Complex dynamics of superconducting neurons

Johanne Hizanidis
03.09.2024

Θερινό Σχολείο - Συνέδριο

“Δυναμικά Συστήματα & Πολυπλοκότητα”

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UNIVERSITY OF CRETE

FOUNDATION FOR RESEARCH AND TECHNOLOGY - HELLAS

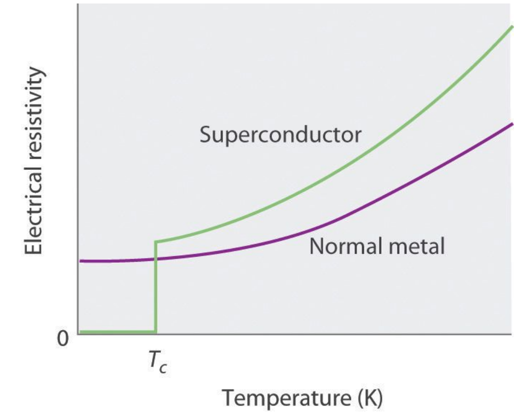
Outline

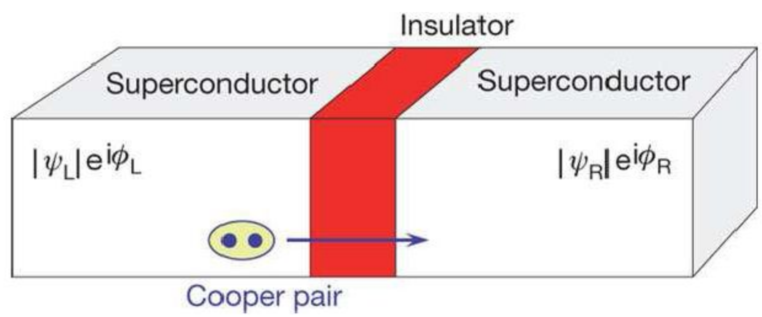
- The Josephson junction
- Single JJ neuron models
- Coupled JJ neuron models
- JJ neuron dynamics
- Neurocomputational properties
- Motivation & Future work

Josephson junction

Consists of 2 superconductors coupled by a weak link.

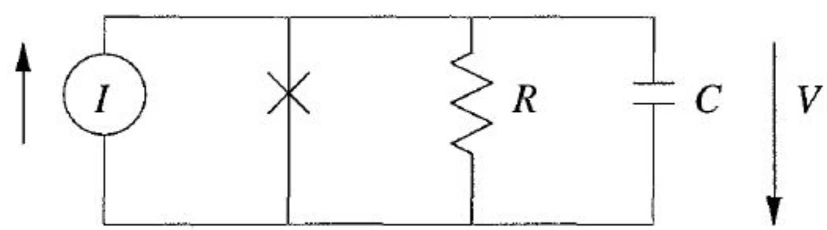
- Phase difference: $\phi = \phi_L - \phi_R$
- Josephson current-phase relation: $I = I_0 \sin\phi$
- Josephson voltage-phase relation: $V = \frac{\hbar}{2e}\frac{d\phi}{dt}$





Resistively Capacitively Shunted Junction model (RCSJ)

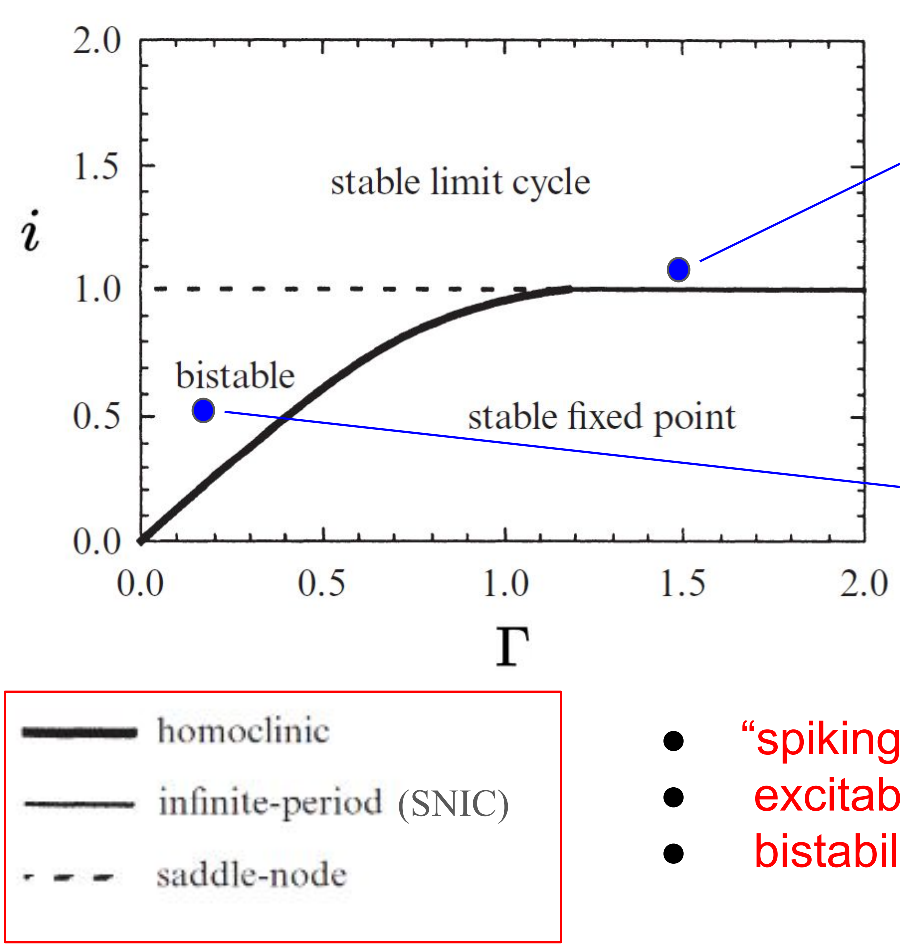
Contributions from displacement and ordinary currents are modelled by the capacitor C and the resistance R.

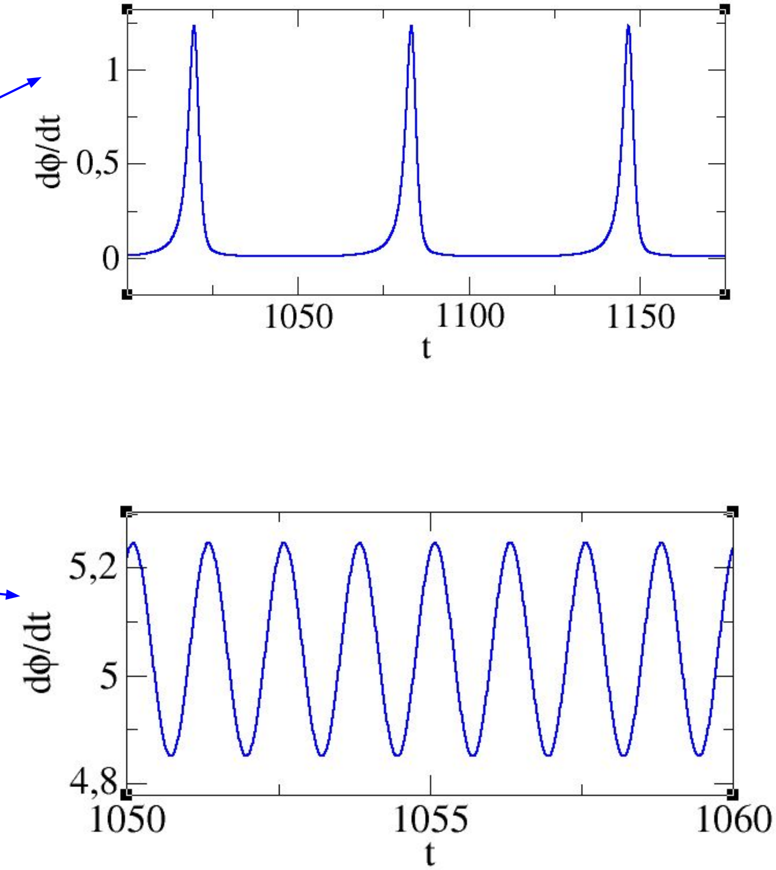


$$\ddot{\phi} + \Gamma\dot{\phi} + \sin\phi = i$$

$$\Phi_0 = h/e, \quad \tau^2 = t^2 \Phi_0 C / 2\pi I_0, \quad \Gamma^2 = \Phi_0 / 2\pi I_0 R^2 C$$

