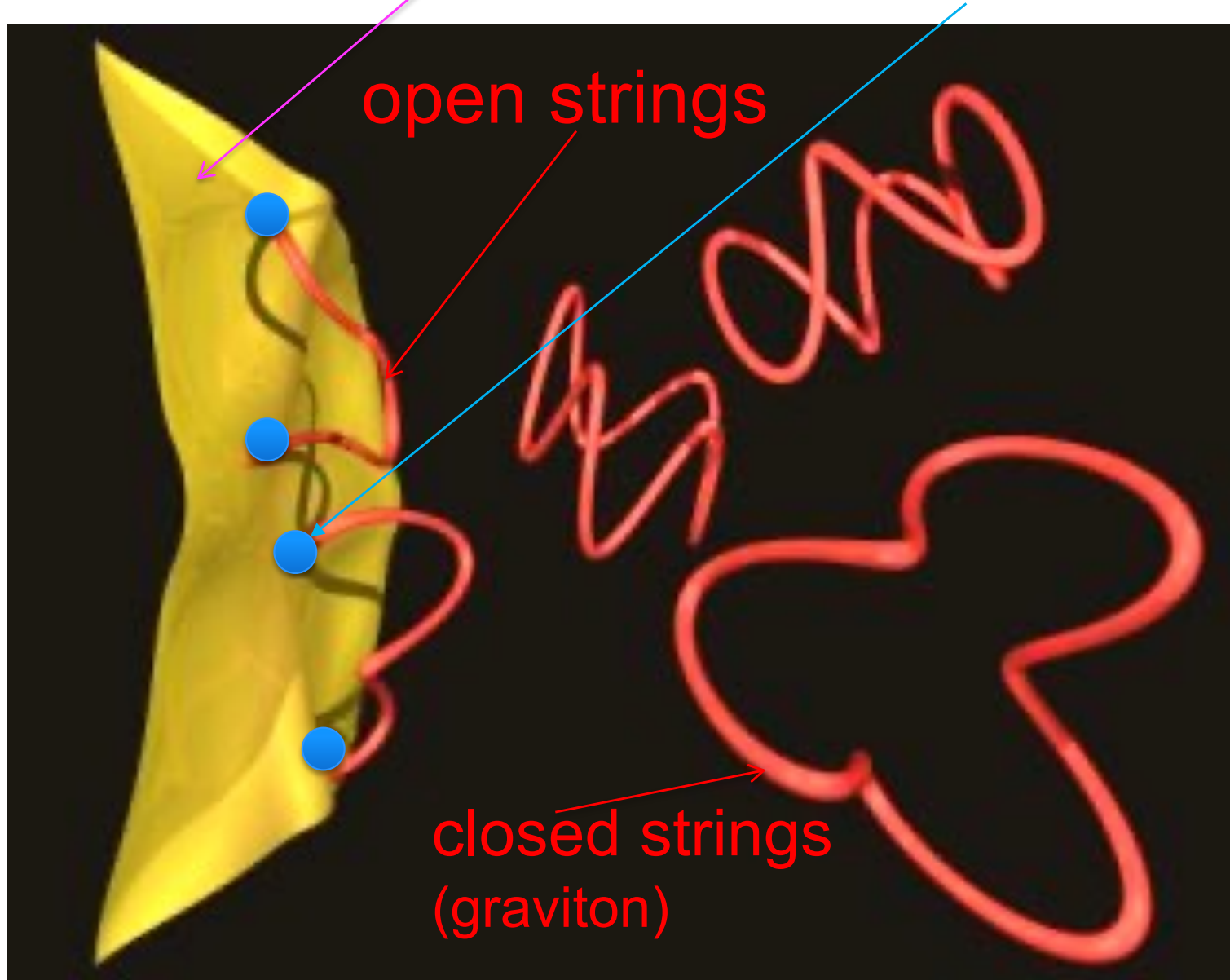
→ Modern String Theory

→ Implications both for particle physics & black hole physics

D(irichlet) - Branes

Polchinski '96

boundaries of open strings with charges at their ends



g_s - small

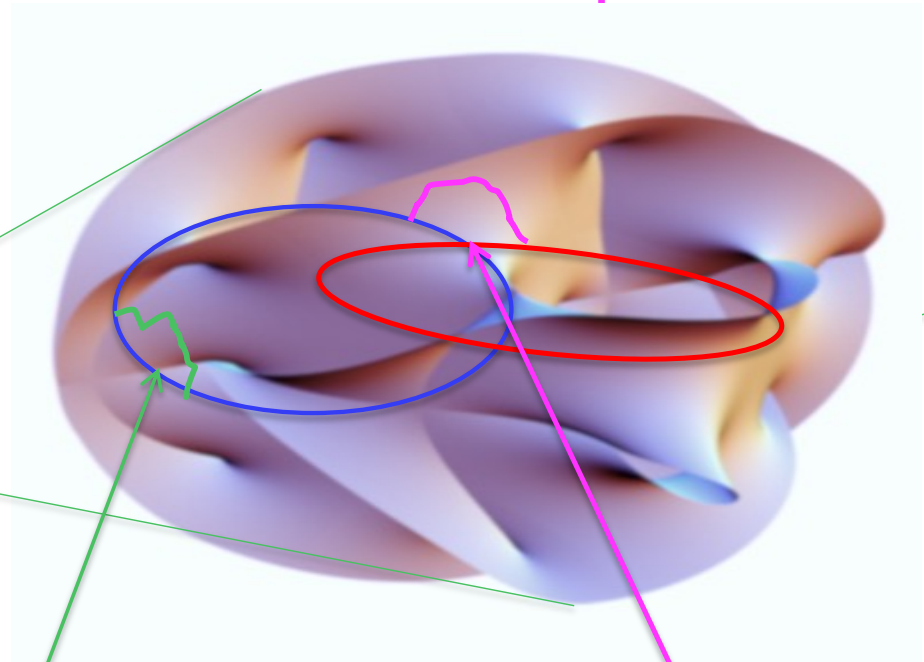
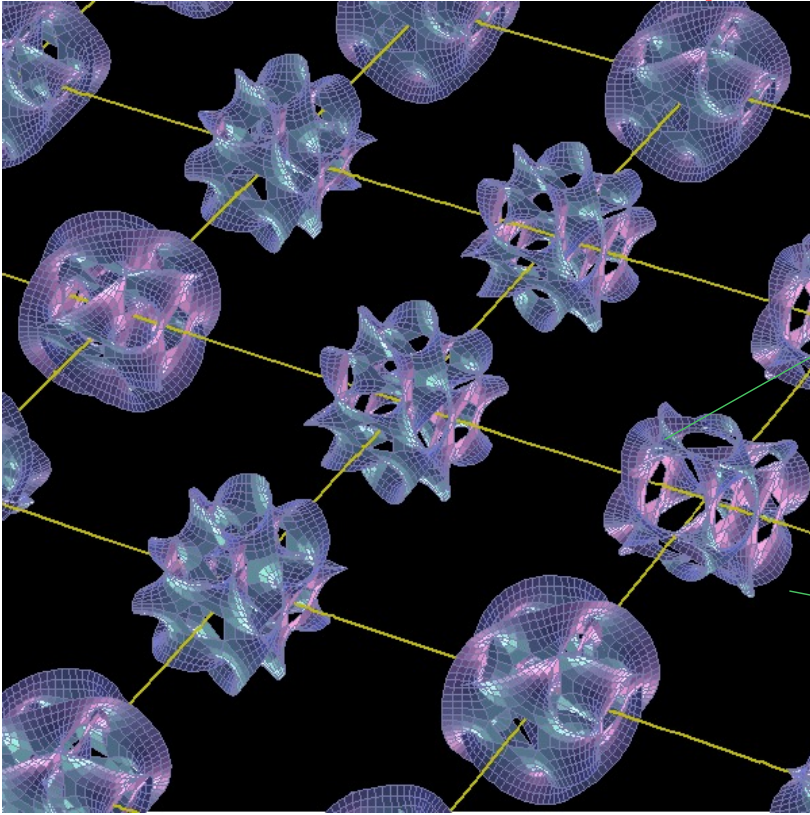
