Cannibalism in Ecological Models as a Response to Declining Availability of Prey

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Content

- Ecological Models
- Lotka-Voltera
- Competion and cannibalism
- Holling and functional response
- Our Model
- Analysis
- Future work
- References

Ecological Models

The study of the biological world around us has always been a matter of wonder and contemplation for mankind. We have historically witnessed emigration patterns and used the knowledge for hunting before even the dawn of the first settlements. This curiosity grew even more as human understanding grew even further.

Ecological Models II

As human knowledge and scientific curiosity evolved, so did our ability to describe the natural world around us . Eventually patterns and motifs slowly fell into mathematical description (like population fluctuations, migration patterns.)

Lotka – Volterra Model

The most well known and studied model in ecological systems is the Lotka Volterra Model as shown here :

$$x' = x(a - by)$$
$$y' = y(-c + dx)$$

This system always has as an equilibrium point the point $\{x \to 0, y \to 0\}$, which corresponds to the collapse of the ecosystem.

Lotka – Volterra Model II

There exists many variations or extensions to the phenomenally simple model :

- Competitive Lotka–Volterra equations
- Nicholson–Bailey model
- Mutualism and the Lotka–Volterra equation
- Rosenzweig–MacArthur
- Arditi–Ginzburg equations

Competion and cannibalism

In the real world the relations between predators and prey are usually more complicated. As mentioned previously , the species might also have some mutuality that is beneficial (parasitic ecosystem for example) or even have competition for resources (multi predator or prey models).

Competion and cannibalism II

Sometimes the competition is intraspecial and expressed in the form of opportunistic or necessary cannibalism , especially when the available prey is reduced.

Holling and functional response

In ecology, a functional response refers to the rate at which a predator takes in food as a function of the food density (availability) in a given ecosystem. This concept is closely related to the numerical response, which describes the change in the reproduction rate of a predator based on the availability of food. The first equation to describe this type of relationship was designed by Holling.

Holling and functional response II

The functional response has 3 types. The first is a linear relationship whilst the second and third one are described by the model :

$$f(R) = \frac{aR^k}{1 + ahR^k}$$

K=1 for Type II and K>1 for Type 3

The Rosenzweig–MacArthur model is a Lotka-Volterra variant model that includes Hollings functional response.

Our Model

Based on the previously discussed models as well as the issue of intraspecies predation and competition, we have designed a model where the total comparative biomass of the prey population and the predator population affect the predation patterns characterized as “cannibalistic” behaviour. The response to the changes of biomass is a major factor to the characterization of the stability of the equilibrium points.

Our Model II

- We use the parameter "relative biomass" in order to study the scarcity of available prey required in order to maintain the predator population. This term will be denoted as "**f**"
- We also use a dampening factor, which show that cannibalistic practices have a rate with which they begin and slowly stop, ensuring that the model is continuous at any point. This parameter is denoted as "**g**" and is always smaller than one.
- Finally, we introduce the intensity of cannibalistic predation as parameter, denoted with "**e**". Although no biological limits can be placed here, it can easily become obvious that the value of this parameter is important in stability of the solutions.

Our Model III

$$\boldsymbol{x' = x(a - by)}$$

$$\boldsymbol{y' = y\left(-c + dc\ - e\, e^{f - g\left(\frac{x}{y}\right)}\right)}$$

Analysis

Equilibrium points :

I. As mentioned in the Lotka-Volterra part of the talk, one of the equilibrium points which we can always find is $\{x \to 0, y \to 0\}$.

II. The second equilibrium point is $\{y \to \frac{a}{b}, x \to constant\}$. This point is difficult to characterize its stability. We have used the Routh Hurwitz Criterion and determined that all points are stable but do not have stability of the same type.

Analysis II

The "constant" value of x is

$$\frac{b\left(\frac{ga}{b}\right)\left(\frac{c}{d}\right)+ProductLog(\left(\frac{e}{d}\right)\left(\frac{ga}{b}\right)e^{(\left(f-\frac{gac}{db}\right))})}{ga}$$

Analysis III

The corresponding eigenvalues are (and the ones used for the Routh-Hurwitz criterion)are:

$$l = \frac{1}{2}\left(-\frac{beg}{a}\; x\; e^{f-\frac{bg}{a}x} \pm \sqrt{z[x]}\right)$$

Where :

$$z[x] = -4\left(ad + bege^{f-\frac{bg}{a}x}\right)x + \left(\frac{beg}{a}e^{f-\frac{bg}{a}x}\; x\right)^2$$