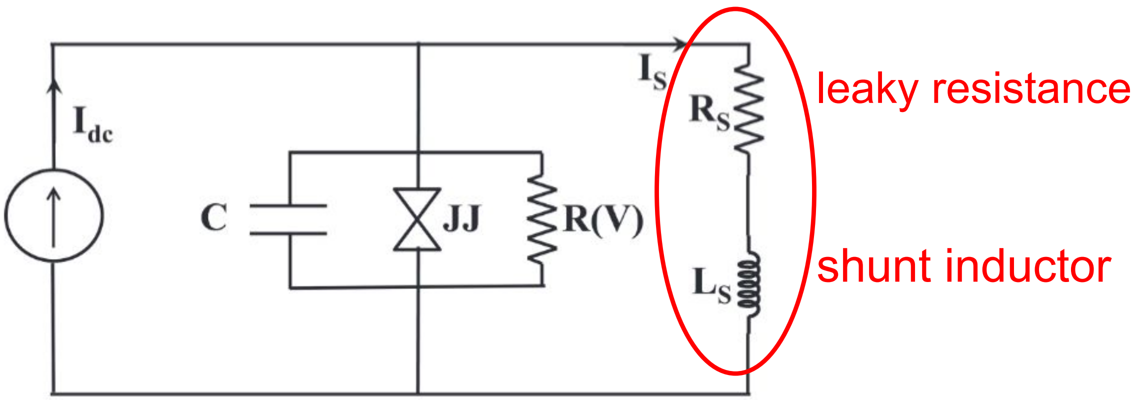
RCSJ model





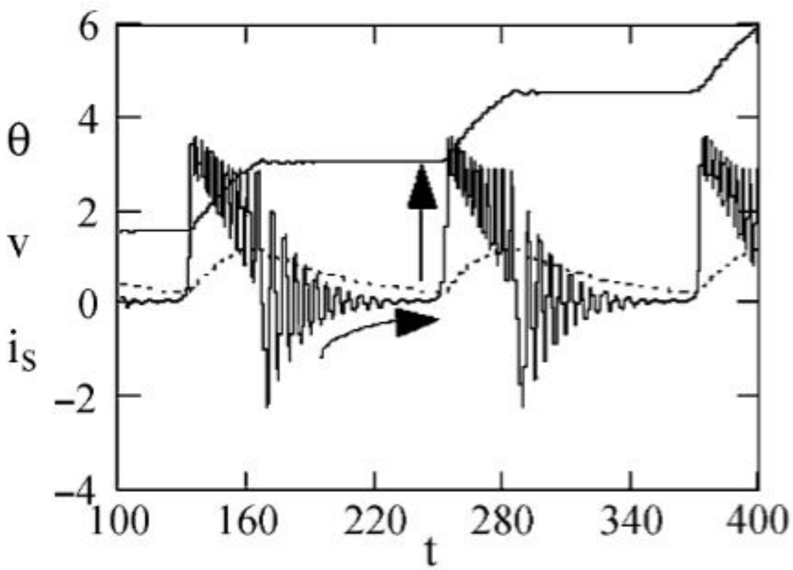
- “spiking”
- excitability type I & II
- bistability

RCLSJ model



$$I_{dc} = C\frac{dV}{dt} + \frac{V}{R(V)} + I_c \sin\theta + I_S,$$

$$V = \frac{\hbar}{2e}\frac{d\theta}{dt} = L_S\frac{dI_S}{dt} + I_S R_S.$$

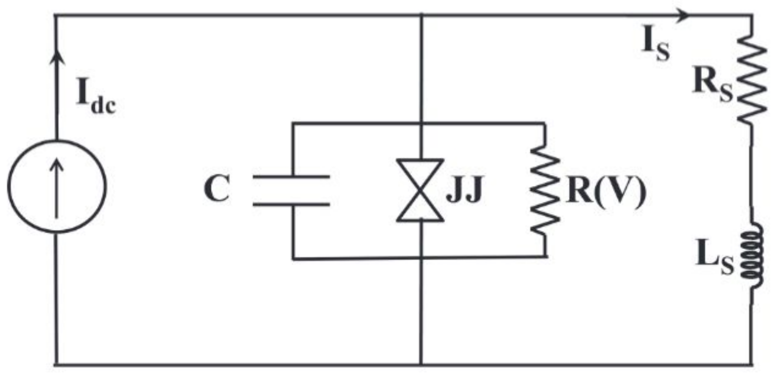


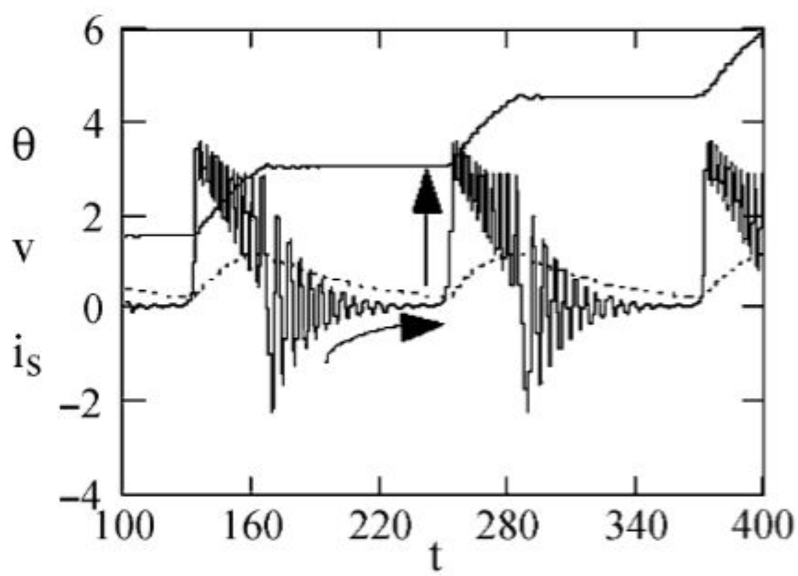
For high inductance and low damping: slow–fast dynamics and thus autonomous bursting.

Biological neuron: Fast spiking of Na+ and K+ ions are controlled by a slow process like Ca++ gated K+ ion movement.

S. K. Dana et al., IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS (2006).
E. Neumann and A. Pikovsky, Eur. Phys. J.(2003).

RCLSJ model



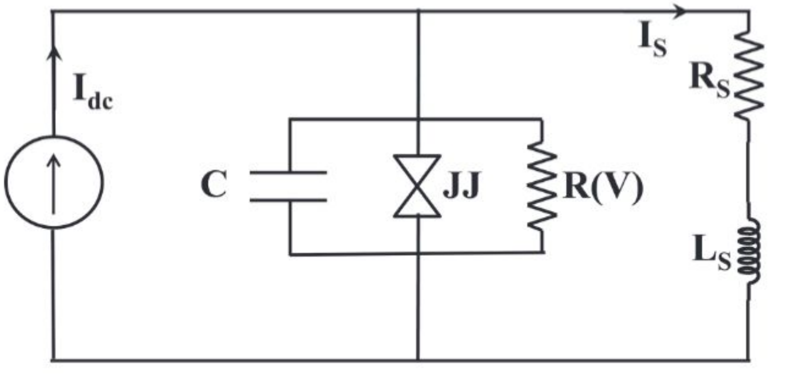


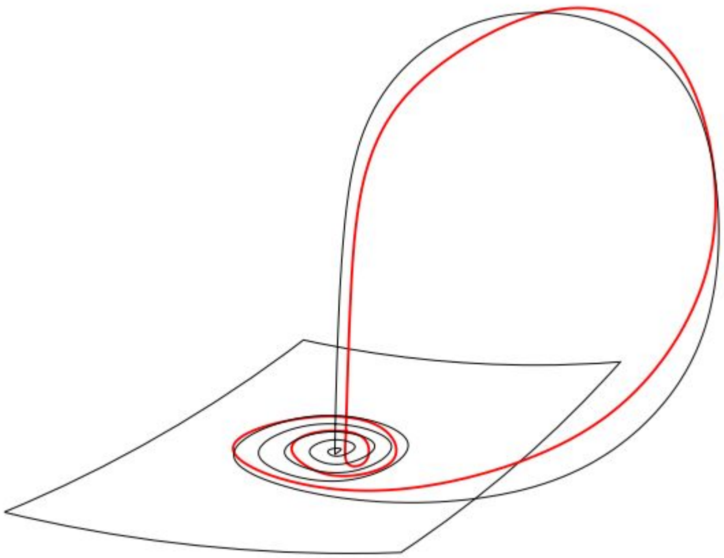
SNIC/homoclinic type bursting (Izhikevich)

- I_s starts growing ⟹ the junction voltage starts spiking via a SNIC bifurcation
- The spiking amplitude grows until I_s starts decaying very slowly
- I_s growing rate is much faster than its decay rate
- During the decaying process of I_s, the junction voltage also starts decaying in a spiral motion into the saddle focus via a homoclinic bifurcation.