Open string excitations charged and "live" on branes

III. Particle Physics Implications

D-Branes fill out (3-space, 1-time) &

D-Branes fill out (3-space, 1-time) &

on Calabi-Yau space



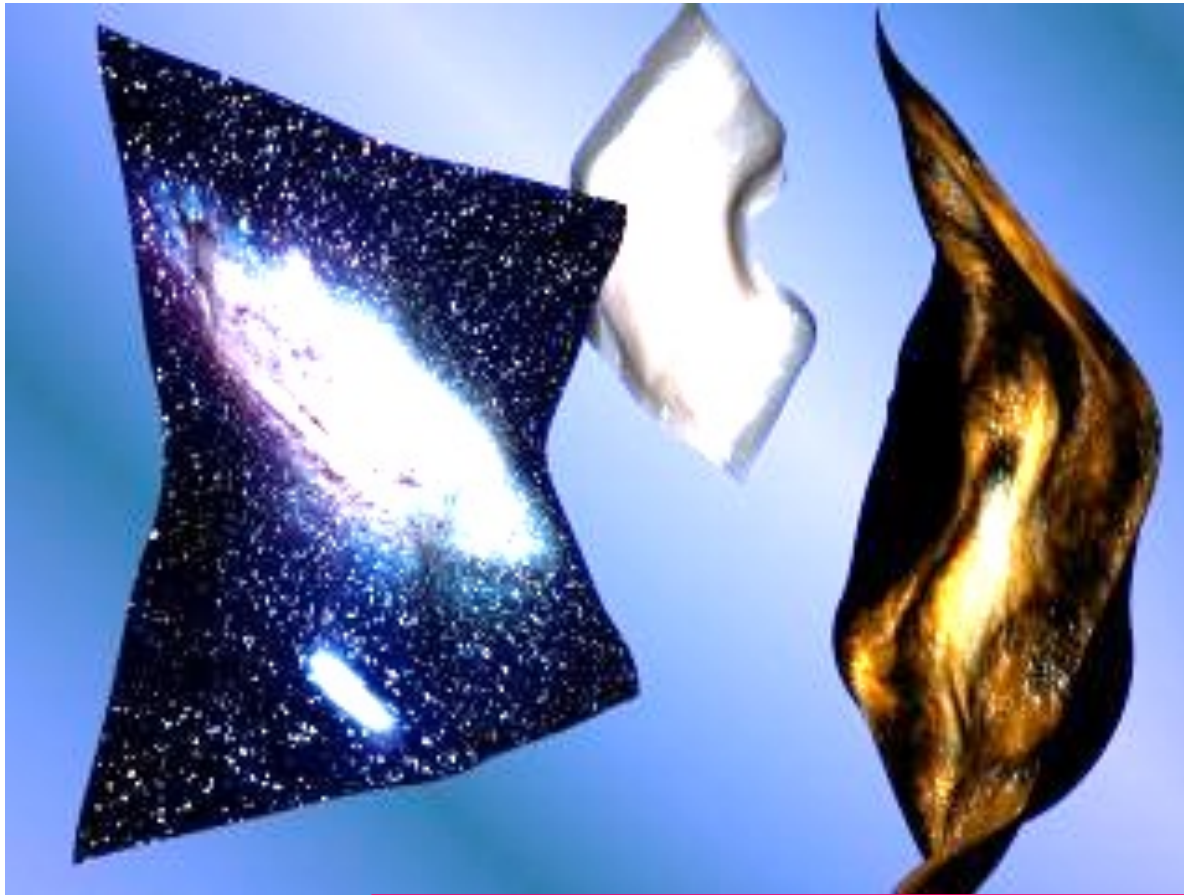
wrap different cycles which intersect
(photon, gluons, W) (matter: quarks, leptons)