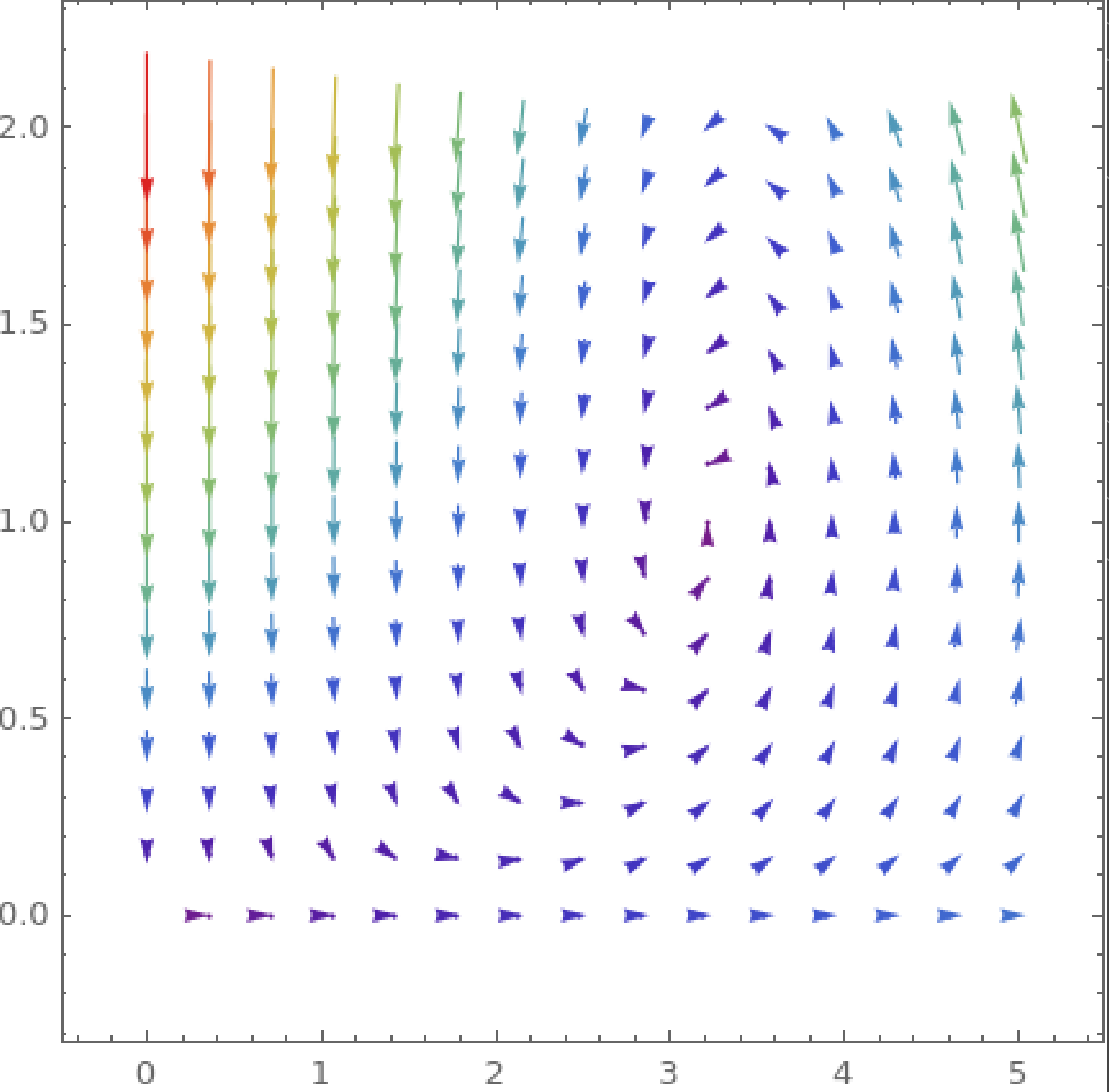
Analysis IV

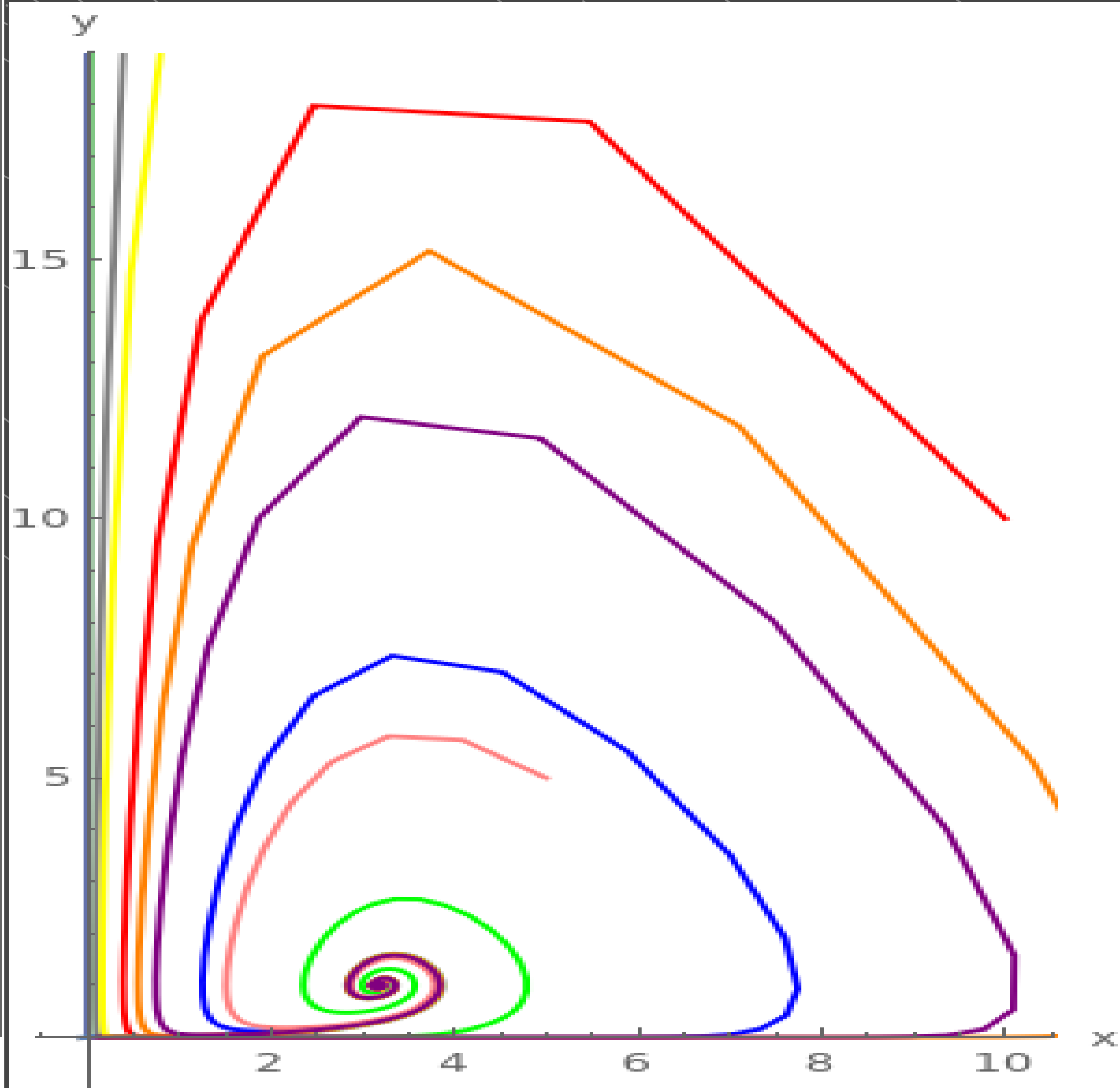
- How and when is x 0?
- One possible differentiation for the system appear if we make the c parameter negative. This would lead to an autotrophic system.
- Some other conditionals on the parameters lead to variants of the classic Lotka-Volterra model, linearly dislocated