S. K. Dana et al., IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS (2006).

RCLSJ model





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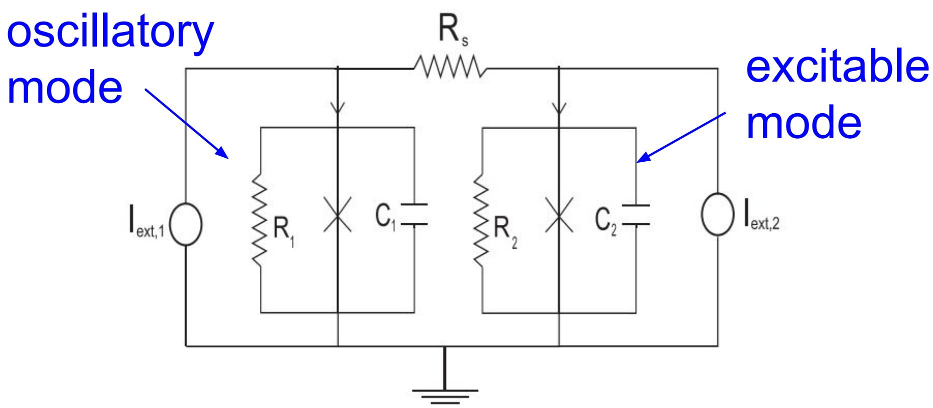
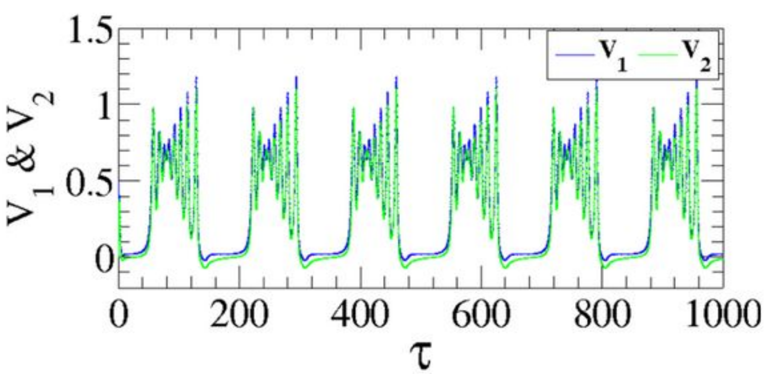
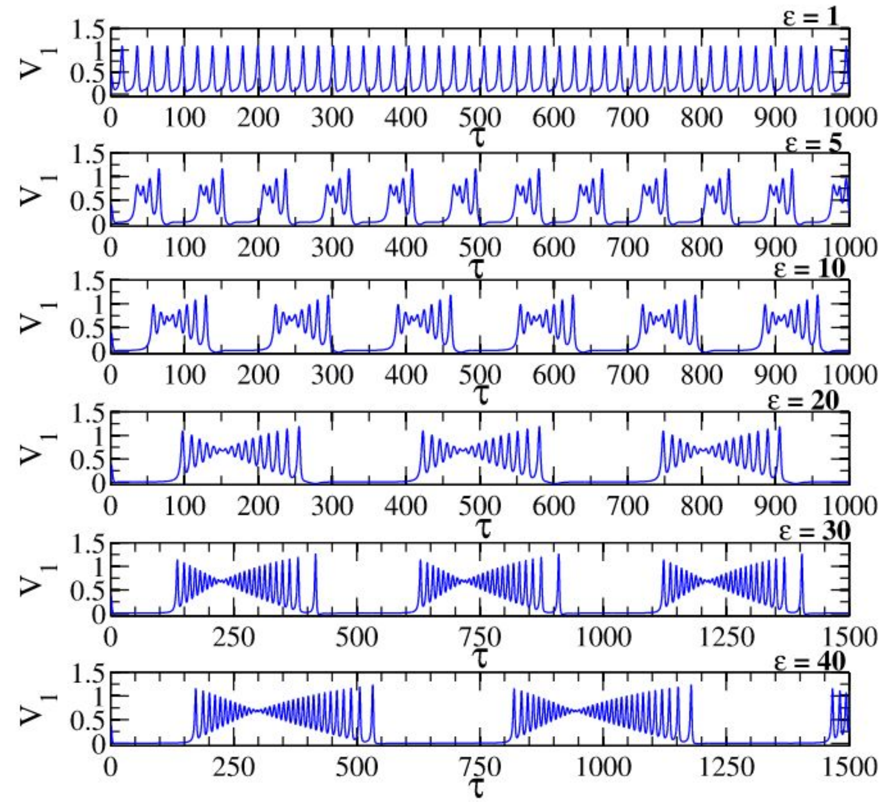
Resistively coupled RCSJ model



$$\ddot{\theta}_1 + \alpha\dot{\theta}_1 + \sin\theta_1 = I_1 + \epsilon(\dot{\theta}_2 - \dot{\theta}_1)$$

$$\ddot{\theta}_2 + \alpha\dot{\theta}_2 + \sin\theta_2 = I_2 + \epsilon(\dot{\theta}_1 - \dot{\theta}_2)$$

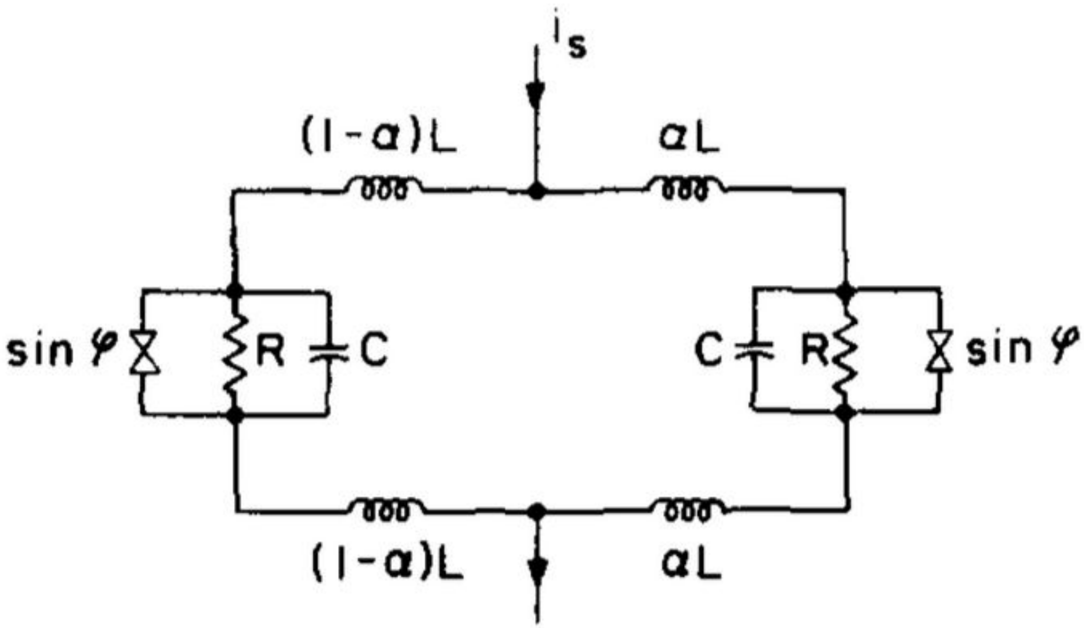


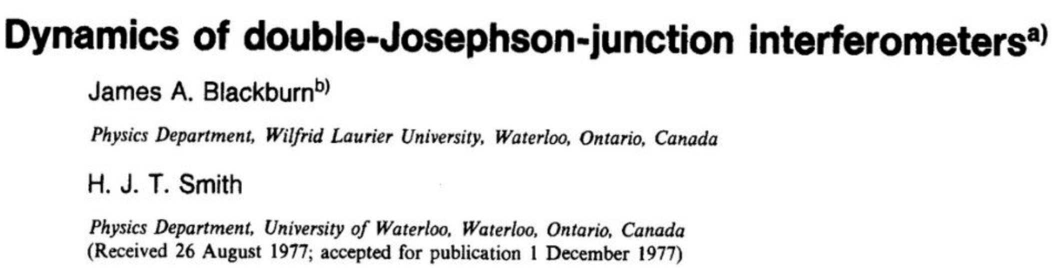


- Bursting due to competition between junctions
- Nearly complete synchronization for strong coupling

Hongray et al., CHAOS (2015)

Inductively coupled RCSJ model (1)

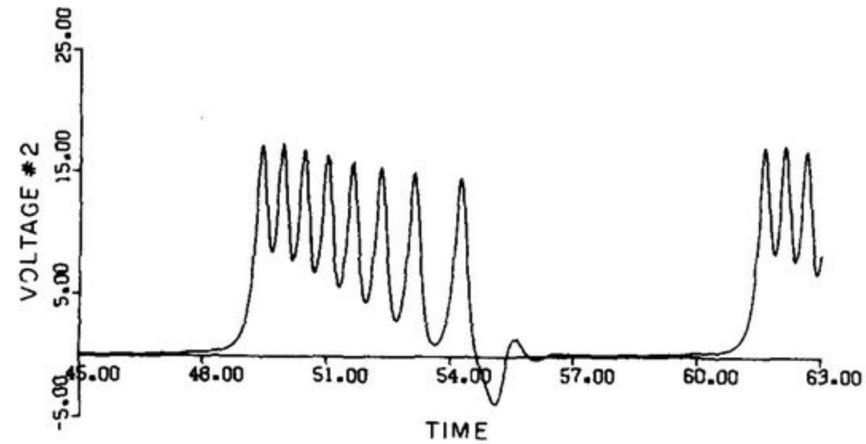


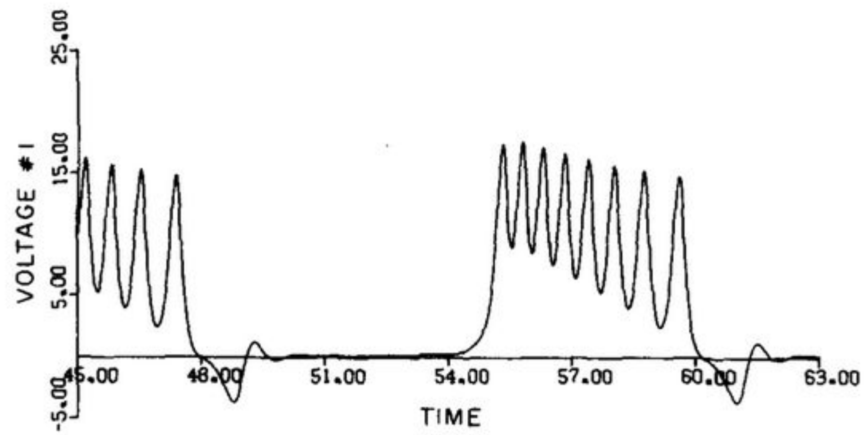


Dynamics of double-Josephson-junction interferometers[a)]