→ Intersecting D-brane solutions of particle physics where gauge symmetry, matter & number of matter families **geometric!**

→ **First three-family Standard Model** M.C., Uranga, Shiu '01...

no time for details

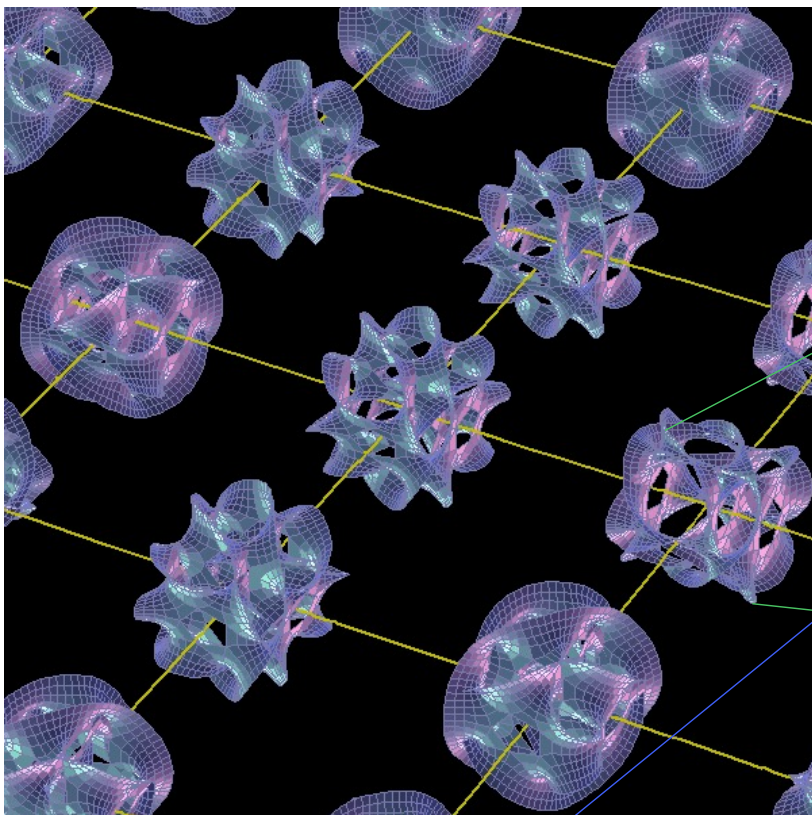
D-branes have dual interpretation:
extended massive sources, curve space-time
("back-reacted" objects at finite-large string coupling g_s)



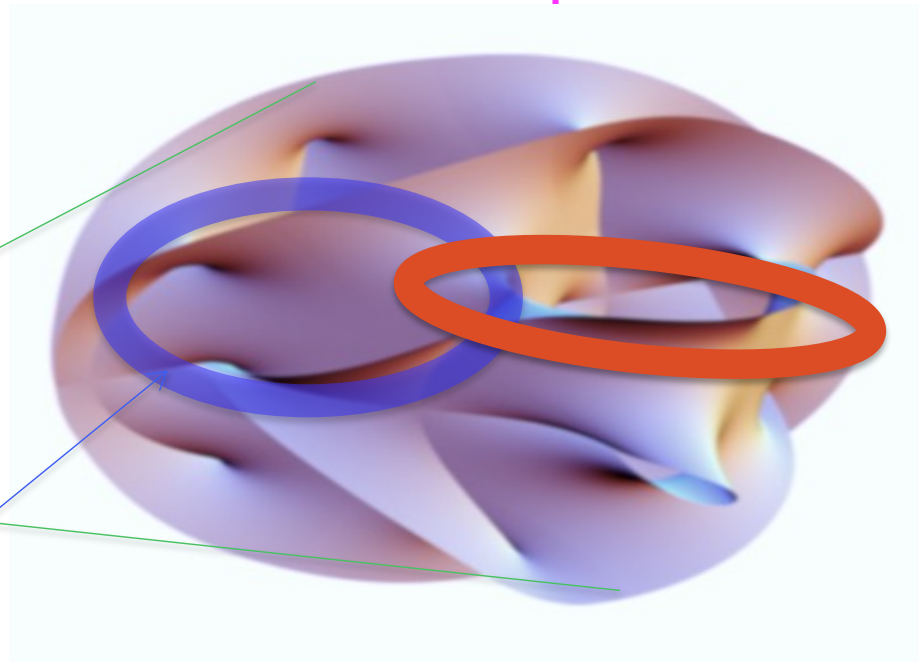
Implications for **particle physics at finite g_s**



D-branes as gravitational objects



on Calabi-Yau space



B

D-branes wrapping cycles “back-react” →
cause highly curved - singular space along cycles ($g_s \rightarrow \infty$)
B – new space with back-reacted D-branes