Analysis V

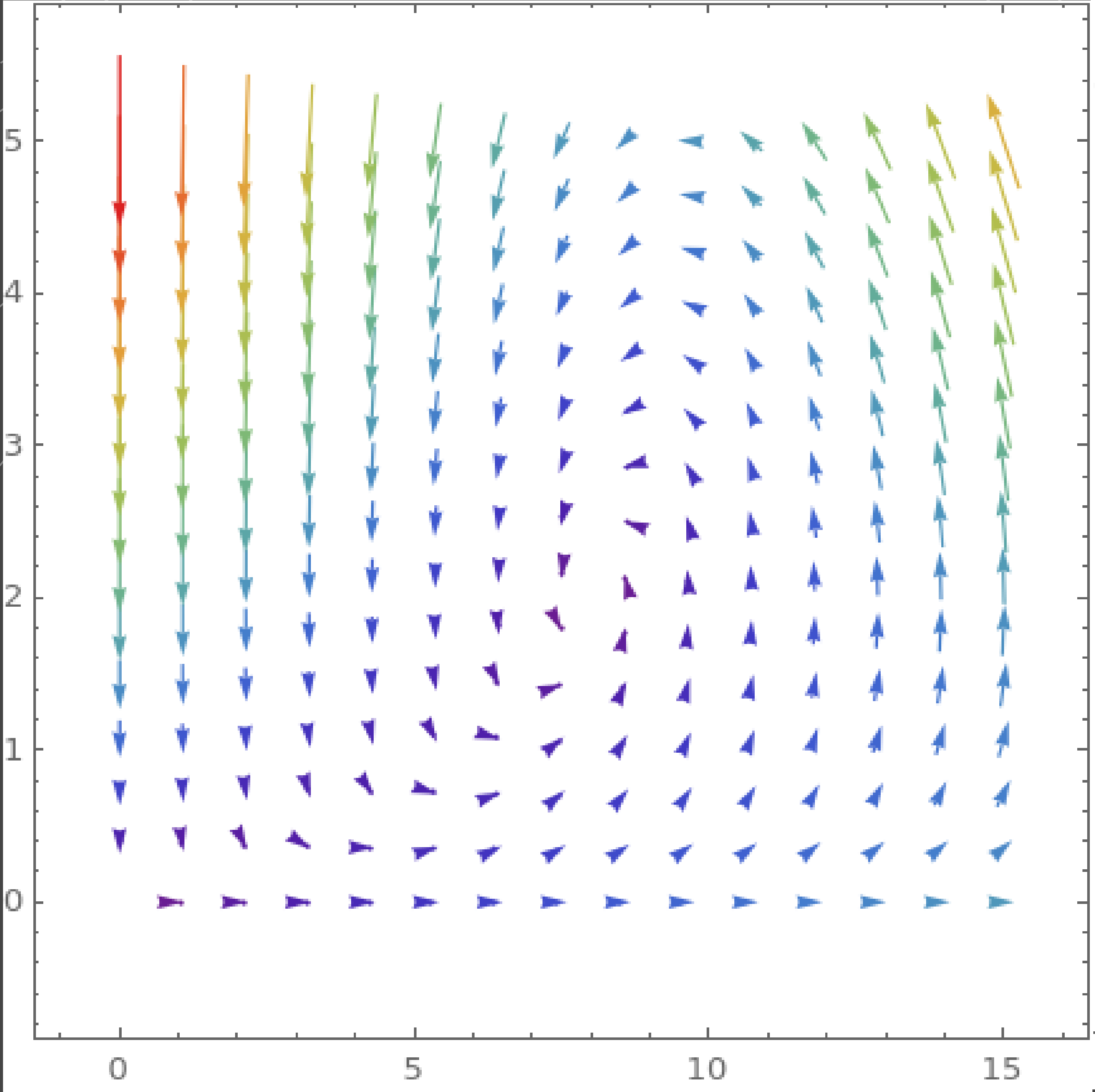
We Have explored 4 different sets of parameters, for 10 trajectories each.