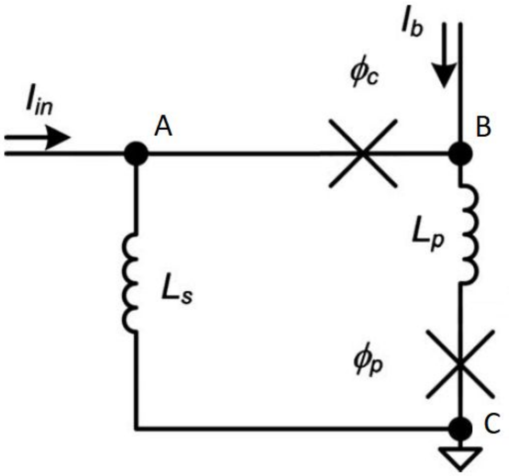
James A. Blackburn[b)]
Physics Department, Wilfrid Laurier University, Waterloo, Ontario, Canada
H. J. T. Smith
Physics Department, University of Waterloo, Waterloo, Ontario, Canada

(Received 26 August 1977; accepted for publication 1 December 1977)





Inductively coupled RCSJ model (2)



"PULSE" JJ

$$\ddot{\phi}_p + \Gamma\dot{\phi}_p + \sin\phi_p = -\lambda(\phi_c + \phi_p) + \Lambda_s i_{in} + (1 - \Lambda_p) i_b = i_p$$

"CONTROL" JJ

$$\ddot{\phi}_c + \Gamma\dot{\phi}_c + \sin\phi_c = -\lambda(\phi_c + \phi_p) + \Lambda_s i_{in} - \Lambda_p i_b = i_c$$

Biological Neuron	JJ neuron
Stimulus	i_{in}
V	$\lambda(\phi_p + \phi_c)$
Na^{+} current	$\dot{\phi}_p$
K^{+} current	$\dot{\phi}_c$

$$\Lambda_s = \frac{L_s}{L_s + L_p}, \quad \Lambda_p = \frac{L_p}{L_s + L_p}$$

$$\lambda = \frac{\hbar}{2e(L_s + L_p) I_0}$$

Crotty et al, "Josephson Junction simulation of neurons", 2010

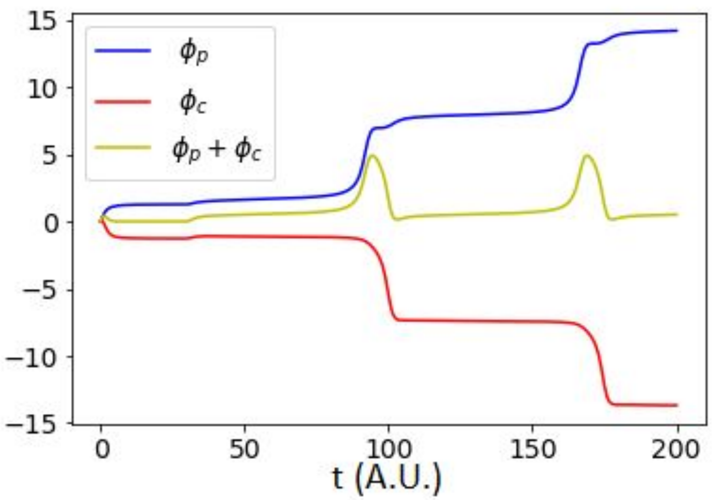
Inductively coupled RCSJ models

$$\ddot{\phi}_p + \Gamma\dot{\phi}_p + \sin\phi_p = -\lambda(\phi_c + \phi_p) + \Lambda_s i_{in} + (1-\Lambda_p)i_b = i_p$$

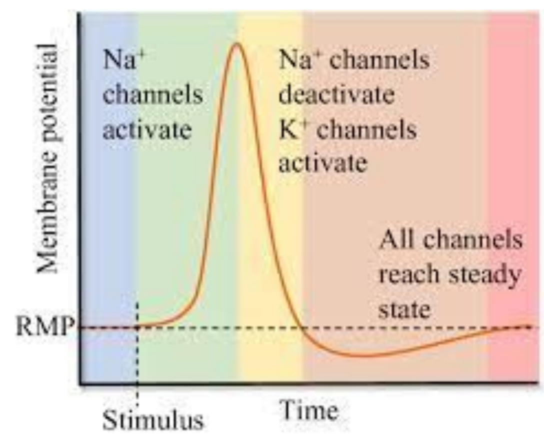
$$\ddot{\phi}_c + \Gamma\dot{\phi}_c + \sin\phi_c = -\lambda(\phi_c + \phi_p) + \Lambda_s i_{in} - \Lambda_p i_b = i_c$$

⇒ $i_p - i_c = i_b$

$\Lambda_p = 0.5$, $\Lambda_s = 0.5$, $i_b = 1.909$, $\Gamma = 1.5$, $\lambda = 0.1$, $i_{in} = 0.22$ ($t > 30$)



The JJ neuron mimics the action potential



Dynamics: Excitability

Fixed parameters: $(\Lambda_s, \Lambda_p, \lambda) = (0.5, 0.5, 0.1)$

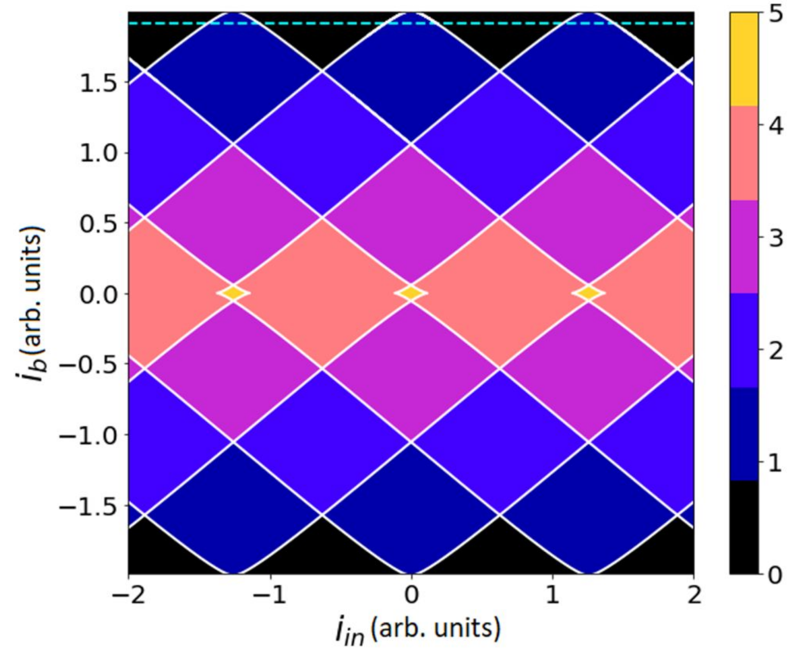
Rewrite equations:

$$\dot{\phi}_p = \omega_p$$
$$\dot{\omega}_p = -\Gamma\omega - \sin\phi_p - \lambda(\phi_c + \phi_p) + \Lambda_s i_{in} + (1 - \Lambda_p) i_b$$
$$\dot{\phi}_c = \omega_c$$
$$\dot{\omega}_c = -\Gamma\omega_c - \sin\phi_c - \lambda(\phi_c + \phi_p) + \Lambda_s i_{in} - \Lambda_p i_b,$$

The fixed points are $(\phi_p^\star, 0, \phi_c^\star, 0)$ where:

$$\sin\phi_p^\star - \sin\left(-\frac{\sin\phi_p^\star}{\lambda} - \phi_p^\star + \frac{\Lambda_s i_{in} + (1-\Lambda_p) i_b}{\lambda}\right) = i_b$$
$$\phi_c^\star = -\frac{\sin\phi_p^\star}{\lambda} - \phi_p^\star + \frac{\Lambda_s i_{in} + (1-\Lambda_p) i_b}{\lambda}$$