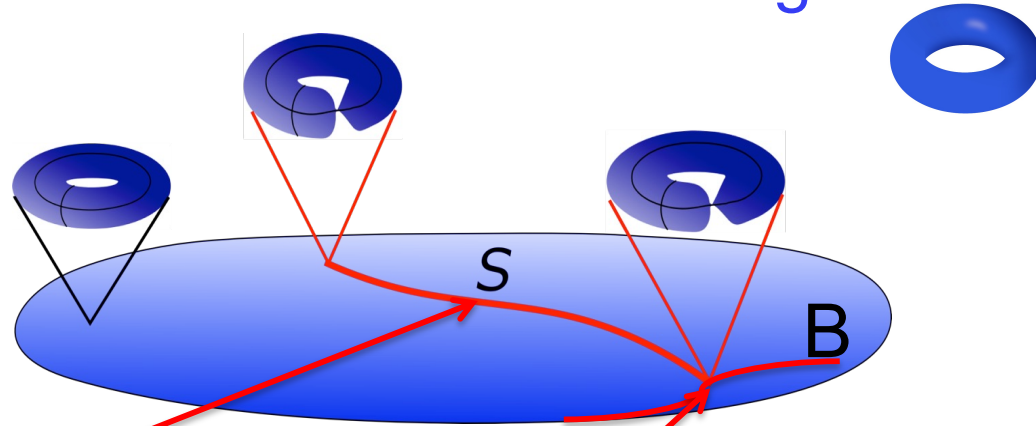
F-theory

Vafa'96...

Key features of F-theory

- F-theory, a powerful framework that geometrizes string coupling $\tau \equiv C_0 + ig_s^{-1}$ as a modular parameter of a torus - "doughnut"

- Compactification on torus-fibered over base **B**
→ singular Calabi-Yau space



- Gauge bosons - encoded by the type of torus singularity over back-reacted D-brane cycles **S**
- Matter - encoded by the type of torus singularity over the intersection of two back-reacted D-brane cycles

Development of geometric techniques

Donagi, Wijnholt'08; Beasley, Heckman, Vafa'08...

 led (for specific torus fibration) to F-theory compactification to 4-dimensions with

M.C., Klevers, Peña, Oehlmann, Reuter '15

Standard Model gauge group

$$\frac{SU(3) \times SU(2) \times U(1)}{\mathbb{Z}_6}$$

with gauge group topology

$$\mathbb{Z}_6$$

(also geometric!) M.C., Lin '17



toric geometry techniques

(toric bases B)

M.C., Halverson, Lin, Liu, Tian '19,

PRL& Scientific American

Quadrillion Standard Models

with 3-chiral families, and gauge coupling unification

Current efforts: determination the exact matter spectra

(including vector pairs & # of Higgs pairs)

Bies, M.C., Donagi, (Liu), (Ong) '21,'22,'23

No time

Recent related developments

- String Theory includes quantum gravity and particle physics with constraints due to geometry of compactified space



- Particle physics with consistent quantum gravity should be subject to additional constraints → Swampland Program

Vafa '06...



- Finding physical conditions (also for new higher symmetries), reflecting geometric constraints of consistent quantum gravity. In 8-dim one-form symmetry constrains gauge symmetry groups.

M.C., Dierigl, Lin, Zhang '20...



- Byproduct: found all string solutions in 8-dimensions

..M.C., Dierigl, Lin, Zhang '22,'23,'24...

→ A long way to 4-dim...

Other recent developments:

Higher Symmetries in Quantum Field Theory

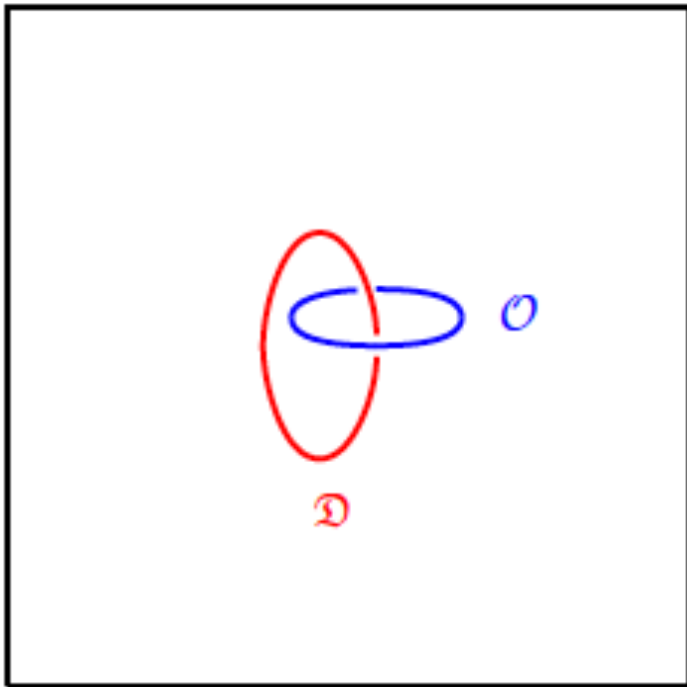
Gaiotto, Kapustin, Seiberg, Willet, 2014

Introduction of **an extended defect \mathcal{D}** and a **symmetry operators \mathcal{O}** acting on the defect

Examples: QFT in $D=3+1$

One-form symmetry generated by a **string defect \mathcal{D}** (1-dim) (Wilson line) and a **symmetry operator \mathcal{O}** (closed loop) acting on it.

[Zero-form symmetry generated by a **point defect \mathcal{D}** (1-dim) and a **symmetry operator \mathcal{O}** acting on it.]