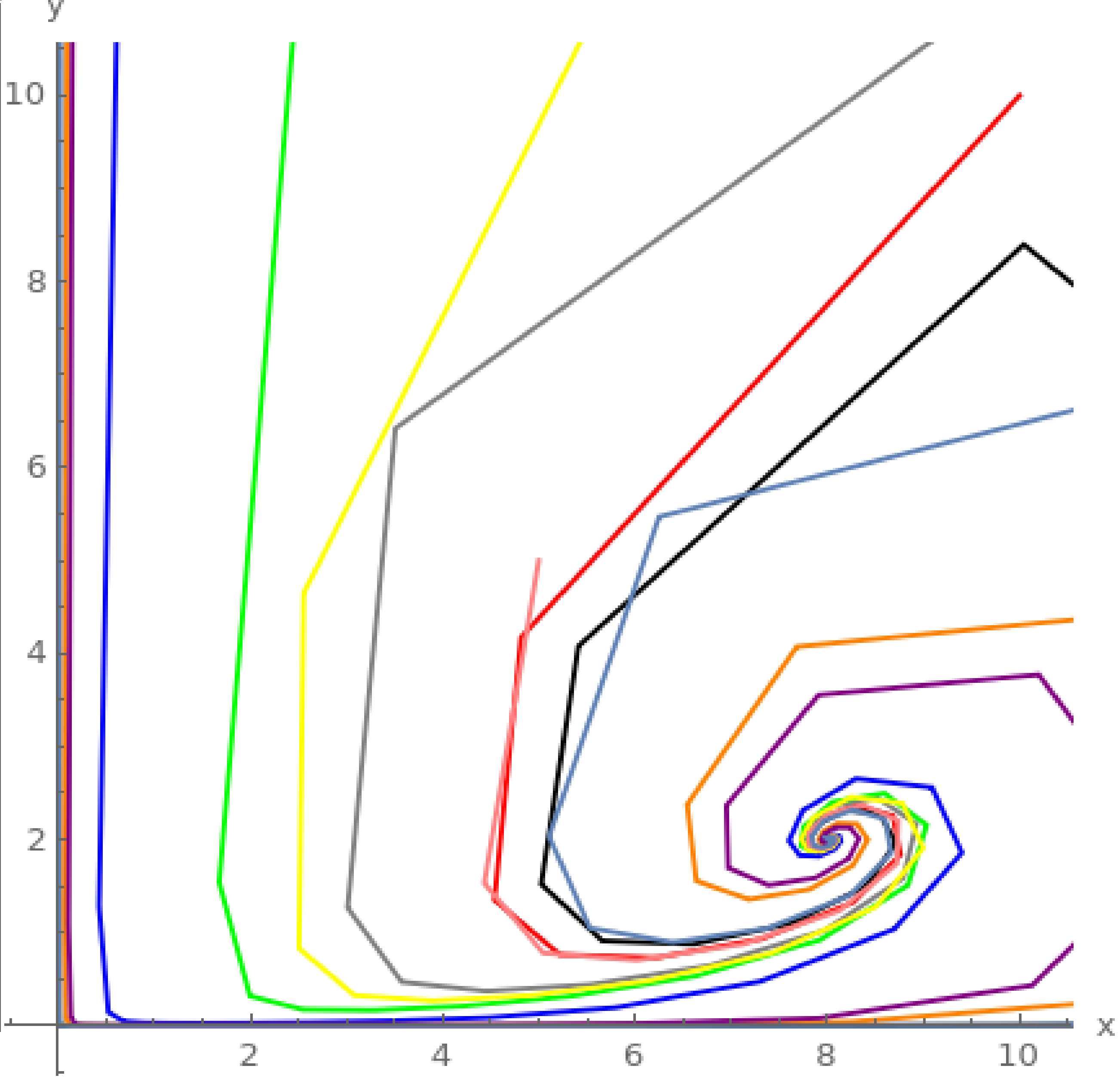
	A	B	C	D
a	*1*	*6*	*2*	*20*
b	*1*	*3*	*3*	*15*
c	*1*	*7*	*4*	*15*
d	*3*	*5*	*5*	*25*
e	*½*	*1/3*	*½*	*1*
f	*3*	*5*	*7*	*10*
g	*1/20*	*1/10*	*1/20*	*1/50*
x	*2,59988*	*8.62425*	*21.9461*	*128.604*
y	*1*	*2*	*2/3*	*4/3*
Ein1	*-0.982537 +0.85489 i*	*-6.64433 + 16.6226 i*	*-170,74*	*-6142.63*
Ein2	*-0.982537 - 0.85489 i*	*-6.64433 - 16.6226 i*	*-3,32458*	*-30,5677*