Number of stable fixed points



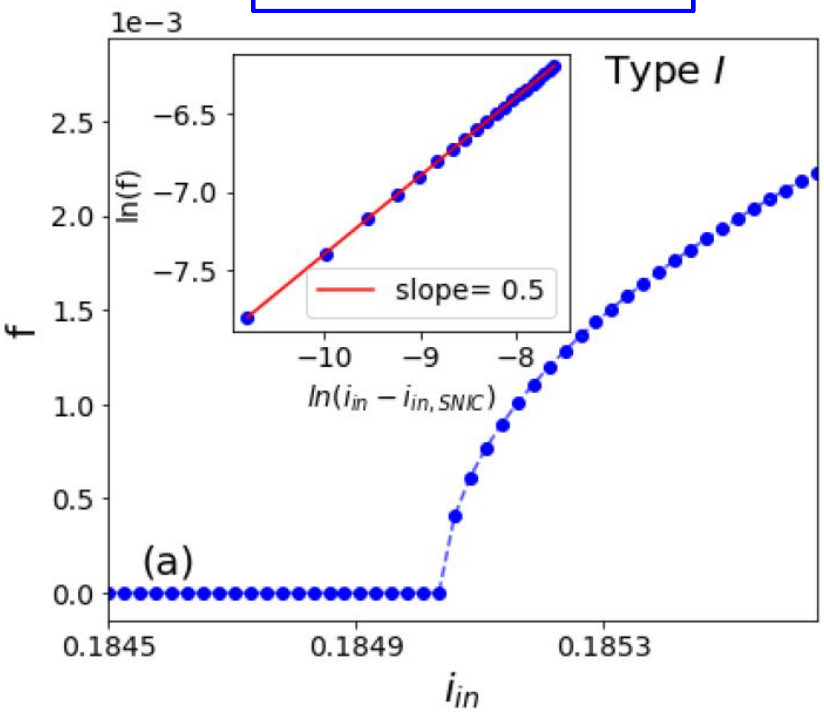
position and stability of fixed points independent of Γ

Dynamics: Excitability

The equilibria disappear through a Saddle-Node on an Invariant Circle (SNIC) bif.

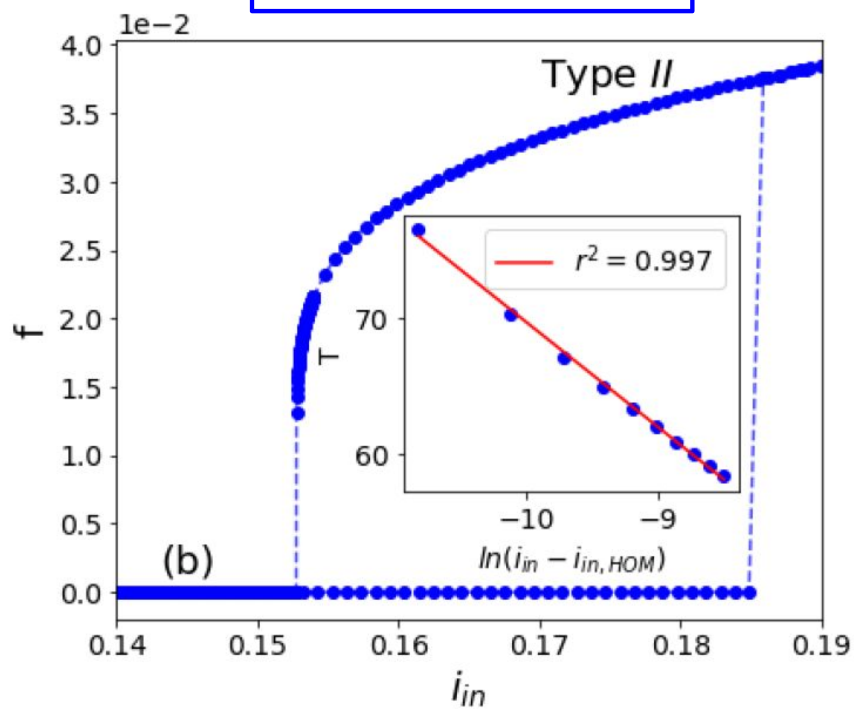
$$f \sim O(\sqrt{i_{in} - i_{in,SN}})$$

$\Gamma > 1$



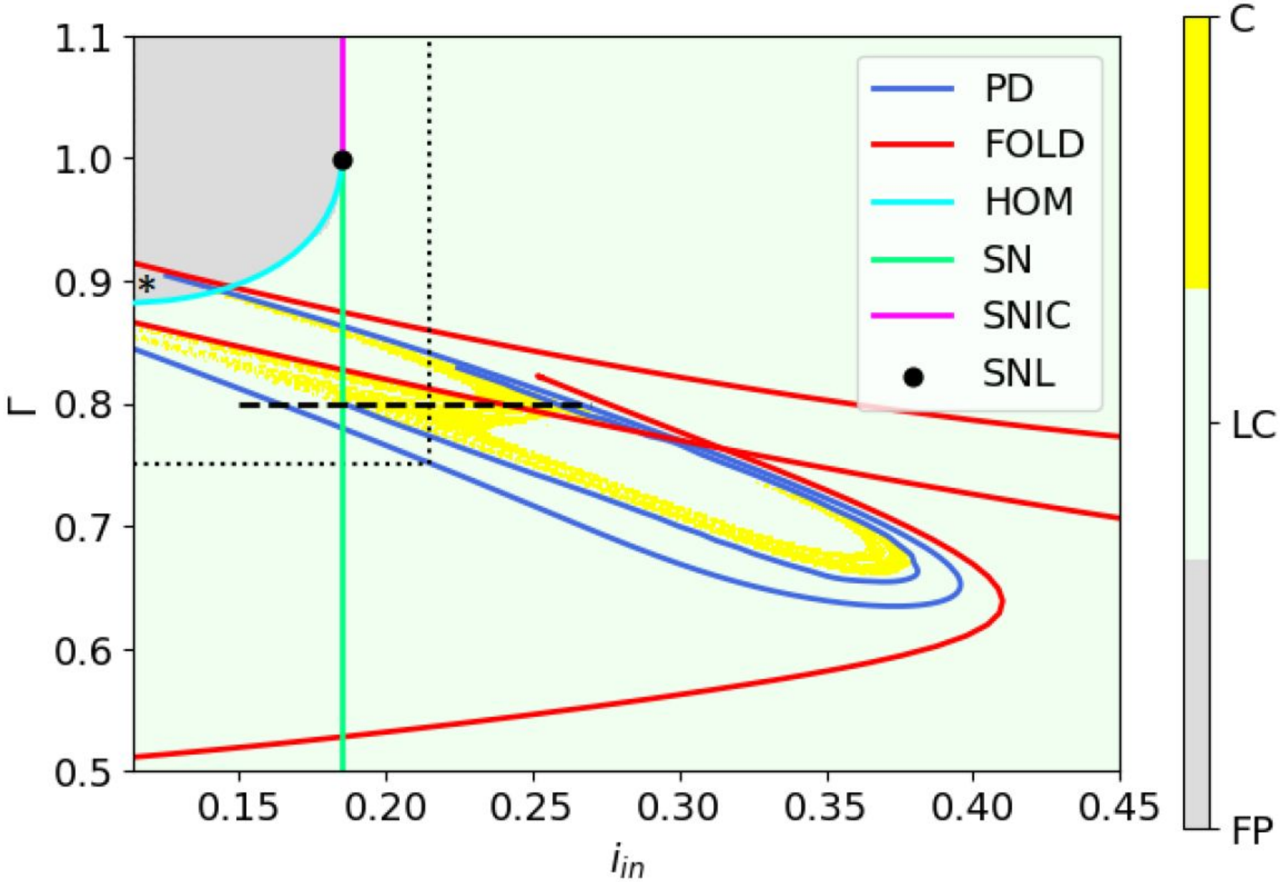
The equilibria disappear through a SN bif. and the trajectory jumps to an existing LC which is born through a homoclinic bif. at $i_{HOM} < i_{SN}$