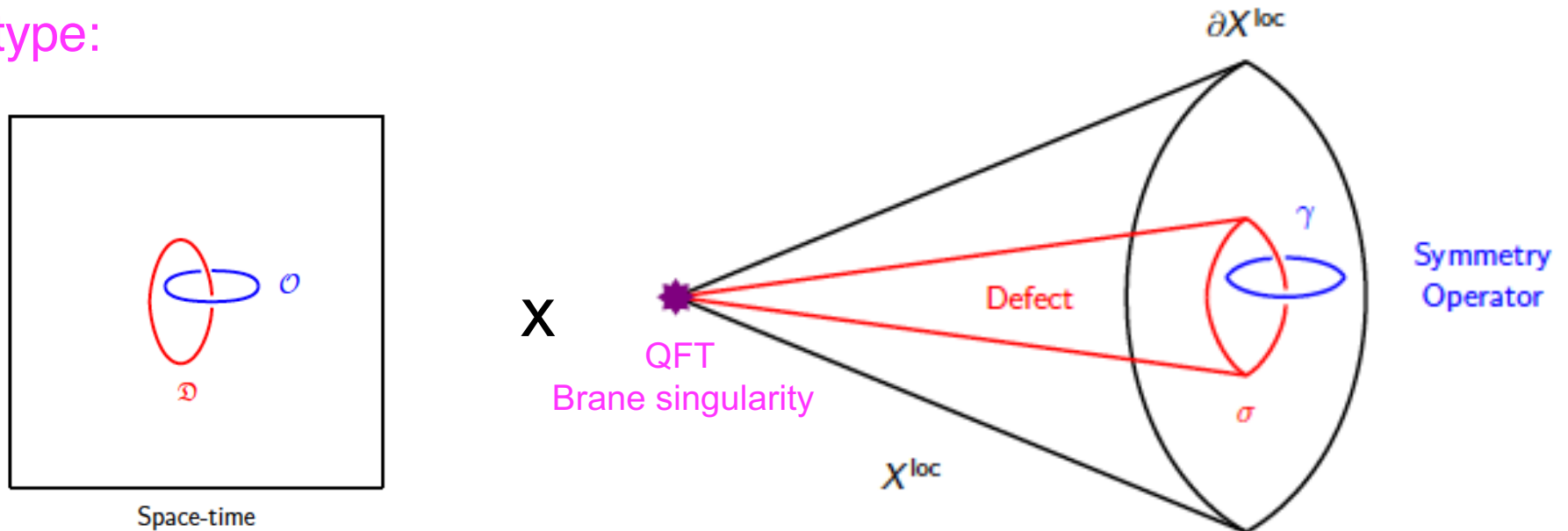
Space-time

Active field of research \rightarrow Implications also condensed matter systems \rightarrow
New phases of quantum matter

Higher Symmetries from String Theory are Natural!

In String Theory, 4-dim QFT construction with additional 6-dim on non-compact X^{loc} .
 For non-compact 6-dim space, gravity decouples in 4-dim with $G_N \rightarrow 0$ & only QFT!

Prototype:



p -dim defects \mathcal{D} associated w/ branes wrapping non-compact cycles σ on X^{loc}
 Symmetry operators \mathcal{O} w/ branes wrapping cycles γ on the boundary ∂X^{loc}

Morrison, Schafer-Nameki, Willett, '20,
 Albertini, Del Zotto, Etxebarria, Hosseini, '20

...

M.C., Heckman, Hübner, Torres, '22,
 Del Zotto, Garcia Etxebarria, Schäfer-Nameki, 2022,

...

M.C., Heckman, Hübner, Torres, Zhang, 2023,
 M.C., Donagi, Heckman, Hübner, Torres, 2024..

M.C., Heckman, Hübner, Torres, '22
M.C., Donagi, Heckman, Hübner, Torres, '24

Numerous examples of X^{loc} were chosen to identify geometric origin of higher-form symmetries (0-form, 1-form & 2-group) by studying the symmetry defects via algebraic topology.

[Via cutting & gluing of singular boundary ∂X^{loc} of the non-compact space X^{loc} (Mayer-Vietoris exact sequences).]

No time for details

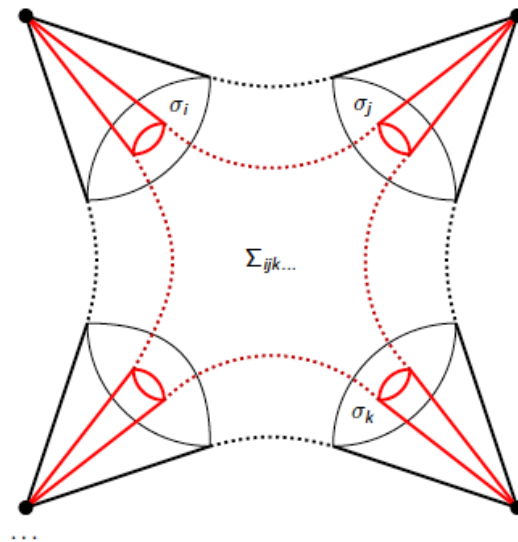
Last, but not least:

M.C., Heckman, Hübner, Torres '23

The fate of global symmetries in compact models

studied via gluing (and cutting) of noncompact examples;
Algebraic Topology (Mayer-Vietoris), again!