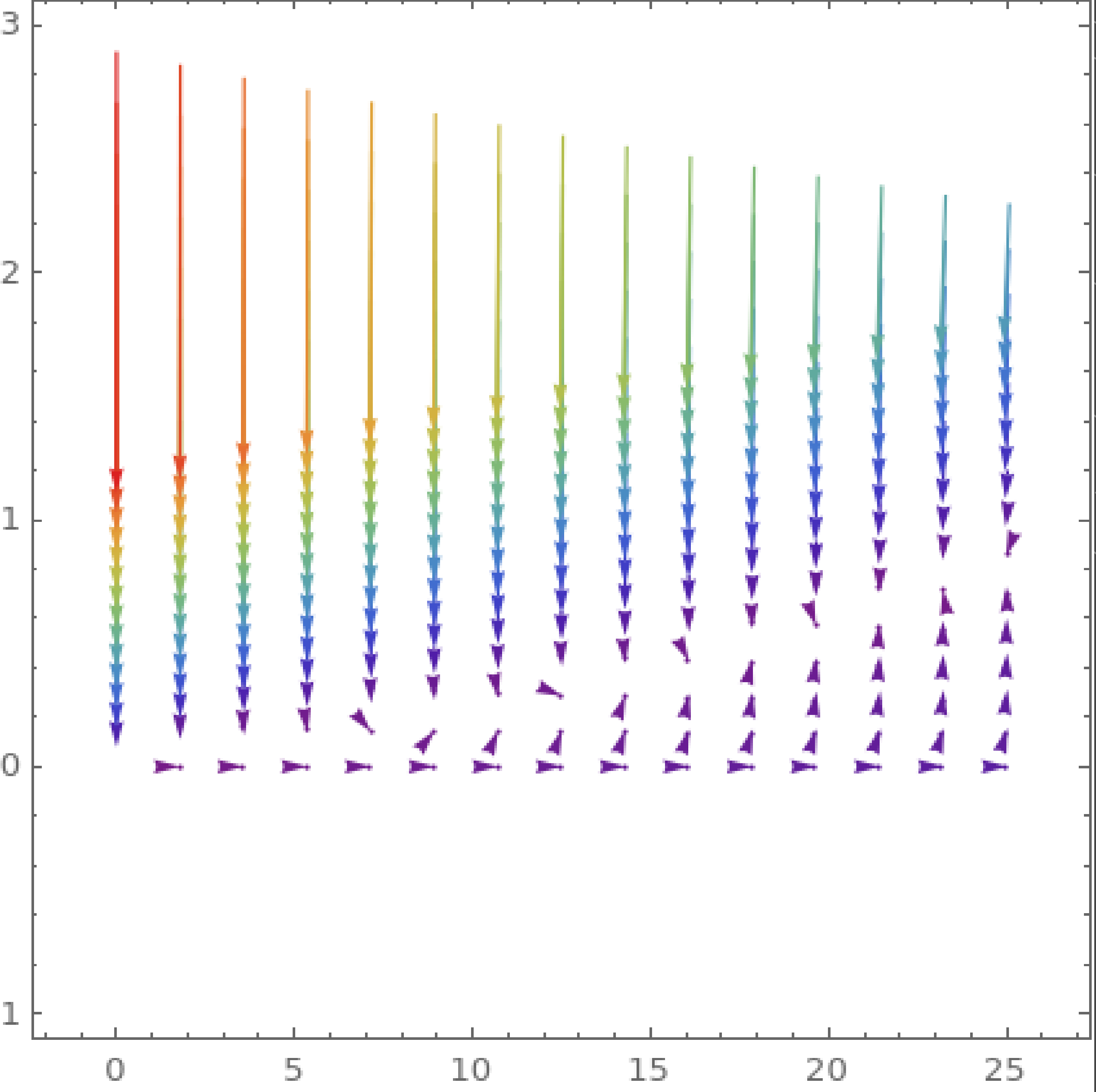


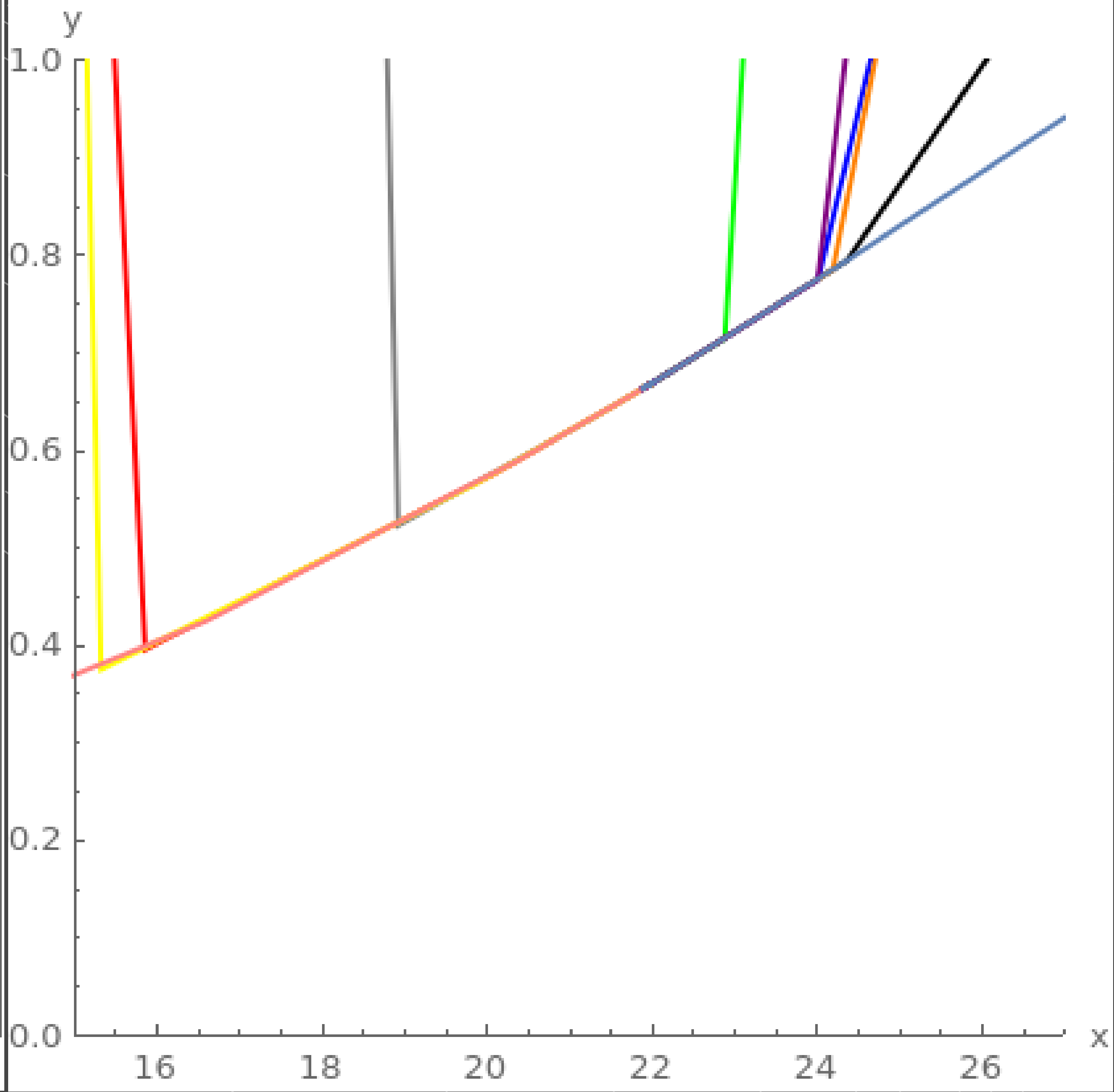
Analysis (Model A)