$$T \sim O(\ln[i_{in} - i_{in,HOM}])$$



$\Gamma < 1$

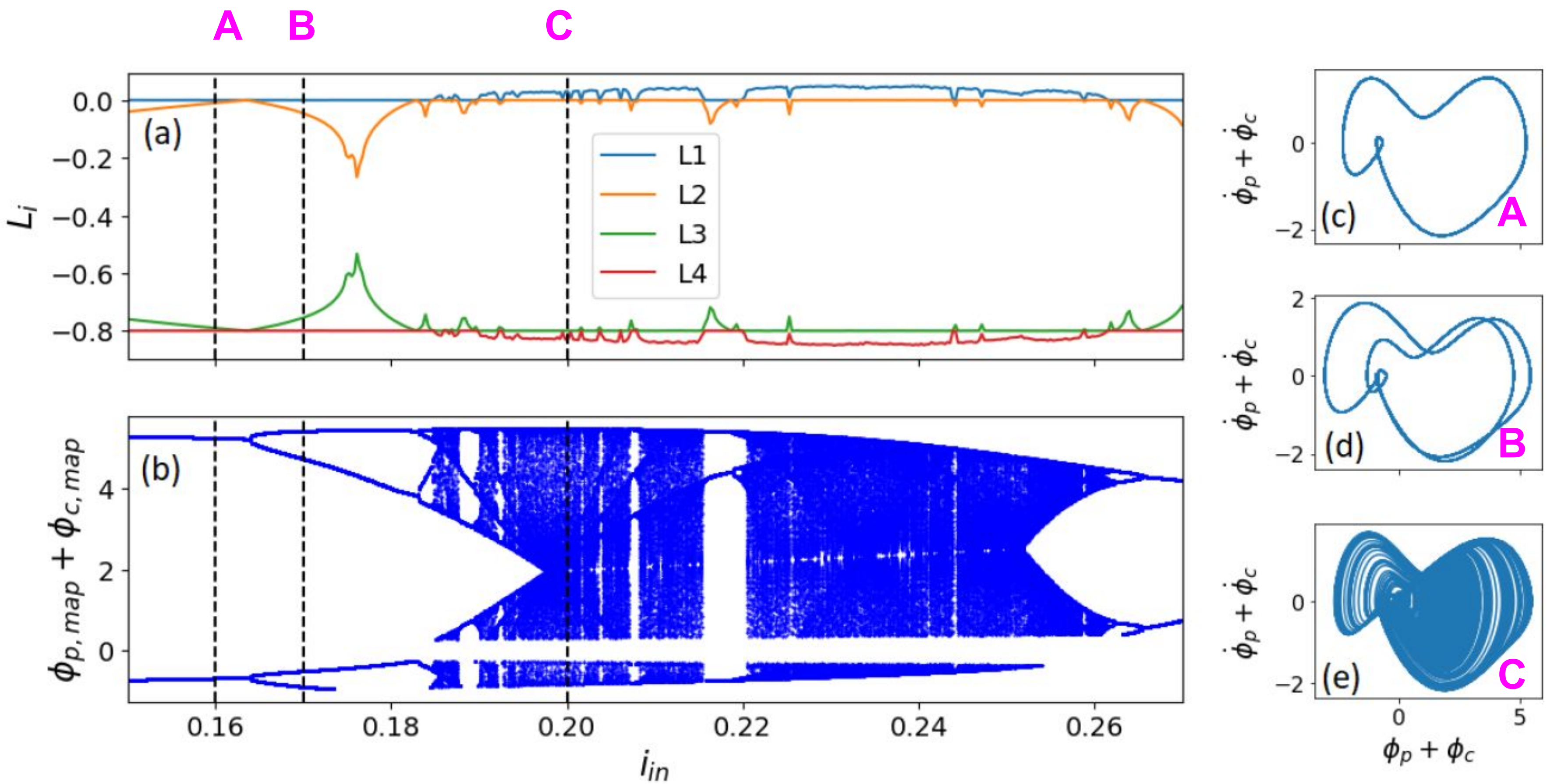
Dynamics: Bifurcations and Chaos



Attractor	L_1	L_2	L_3, L_4
Fixed point (FP)	−	−	−
Limit Cycle (LC)	0	−	−
Quasiperiodic (QP)	0	0	−
Chaotic (C)	+	0	−

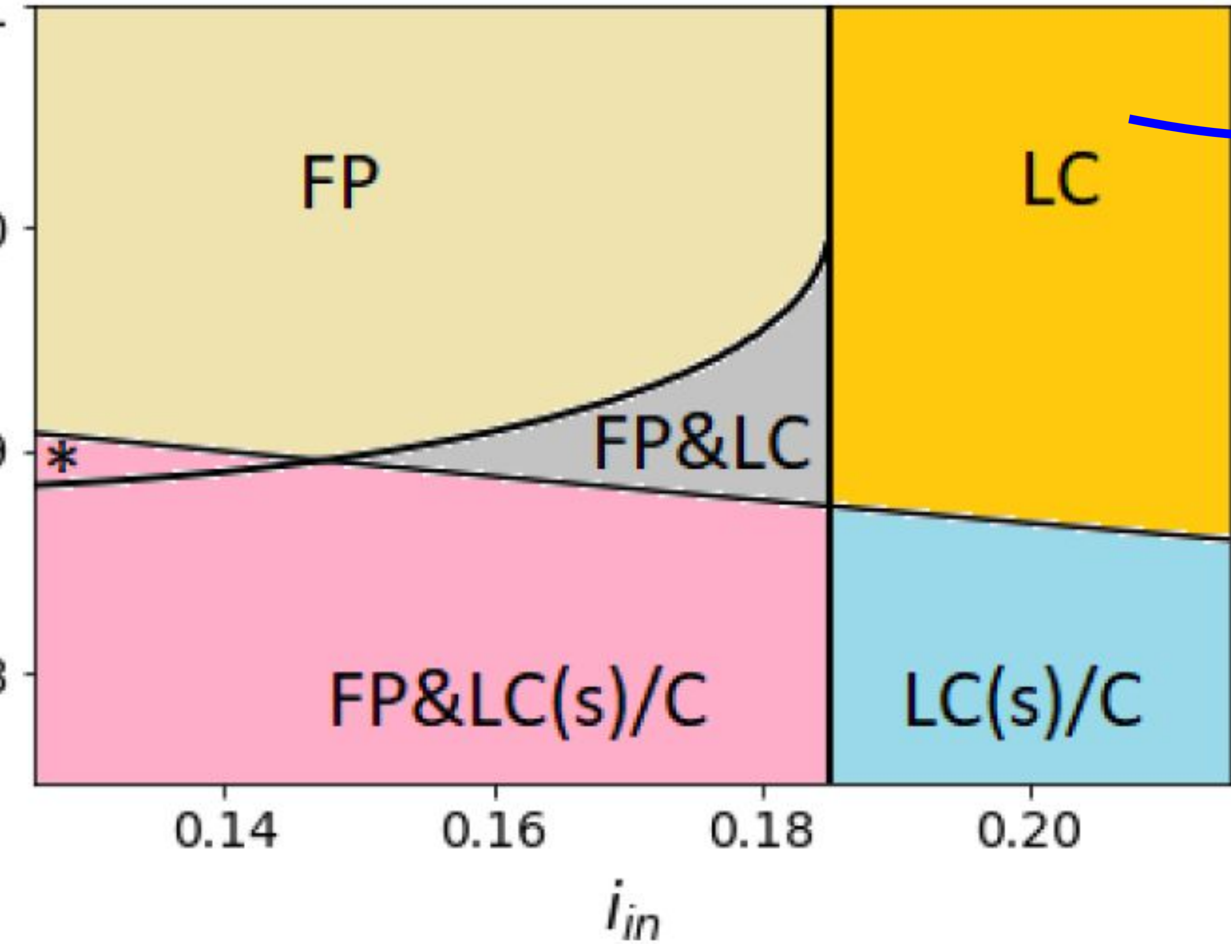
D. Chalkiadakis & J. Hizanidis, PRE (2022)

Dynamics: Route to chaos (Γ=0.8)



D. Chalkiadakis & J. Hizanidis, PRE (2022)

Dynamics: mapping of regimes



most works on JJ neurons focus on this regime where the system exhibits only spiking

D. Chalkiadakis & J. Hizanidis, PRE (2022)

Neurocomputation properties: **Bursting**

A burst is two or more spikes followed by a period of quiescence. Bursting occurs due to the interplay of fast currents responsible for spiking activity and slow currents that modulate the activity.

Izhikevich, Computational Neuroscience 2007

In the **bistable regime** small perturbations **ξ(t)** of the stimulus can switch spike trains on and off

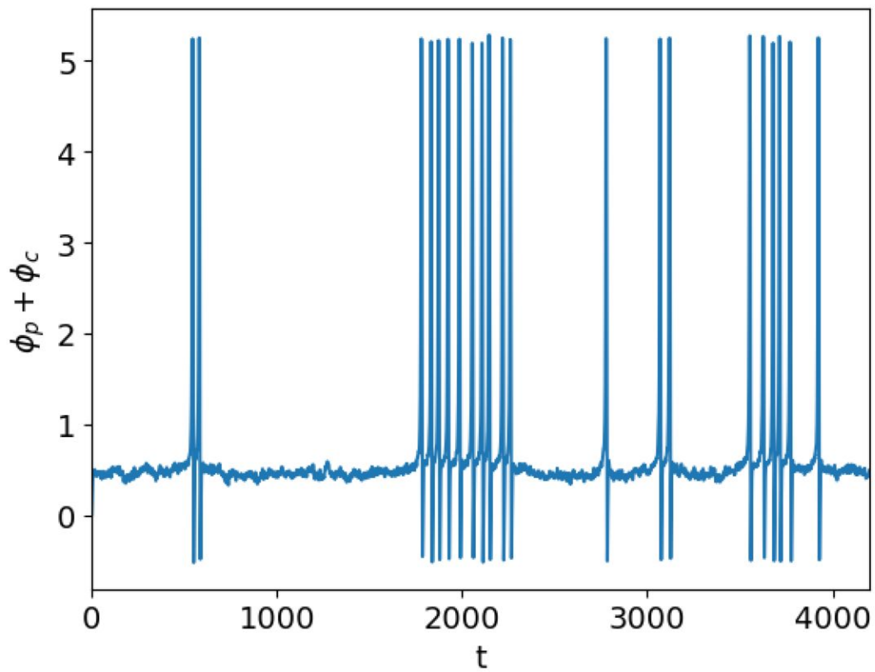
$$i_{in} + \xi(t)$$

Gaussian white noise with zero mean and autocorrelation function:

$$\langle \xi(t)\xi(\tau) \rangle = \sigma^2 \delta(t-\tau)$$

D. Chalkiadakis & J. Hizanidis, PRE (2022)

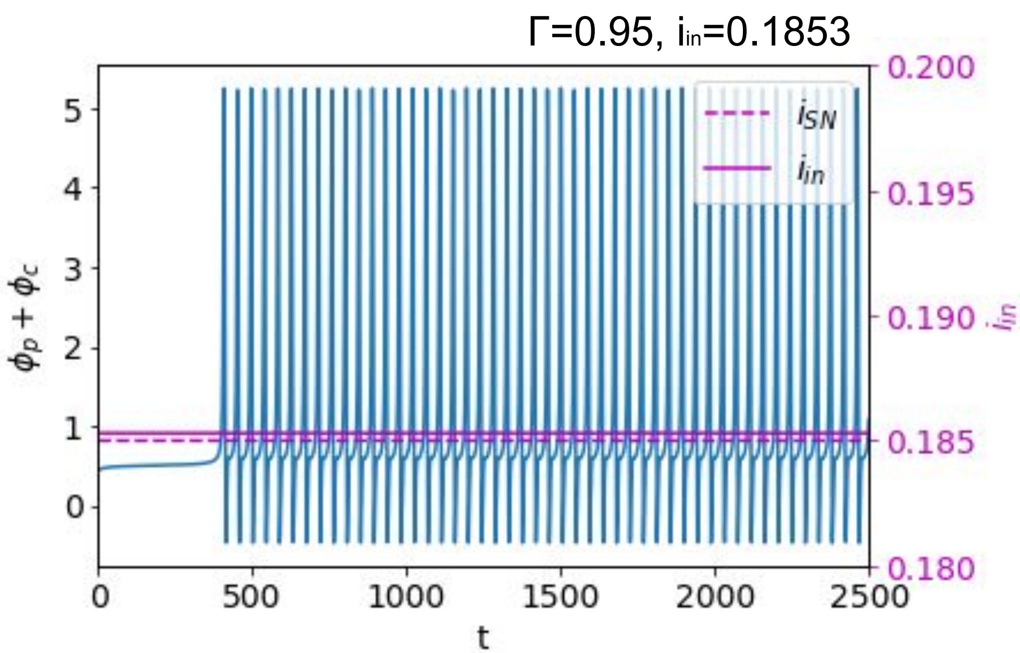
Γ=0.95, i_{in}=0.182, σ=0.06

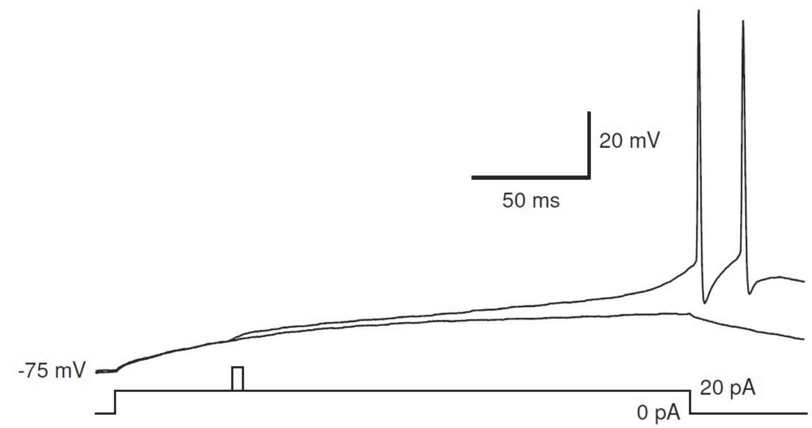


Neurocomputation properties: **Spike latency**

A barely **superthreshold** stimulation evokes action potentials with a significant **delay.**

Izhikevich, Computational Neuroscience 2007





Long latencies of neuron recorded in vitro of rat motor cortex.

Different JJ neuron implementations

- Single RCSJ model shows simple **spiking**
- Single RCLSJ model shows **bursting** similar to slow-fast ionic mechanism in real neurons
- Resistively coupled RCSJ neurons exhibit **bursting** based on competition of excitable and oscillatory neuron
- Inductively coupled RCSJ neuron (1) shows **bursting** but mechanism has not been studied

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} voltage variable

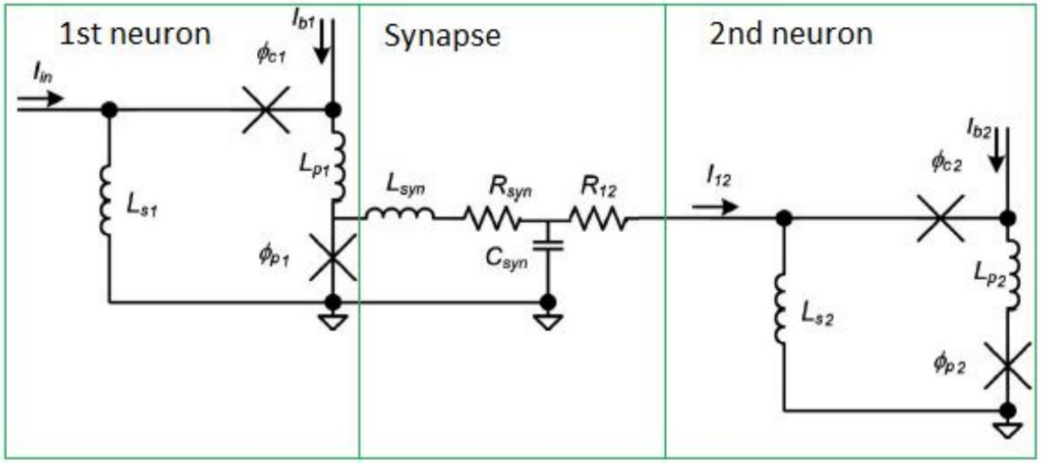
Different JJ neuron implementations

- Single RCSJ model shows **simple spiking**
- Single RCLSJ model shows **bursting** similar to slow-fast ionic mechanism in real neurons
- Resistively coupled RCSJ neurons exhibit **bursting** based on competition of excitable and oscillatory neuron
- Inductively coupled RCSJ neuron (1) shows **bursting** but mechanism has not been studied
- Inductively coupled RCSJ neuron (2) mimics the exact neuron-like spiking, bistability, chaos, noise-induced bursting and spike latency

voltage variable

phase difference

Implementation of a synapse



$$\ddot{\phi}_{p1} + \Gamma\dot{\phi}_{p1} + sin(\phi_{p1}) = i_{p1} = -\lambda(\phi_{c1} + \phi_{p1}) + \Lambda_{s1} i_{in1} + (1 - \Lambda_{p1}) i_{b1} - i_{12} - \lambda \frac{v_{out}}{\Lambda_{syn}\omega_0^2}$$

$$\eta\left[\ddot{\phi}_{c1} + \Gamma\dot{\phi}_{c1} + sin(\phi_{c1})\right] = i_{c1} = -\lambda(\phi_{c1} + \phi_{p1}) + \Lambda_{s1} i_{in1} - \Lambda_{p1} i_{b1}$$

$$\ddot{\phi}_{p2} + \Gamma\dot{\phi}_{p2} + sin(\phi_{p2}) = i_{p2} = -\lambda(\phi_{c2} + \phi_{p2}) + \Lambda_{s2} i_{12} + (1 - \Lambda_{p2}) i_{b2}$$

$$\eta\left[\ddot{\phi}_{c2} + \Gamma\dot{\phi}_{c2} + sin(\phi_{c2})\right] = i_{c2} = -\lambda(\phi_{c2} + \phi_{p2}) + \Lambda_{s2} i_{12} - \Lambda_p i_{b2}$$

$$\frac{1}{\Omega_0^2}\ddot{v}_{out} + \frac{Q}{\Omega_0}\dot{v}_{out} + v_{out} = v_p - \frac{Q\Omega_0\Lambda_{syn}}{\lambda} i_{12} - \frac{\Lambda_{syn}}{\lambda}\dot{i}_{12}$$

$$\frac{\Lambda_{syn}(1-\Lambda_{syn})}{\lambda}\dot{i}_{12} + \frac{r_{12}}{\Gamma} i_{12} = v_{out} - \Lambda_{syn}(\dot{\phi}_{c2} + \dot{\phi}_{p2})$$

The output is taken across the capacitor and sent through a resistor to the input of a postsynaptic neuron.

If the bias current applied to the JJ neuron is positive (negative) with respect to ground, then the synapse is excitatory (inhibitory).

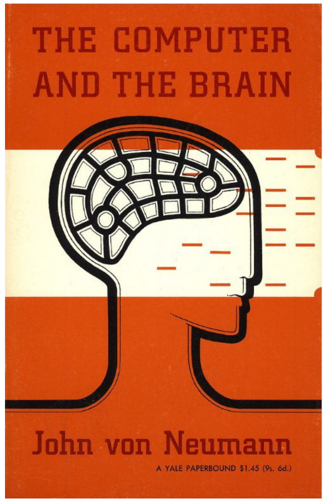
- Desynchronization
- In-phase & anti-phase synchronization

Segall et al, "Synchronization dynamics on the picosecond time scale in coupled Josephson junction neurons", PRE (2017).

"SuperMind: a survey of the potential of superconducting electronics for neuromorphic computing", Schneider et al, Supercond. Sci. Technol. (2022).