Schematically:



Defects σ_i :

Some cannot be glued consistently \rightarrow higher symmetry broken

Some are glued into compact cycles \rightarrow higher symmetry gauged

By-product: String theory on all 4-dim and some 6-dim toroidal orbifolds

Determine full resulting gauge group symmetry

Example: T^4 orbifolds:

$$T^4/\mathbb{Z}_2 : \quad G = \frac{(SU(2)^{16}/\mathbb{Z}_2^5) \times U(1)^6}{\mathbb{Z}_2^6}$$

$$T^4/\mathbb{Z}_3 : \quad G = \frac{(SU(3)^9/\mathbb{Z}_3^3) \times U(1)^4}{\mathbb{Z}_3^3}$$

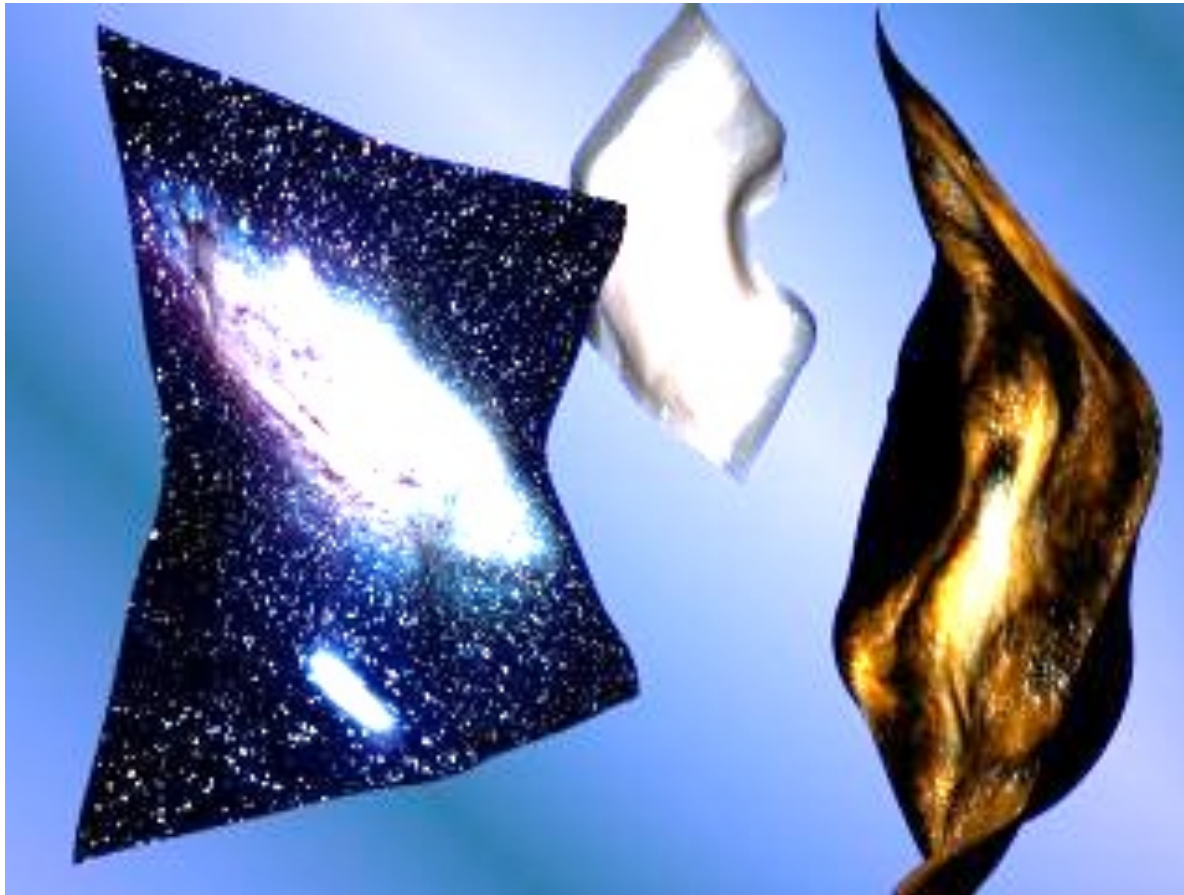
$$T^4/\mathbb{Z}_4 : \quad G = \frac{(SU(4)^4/\mathbb{Z}_4 \times \mathbb{Z}_2^2) \times SU(2)^6 \times U(1)^4}{\mathbb{Z}_4^2 \times \mathbb{Z}_2^2}$$

$$T^4/\mathbb{Z}_6 : \quad G = \frac{([SU(6) \times SU(3)^4 \times SU(2)^5]/\mathbb{Z}_3 \times \mathbb{Z}_2) \times U(1)^4}{\mathbb{Z}_6^3 \times \mathbb{Z}_2}$$

Dual D-brane interpretation:

extended massive sources, curve space-time

(“back-reacted” objects at finite-large string coupling g_s)