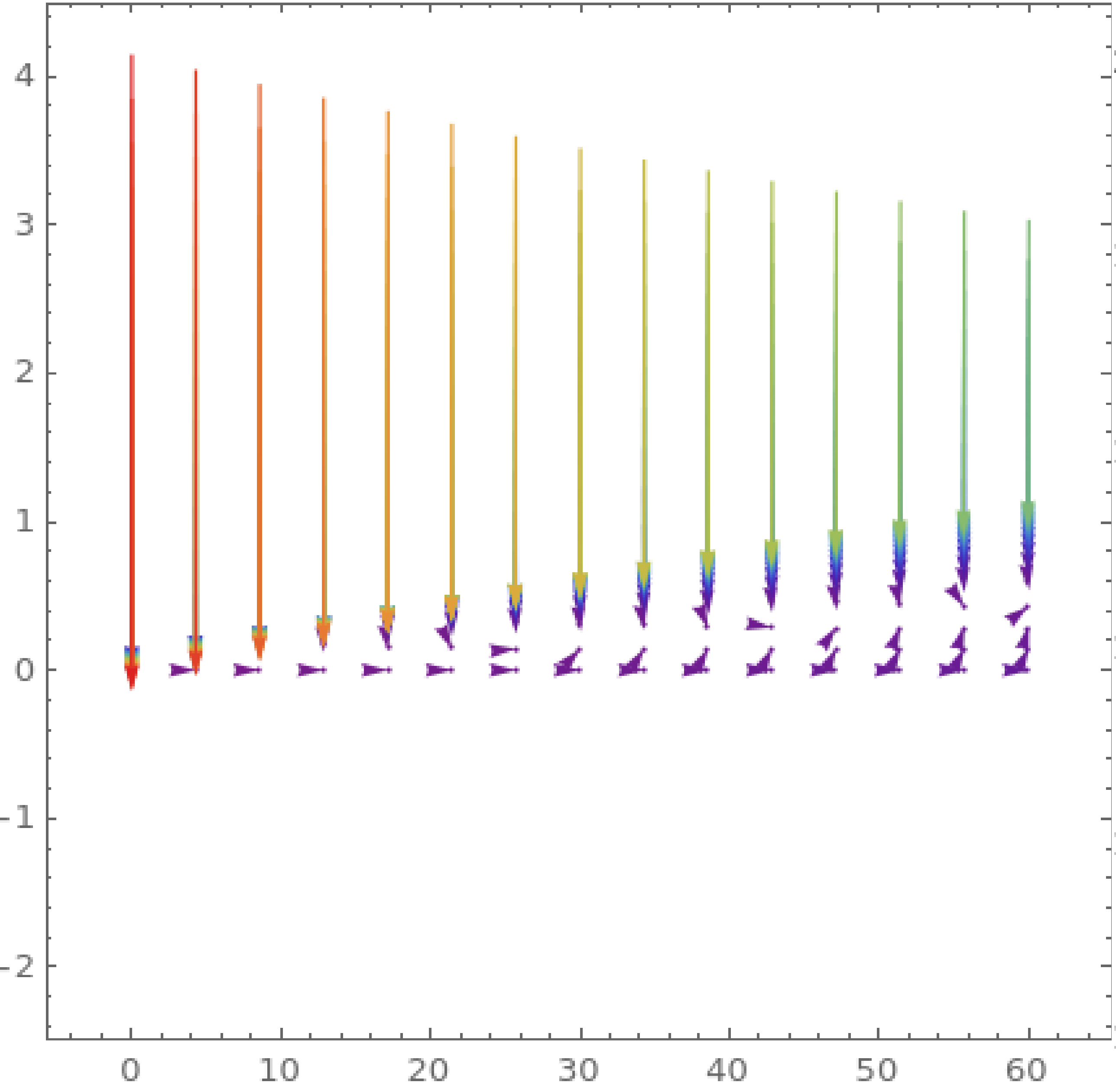


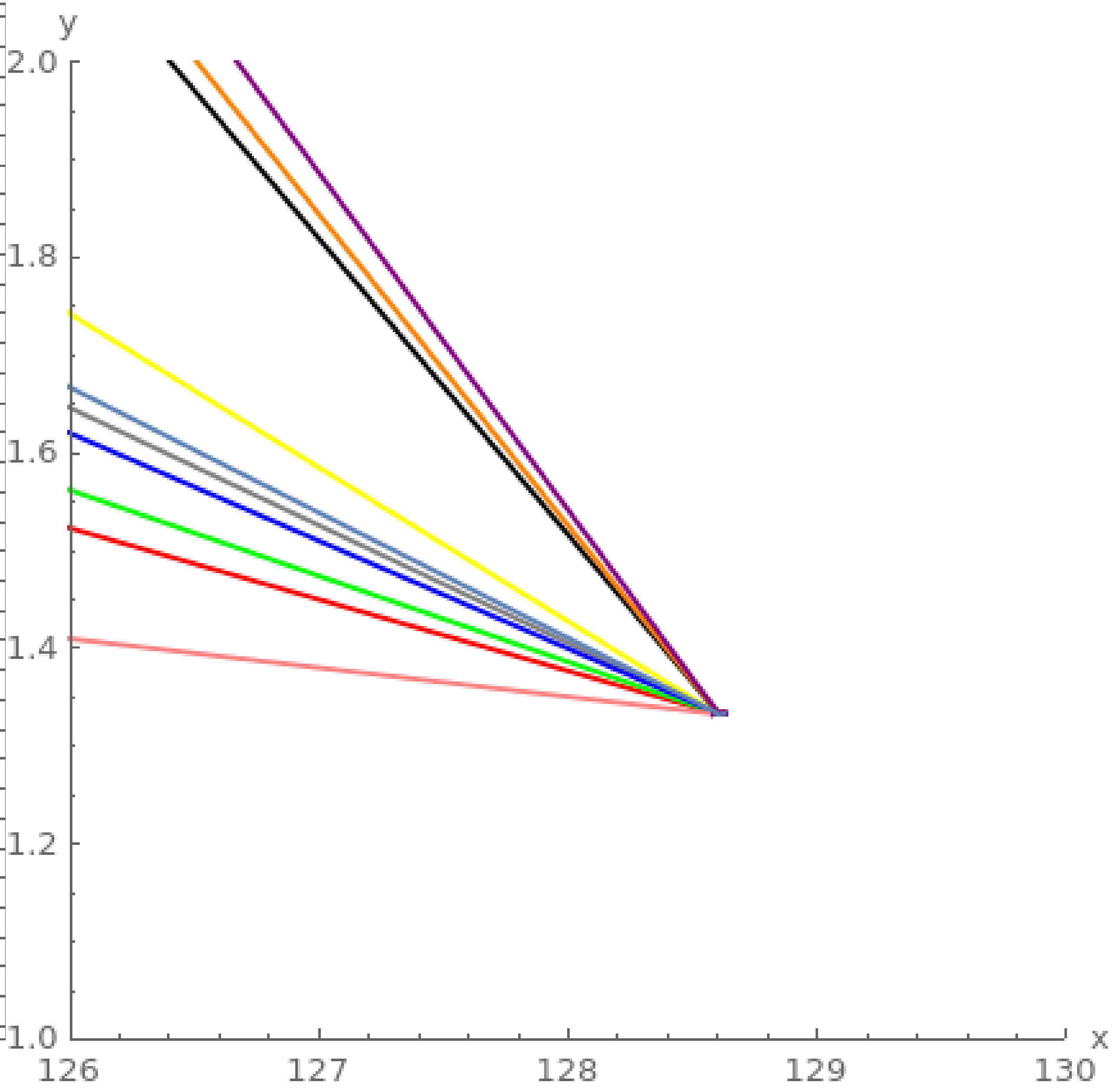
Analysis (Model B)





Analysis (Model C)





Analysis (Model D)

Future works

Our team is also working on a similar non autonomous model, where seasonal scarcity affects the system instead of the relative biomass. It will be presented at the **"XXIInd HyperComplex Seminar ,Lodz, Poland"** later this month.

We would also like to combine the models and study the solutions that we can find, their natural explanation or search for any observation that can confirm it, as well as include other forms of cannibalism (larval/many stage predators, intraspecies predations, opportunistic etc).

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Thank you for your attention