Brains have been inspiring computers for decades

John von Neumann



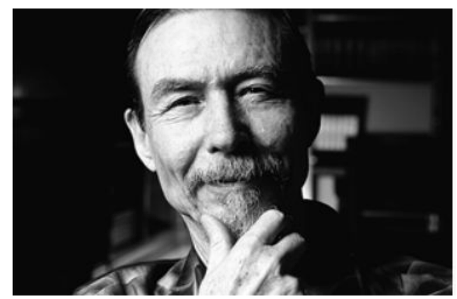
Alan Turing

Neuromorphic Electronic Systems

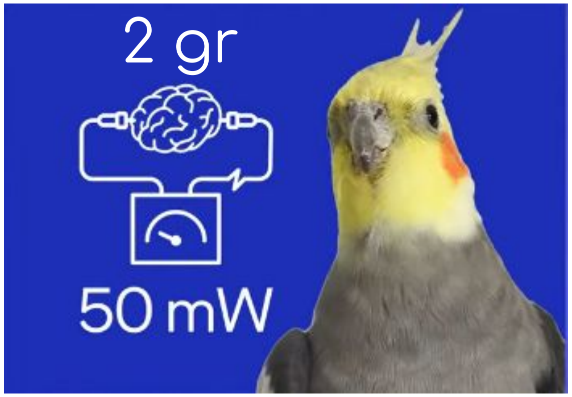
CARVER MEAD

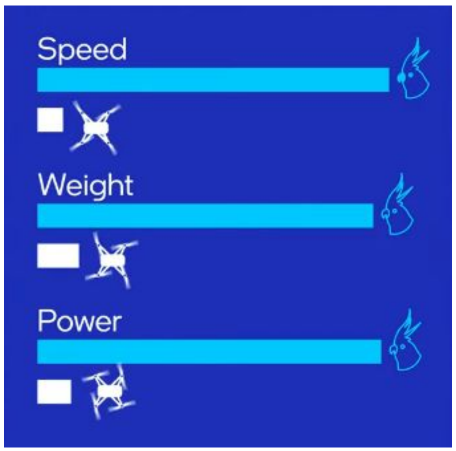
Invited Paper

PROCEEDINGS OF THE IEEE, VOL. 78, NO. 10, OCTOBER 1990



Motivation behind Neuromorphic Computing



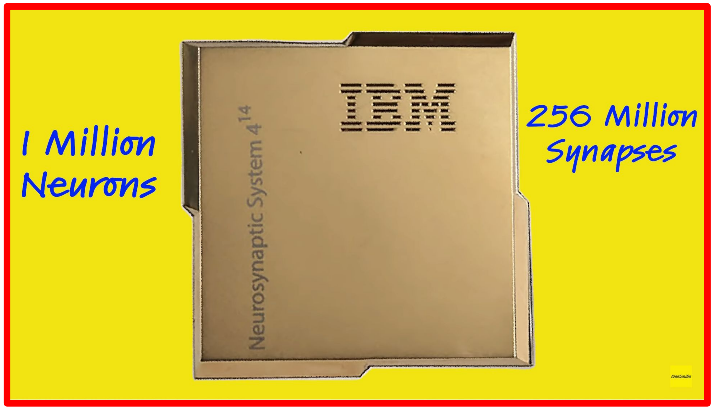


The parrot’s brain far outperforms today’s state-of-the-art computer architectures (in speed, weight, power) by orders of magnitude.

Architecture All Access: Neuromorphic Computing (Intel Technology)

Brain	vs	Conventional Computer
co-location of memory & processing		memory and computing are separate (“von Neumann bottleneck”)
neurons (~10^11) & synapses (~10^15) ~10.000 synapses per neuron		transistor area and wiring resources are behind many orders of magnitude
stochasticity of cells, low energy		**high-precisions reliable circuits**
plasticity due to adaptive synapses		limited reconfigurability
operation speeds: ms, kHz		**ns, GHz very fast**

Existing neuromorphic machines



TrueNorth Chip
(IBM)

Machine learning applications
(image recognition)

Loihi 2 (Intel)

1 million neurons per chip
Applications: robotic skin with a sense of touch
National University of Singapore (NUS)

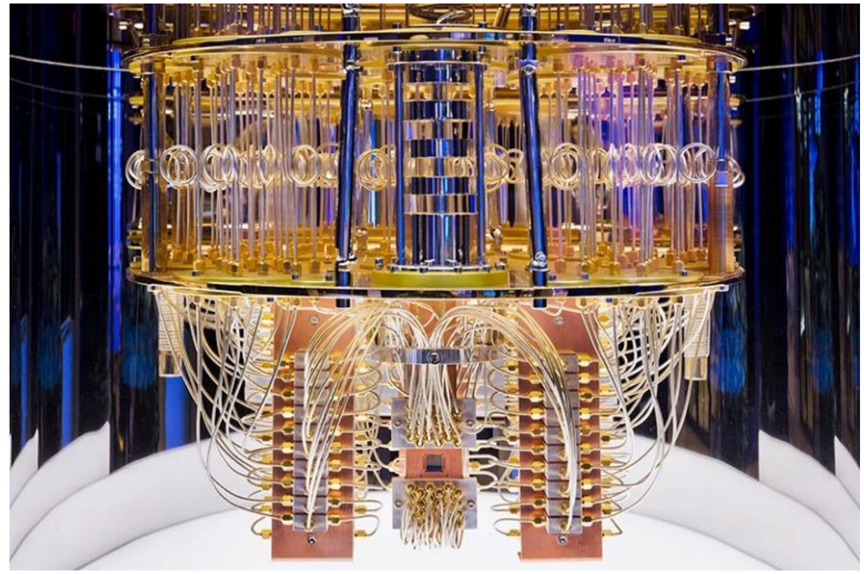
Different neuromorphic systems: comparison

Technology	CMOS synapses and neurons	Resistive switching synapses with CMOS neurons	Photonic synapses and neurons	Spintronic synapses and neurons	Superconductive synapses and neurons
Connections	Wires	Wires	Light	Microwaves	Wires or microwaves
Minimum lateral size of neuron	10 µm	10 µm	100 µm	10 nm	20 nm
Minimum lateral size of synapse	10 µm	10 nm	1 µm	10 nm	20 nm
Advantages	Commercially available	Nanoscale synapse, technology ready	Wavelength multiplexing, can be completely passive (low energy consumption[150,151])	Nanoscale synapses and neurons, almost commercial technology	Low energy consumption beside cryogenic requirements, all identical spikes
Disadvantages	Size of neurons and synapses, no in-memory computing	Size of neurons, complex wiring	Size of neurons and synapses, dissipation required for nonlinearity	Scalability yet to be demonstrated	Scalability yet to be demonstrated
Chip capabilities	Inference and learning	Inference	No chip	No chip	No chip

Josephson Junctions

Danijela Marković et al. "Physics for neuromorphic computing", Nature Reviews (2020)

High Performance Computing applications



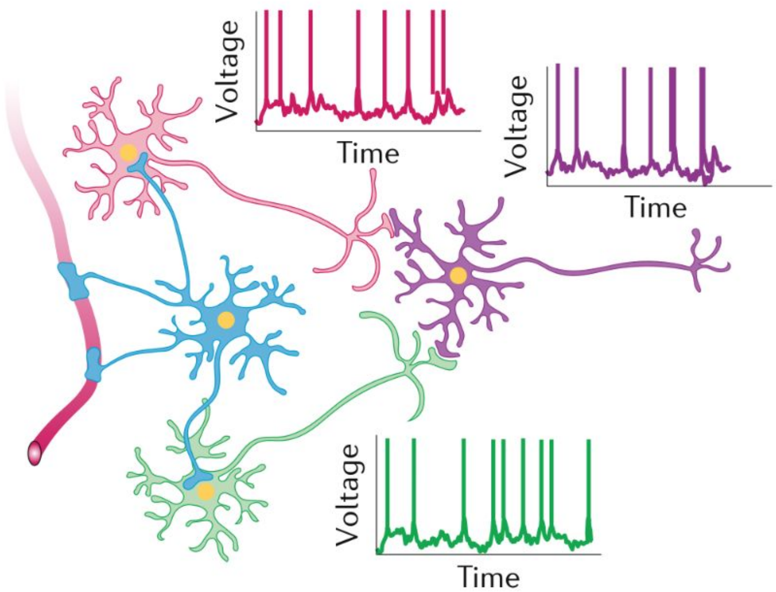
IBM quantum computer

Once you are working cryogenically already for Quantum Computing, why not build extra neuromorphic devices using **superconductors**?

Neuromorphic computing: two approaches

1. **Map AI algorithms to physical systems**: Develop hardware (beyond GPUs and TPUs) that is better suited to run current neural networks (physical reservoir computing)

2. **Match neuroscience-inspired concepts to hardware and software**:



Implement neural networks that spike (SNNs), feature memory, are stochastic, can oscillate and synchronize (plasticity), are excitable, exhibit bursting, and chaos….

DYNAMICS & COMPLEXITY

Work in progress and future ideas

- Dynamical properties and synchronization of bursting patterns in coupled RCLS neurons (model 1) (with ECE AUTH student Giorgos Baxevanis)

- Study of JJ autapse known to be responsible for excitability switching

- Reservoir computing with JJ neurons/autapse (collaborator: Prof. Kathy Luedge TU Ilmenau, Germany)

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