Implications for **particle physics** at finite g_s & **black holes**



IV. Implications for Black Holes

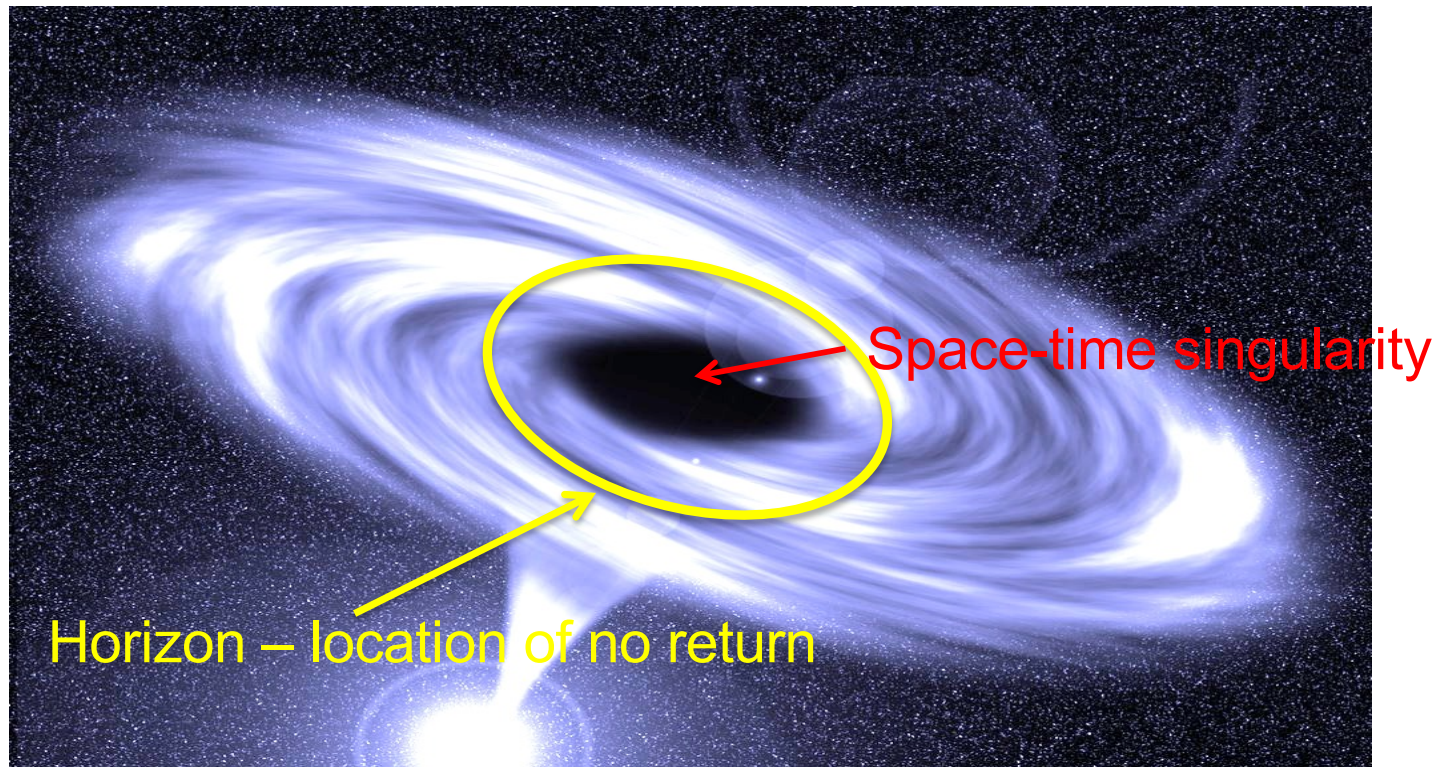
No time

Black hole entropy: $S_{\text{thermo}} = \frac{1}{4} A$

A = area of the black hole horizon

How to relate it to its **statistical entropy** $S_{\text{stat}} = \log N_i$?

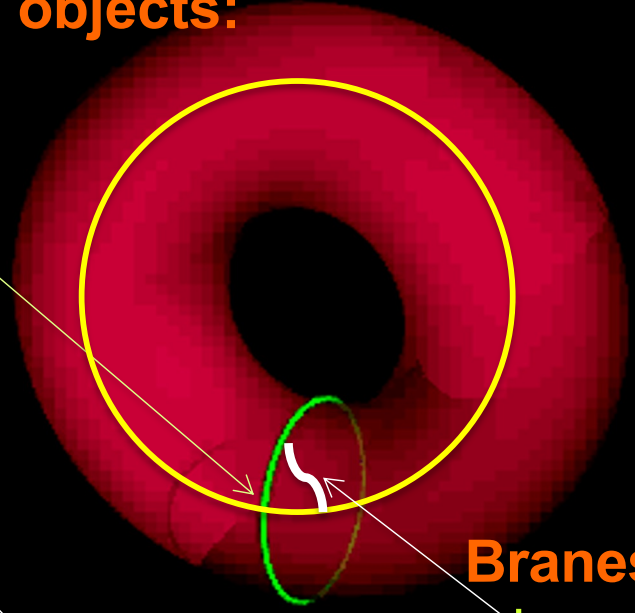
Where do black hole microscopic degrees N_i come from?



String theory insights into black hole microscopics

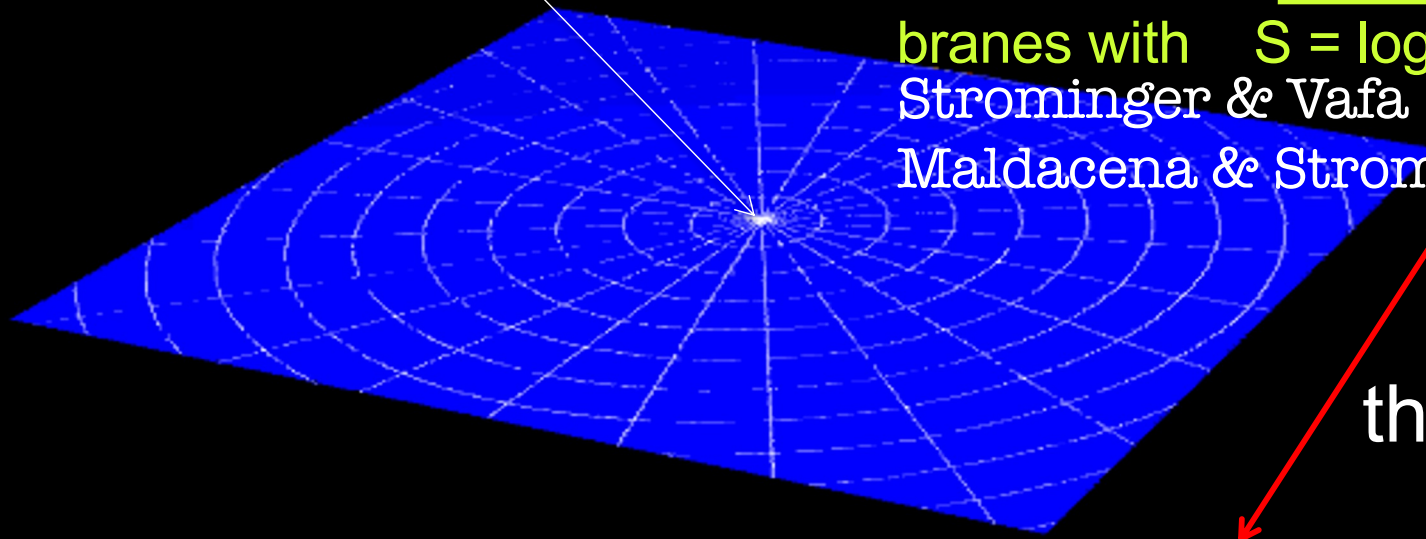
Branes as gravitational objects:

wrap intersecting cycles
in internal dimensions &
charged black holes
in 4-dimensions
(with each brane cycle
sourcing charge Q_i)



Branes boundaries of strings:
microscopic degrees N_i are string
excitations on intersecting

branes with $S = \log N_i$
Strominger & Vafa 9601029
Maldacena & Strominger 9603060



the same!

Prototype: four-charge black hole w/ $S = 2\pi\sqrt{Q_1Q_2Q_3Q_4}$

M.C. & Youm 9507090

- Approach explains **microscopic origin of entropy** of special multi-charged black holes with (schematically)

$$M = Q_1 + Q_2 + Q_3 + Q_4 - \text{extremal black holes}$$

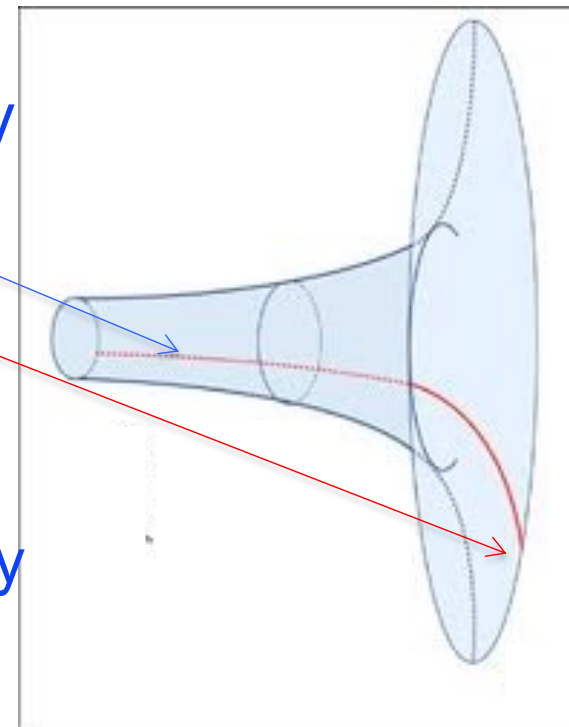
M-mass, Q_i – different charges (sourced by branes)

- Systematic study of microscopic degrees formalized via **Gravity/Field Theory [AdS/CFT] correspondence**

Maldacena '97

A string theory on a specific **Curved Space-Time** related to a specific **Field Theory** on its boundary

Microscopic studies for **extremal black holes**: horizon geometry as **anti-deSitter space [AdS]** & dual to **conformal field theory (CFT)** on its boundary



Further Developments

- Microscopic counting expanded to numerous examples of extremal black holes in diverse dimensions (typically: $\text{AdS}_3/\text{CFT}_2$, $\text{AdS}_2/\text{CFT}_1$)
- AdS/CFT correspondence grown into broad field with numerous applications to strongly interacting systems, including condensed matter
- Important current advances in dynamic studies of extremal black holes: entanglement entropy, information theory, late-time black hole evolution...

No time

How about non-extremal black holes in string theory?

with $M > Q_1 + Q_2 + Q_3 + Q_4$ & non-zero J

M - mass; Q_i - charges; J - angular momentum

- **Prototype: black holes** of a sector of string theory compactified on torii - ``doughnuts''

M.C., Youm '96
Chong, M.C., Lü, Pope '04

[The extremal black hole earlier is an extremal black hole of this sector.]

Approaches to microscopics of non-extremal black holes

- Compelling thermodynamics, suggestive of dual 2-dimensional conformal field theory (CFT_2)!

M.C., Youm '96; Larsen '97 M.C., Larsen '11; M.C., Gibbons, Lü, Pope '18...

- Introduction of “subtracted geometry” → “black hole in a box” with CFT_2 manifest!

M.C., Larsen '11; M.C., Gibbons '12...



- Relation to 2-dimensional gravity theory and AdS_2/CFT_1 [Connection to dual Sachdev-Ye-Kitaev (SYK) model]

M.C., Papadimitriou '16...

- “Subtracted geometry” of Kerr black hole captures the internal structure of astrophysical black holes → Love numbers & quasi-normal modes.

M.C., Rodriguez, Varela '24;
M.C., Perry, work in progress

No time

V. Concluding remarks

- Highlighted modern developments in string theory & insights they shed into unifying origin of particle physics and quantum gravity
- Presented highlights for
 - Geometric origin of particle physics and higher symmetries in quantum field theory
 - Microscopic interpretation of black hole entropy (including some insights into non-extremal ones)

No time

Final Message

- We have gained important insights into our quest of unifying forces of nature, including quantum gravity
- Uncovering deep links between Geometry and Physics
- Many open questions
→ “a work in progress”

Thank you

&

*Congratulations to Marko on his
70th birthday
and to many more productive,
scientific contributions!*